

The Tricept Robot: Dynamics and Impedance Control

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Abstract

The Tricept is a novel industrial robot characterized by a hybrid kinematic design featuring a 3-degree-of-freedom (3-DOF) structure of parallel type and a 3-DOF spherical wrist. In this work the authors focus on the derivation of a dynamic model to be used for both simulation and control purposes. Two different approaches are discussed and compared in terms of inverse dynamics computation. Then, a model-based control is derived aimed at enforcing a 6-DOF impedance behaviour at the end-effector to manage interaction with the environment. Simulation results are presented to evaluate the accuracy of an approximate dynamic model computation as well as to test the effectiveness of the proposed impedance control strategy.

I. INTRODUCTION

Parallel robots are constituted by a fixed base and a mobile base, connected by a number of independent kinematic chains [9]. They have lately been receiving quite a deal of attention not only in the academic community but also in the industrial community; their main advantage regards the possibility to obtain high structural stiffness and perform high-speed motions. Two recent examples of parallel structures for industrial robots are the Tricept robot designed by Neumann [10] and the Flexpicker robot based on the Delta design by Clavel [5].

In this work the class of parallel robots having the kinematic structure of the Tricept robot is considered (Fig. 1). This class is characterized by having a radial link of variable length connected to the mobile base. A recent work has analyzed the kinematics and kineto-statics of a Tricept structure [13]. On the other hand, for analysis and control purposes it is of the utmost importance to compute inverse and direct dynamics based on suitable mathematical models.

Previous research efforts aimed at obtaining dynamic models for parallel robots can be found in, e.g., [6], where a Newton-Euler formulation is adopted and [8],[11], where a Lagrangian formulation is adopted. The resulting inverse dynamics is computed numerically in view of the difficulties concerned with the derivation of a complete symbolic solution. On the other hand, the availability of a symbolic, yet approximate, dynamic model is crucial for computation of direct dynamics and ultimately for parameter identification and model-based dynamic control purposes [12].

In this work two different approaches to dynamic model derivation are pursued; namely, one based on the application of the virtual work principle leading to an exact solution, and another based on the Lagrangian formulation under simplifying assumptions leading to an approximate solution. Simulations for the inverse dynamics on a test trajectory have been carried out and the results are discussed in terms of model accuracy.

As a further contribution of the present work, the simplified symbolic model is adopted to develop a model-based impedance control strategy to manage the interaction of the robot with the environment. The impedance concept was proposed in [7] in order to avoid large values of the contact forces and moments due to the robot/environment interaction. This is achieved by enforcing a compliant behaviour of the end effector with suitable dynamic features, characterized by a desired inertia, damping and stiffness. As shown in [2], the typical six-DOF impedance control based on Euler angles may give rise to task geometric inconsistency due to the dependence of the rotational stiffness upon the actual orientation; such a drawback can be overcome by defining a spatial impedance based on the unit quaternion.

Then, the spatial impedance concept is applied to the class of Tricept parallel robots. The control

scheme is designed by resorting to an inner position/orientation loop, ensuring good robustness to disturbances and modelling approximations, and an outer loop, which confers the desired impedance behaviour to the system. A case study for a Tricept structure in contact with a planar surface has been simulated in order to show the effectiveness of the proposed approach.

II. KINEMATICS

In this Section the kinematic model of the Tricept robot is briefly reviewed. More details can be found in [13].

The Tricept robot is composed by a 3-DOF structure of parallel type and a 3-DOF spherical wrist. A sketch of the parallel kinematic structure is illustrated in Fig. 2. This consists of three identical outer links with actuated prismatic joints q_1, q_2, q_3 . Each outer link is composed by a cylinder and a piston. The fixed base (equilateral triangle $A_1A_2A_3$) is part of the supporting structure of the manipulator. Each link is connected to the fixed base by means of a Cardan joint allowing the link to rotate about two axes that are both orthogonal to the link. On the other end, the links are connected to the mobile base (equilateral triangle $B_1B_2B_3$) by three spherical joints. Besides the above three outer links, a radial link is present: This is connected to the fixed base by a 3-DOF joint allowing the link to rotate by the angles α, β about two mutually orthogonal axes (one of which is orthogonal itself to the link), and to translate by r along a through-hole axis; the connection of the radial link to the mobile base is fixed and orthogonal so as to avoid axial rotations and determine the reference point for the attachment of the spherical wrist.

It is easy to recognize that the parallel structure has 3 DOF's which can be effectively described by the vector of spherical coordinates $\mathbf{x}_p = [r \ \alpha \ \beta]^T$.

The above coordinates, together with the vector of the three wrist joint angles $\mathbf{x}_o = [q_4 \ q_5 \ q_6]^T$, constitute the 6 DOF's of the robot that can be collected into the vector of task variables $\mathbf{x} = [\mathbf{x}_p^T \ \mathbf{x}_o^T]^T$. Hence, the kinematic model of the manipulator can be seen as that of an equivalent open-chain spherical arm, characterized by the three joint coordinates r, α, β , with a spherical wrist, characterized by the three joint coordinates q_4, q_5, q_6 (i.e., a Stanford manipulator).

A Jacobian matrix $\mathbf{J}$ relates the task velocity vector $\dot{\mathbf{x}}$ to the joint velocities $\dot{q}_i$ ($i = 1, \dots, 6$) collected into the vector $\dot{\mathbf{q}}$:

$$\dot{\mathbf{q}} = \begin{bmatrix} \mathbf{J}_p^{-1}(\mathbf{x}_p) & \mathbf{O}_3 \\ \mathbf{O}_3 & \mathbf{I}_3 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}}_p \\ \dot{\mathbf{x}}_o \end{bmatrix} = \mathbf{J}^{-1}(\mathbf{x})\dot{\mathbf{x}}, \quad (1)$$

where the Jacobian $\mathbf{J}_p$ can be computed through the differential kinematics of the structure [13].

Let $\mathbf{p}_e$ be the position of a frame attached to the end effector, and $\mathbf{R}_e$ the rotation matrix expressing the orientation of the frame. Then, the differential kinematic model is completed by expressing the

relationship between $\dot{\mathbf{x}}$ and the linear and angular velocity of the end-effector frame $\mathbf{v}_e = [\dot{\mathbf{p}}_e^T \ \dot{\boldsymbol{\omega}}_e^T]^T$, i.e.

$$\mathbf{v}_e = \mathbf{J}_e(\mathbf{x})\dot{\mathbf{x}} \quad (2)$$

where $\mathbf{J}_e$ coincides with the geometric Jacobian of the equivalent open-chain manipulator defined above, which can be computed via standard techniques [12].

III. DYNAMICS

In this Section an exact dynamic model is derived based on the application of the virtual work principle [15]. Then, an approximate dynamic model is determined based on the Lagrangian formulation under suitable simplifying assumptions. The models are derived exclusively for the 3-DOF parallel structure, since a standard Lagrangian or Newton-Euler approach can be pursued to compute the remaining part of the model.

A. Exact Model

The proper link velocities and accelerations can be computed from the kinematic model: The angular velocity and acceleration of the radial link $(\boldsymbol{\omega}_m, \dot{\boldsymbol{\omega}}_m)$, the acceleration of the center of mass of the radial link $(\ddot{\mathbf{p}}_R)$, the angular velocity and acceleration of the i -th cylinder $(\boldsymbol{\omega}_i, \dot{\boldsymbol{\omega}}_i)$, the acceleration of its center of mass $(\ddot{\mathbf{p}}_{C_i})$ and the acceleration of the center of mass of the i -th piston $(\ddot{\mathbf{p}}_{P_i})$. Details on the calculation of these quantities can be found in [4].

On the assumption of frictionless contact between rigid bodies and time-invariant holonomic constraints, the principle of virtual works yields

$$dW' + dW'' = 0, \quad (3)$$

where dW' is the elementary work of the actuating forces and dW'' is the elementary work of the inertial and gravitational forces, with null external forces and moments.

The first term in (3) can be computed as

$$dW' = d\mathbf{q}_p^T \boldsymbol{\tau}_p = d\mathbf{x}_p^T \mathbf{J}_p^{-T} \boldsymbol{\tau}_p \quad (4)$$

where $\boldsymbol{\tau}_p$ is the vector of joint actuating forces and the Jacobian in (1) has been used to express the elementary displacement in terms of task space variables.

By accounting for the contributions of the radial link and of the piston-cylinder pairs of the three outer links, the second term in (3) can be computed as

$$dW'' = d\mathbf{x}_p^T \boldsymbol{\gamma}_p \quad (5)$$

where

$$\gamma_p = \mathbf{J}_{R,p}^T \mathbf{f}_R + \mathbf{J}_{R,o}^T \boldsymbol{\mu}_R + \sum_{i=1}^3 \left(\mathbf{J}_{C_i,p}^T \mathbf{f}_{C_i} + \mathbf{J}_{C_i,o}^T \boldsymbol{\mu}_{C_i} + \mathbf{J}_{P_i,p}^T \mathbf{f}_{P_i} + \mathbf{J}_{P_i,o}^T \boldsymbol{\mu}_{P_i} \right). \quad (6)$$

In the above equation, $\mathbf{f}_R$ ($\mathbf{f}_{C_i}$ or $\mathbf{f}_{P_i}$) and $\boldsymbol{\mu}_R$ ($\boldsymbol{\mu}_{C_i}$ or $\boldsymbol{\mu}_{P_i}$) denote the vectors of inertial and gravitational force and moment exerted on the radial link (the i -th cylinder or the piston). Suitably defined Jacobians ($\mathbf{J}_{R,p}$, $\mathbf{J}_{R,o}$, $\mathbf{J}_{C_i,p}$, $\mathbf{J}_{C_i,o}$, $\mathbf{J}_{P_i,p}$ and $\mathbf{J}_{P_i,o}$) for the position and orientation part are used to express the work in terms of the task space elementary displacement. The expression of the Jacobians can be found in [4].

Combining (4) with (5) yields the equations of motion in the form

$$\boldsymbol{\tau}_p = \mathbf{J}_p^T \gamma_p \quad (7)$$

with γ_p as in (6).

The force and moment contributions in (6) can be computed as:

$$\mathbf{f}_R = -m_R(\ddot{\mathbf{p}}_R - \mathbf{g}_0) \quad (8)$$

$$\boldsymbol{\mu}_R = -\mathbf{I}_R \dot{\boldsymbol{\omega}}_m - \boldsymbol{\omega}_m \times \mathbf{I}_R \boldsymbol{\omega}_m - (r - \ell_R) \mathbf{f}_R \times \mathbf{z}_m \quad (9)$$

$$\mathbf{f}_{C_i} = -m_C(\ddot{\mathbf{p}}_{C_i} - \mathbf{g}_0) \quad (10)$$

$$\boldsymbol{\mu}_{C_i} = -\mathbf{I}_C \dot{\boldsymbol{\omega}}_i - \boldsymbol{\omega}_i \times \mathbf{I}_C \boldsymbol{\omega}_i + \ell_C \mathbf{f}_{C_i} \times \mathbf{z}_i \quad (11)$$

$$\mathbf{f}_{P_i} = -m_P(\ddot{\mathbf{p}}_{P_i} - \mathbf{g}_0) \quad (12)$$

$$\boldsymbol{\mu}_{P_i} = -\mathbf{I}_P \dot{\boldsymbol{\omega}}_i - \boldsymbol{\omega}_i \times \mathbf{I}_P \boldsymbol{\omega}_i - (q_i - \ell_P) \mathbf{f}_{P_i} \times \mathbf{z}_i \quad (13)$$

where m_R (m_C or m_P) is the mass of the radial link (the cylinder or the piston), $\mathbf{I}_R$ ($\mathbf{I}_C$ or $\mathbf{I}_P$) is the inertia tensor of the radial link (the cylinder or the piston) and $\mathbf{g}_0$ denotes the vector of gravity acceleration.

Having derived the expressions of all the terms in (7), the dynamic model of the parallel robot can be computed. However, it should be stressed that a symbolic form of the equations of motion, i.e. of the inertial, Coriolis and centrifugal, and gravitational forces, is heavy to obtain, since it would require the computation of the angles of the cardan joints in terms of $\mathbf{x}_p$. Therefore, this solution is efficient only for the computation of the inverse dynamics of the structure, i.e. given the joint positions, velocities and accelerations, compute the joint torques in a numerical fashion.

B. Approximate Model

An approximate model can be derived on the basis of the following simplifying assumptions:

- the center of mass of the cylinder of the outer link i coincides with A_i ;

- the center of mass of the piston of the outer link i coincides with B_i ;
- the rotational contribution to the kinetic energy of the outer links is neglected.

In such a case, it is convenient to refer directly to a Lagrangian formulation. Choosing $\mathbf{x}_p$ as the vector of generalized coordinates, the equations of motion are

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{x}}_p} \right)^T - \left(\frac{\partial \mathcal{L}}{\partial \mathbf{x}_p} \right)^T = \boldsymbol{\gamma}_p, \quad (14)$$

where the Lagrangian function $\mathcal{L} = \mathcal{T} - \mathcal{U}$ is the difference between the kinetic energy $\mathcal{T}$ and the potential energy $\mathcal{U}$ of the system, and $\boldsymbol{\gamma}_p$ is the vector of the generalized forces in (6).

The kinetic energy can be computed by adding the contributions of the radial and outer links in the form

$$\mathcal{T} = \frac{1}{2} \mathbf{x}_p^T \mathbf{B}_p(\mathbf{x}_p) \mathbf{x}_p \quad (15)$$

where

$$\mathbf{B}_p = m_R \hat{\mathbf{J}}_{R,p}^T \hat{\mathbf{J}}_{R,p} + \mathbf{J}_{R,o}^T \mathbf{I}_R \mathbf{J}_{R,o} + m_P \sum_{i=1}^3 \mathbf{J}_i^T \mathbf{J}_i \quad (16)$$

is the positive definite symmetric inertia matrix, and $\hat{\mathbf{J}}_{R,p}$ is the Jacobian that can be obtained from $\mathbf{J}_{R,p}$ by replacing the quantity r with $(r - \ell_R)$. Similarly, the potential energy can be computed as

$$\mathcal{U} = -\mathbf{g}_0^T \left(m_R \mathbf{p}_R + m_P \sum_{i=1}^3 \mathbf{p}_i \right). \quad (17)$$

Folding (15) and (17) into (14) and accounting for (6), yields the dynamic model in symbolic form

$$\boldsymbol{\tau}_p = \mathbf{J}_p^T(\mathbf{x}_p) \left(\mathbf{B}_p(\mathbf{x}_p) \ddot{\mathbf{x}}_p + \mathbf{C}_p(\mathbf{x}_p, \dot{\mathbf{x}}_p) \dot{\mathbf{x}}_p + \mathbf{g}_p(\mathbf{x}_p) \right). \quad (18)$$

The inertia matrix turns out to be diagonal. The matrix $\mathbf{C}_p$, which characterizes the Coriolis and centrifugal forces, can be computed from the elements of the inertia matrix using the Christoffel symbols of the first type [12]. The vector $\mathbf{g}_p$ represents the effect of gravity force.

Finally, the complete approximate dynamic model of the Tricept robot, including the spherical wrist, can be written in the form

$$\mathbf{B}(\mathbf{x}) \ddot{\mathbf{x}} + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}}) \dot{\mathbf{x}} + \mathbf{g}(\mathbf{x}) = \boldsymbol{\gamma} - \mathbf{J}_e^T(\mathbf{x}) \mathbf{h} \quad (19)$$

with obvious meaning of the symbols $\mathbf{B}$, $\mathbf{C}\dot{\mathbf{x}}$, $\mathbf{g}$, while $\mathbf{h}$ is the vector of end-effector contact forces and $\mathbf{J}_e$ is given in (2). Further in (19), $\boldsymbol{\gamma}$ is the vector of generalized forces related to the joint actuating forces via the kinetostatic mapping $\boldsymbol{\tau} = \mathbf{J}^T \boldsymbol{\gamma}$, where $\mathbf{J}$ is the Jacobian matrix in (1).

C. Comparison

The inverse dynamics for the Tricept robot is computed on a test trajectory for the task variables interpolating the points $\mathbf{x}_{p1} = [0.8 \ 0 \ 0]^T$, $\mathbf{x}_{p2} = [1.1 \ -\pi/6 \ 0]^T$, $\mathbf{x}_{p3} = [1.1 \ 0 \ \pi/6]^T$ and $\mathbf{x}_{p4} = \mathbf{x}_{p1}$ with equal time intervals. CAD estimates of the dynamic parameters have been used. The simulation results are summarized in Tab. I where the average and the maximum percent error on the joint torques is listed for a different duration of the time interval. In particular, the first entry in the table corresponds to achieving nearly the maximum velocity for the prismatic joints of the robot. It can be recognized that the impact of the simplifying assumptions on model accuracy decreases as the motion becomes slower, with a sort of saturation effect when the contribution of the inertial forces becomes negligible with respect to that of the gravitational forces.

IV. IMPEDANCE CONTROL

Because of the heavy computational burden, the exact numerical model is not suitable for control purposes. Hence, in the following the approximate symbolic model is adopted in the design of an impedance control.

With reference to (19), according to the well-known inverse dynamics strategy, the vector of generalized forces can be chosen as [12]

$$\boldsymbol{\gamma} = \mathbf{B}(\mathbf{x})\mathbf{y} + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{g}(\mathbf{x}) + \mathbf{J}_e^T(\mathbf{x})\mathbf{h} \quad (20)$$

where $\mathbf{y}$ is a resolved acceleration given by $\mathbf{y} = \mathbf{J}_e^{-1}(\mathbf{x}) \left(\mathbf{a} - \dot{\mathbf{J}}_e(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} \right)$, with $\mathbf{a} = [\mathbf{a}_p^T \ \mathbf{a}_o^T]^T$. Folding (20) in (19), taking into account the expression of $\mathbf{y}$, $\mathbf{a}$ and the time derivative of (2), yields

$$\ddot{\mathbf{p}}_e = \mathbf{a}_p \quad (21)$$

$$\dot{\boldsymbol{\omega}}_e = \mathbf{a}_o. \quad (22)$$

In order to track a position trajectory for a reference frame c , $\mathbf{a}_p$ can be selected as

$$\mathbf{a}_p = \ddot{\mathbf{p}}_c + \mathbf{K}_{Dp}\Delta\dot{\mathbf{p}}_{ce} + \mathbf{K}_{Pp}\Delta\mathbf{p}_{ce} \quad (23)$$

where $\mathbf{K}_{Dp}$, $\mathbf{K}_{Pp}$ are suitable positive definite matrix gains and $\Delta\mathbf{p}_{ce} = \mathbf{p}_c - \mathbf{p}_e$ is the position error between the reference and the end-effector frame. On the other hand, tracking of an orientation trajectory for the reference frame c is achieved by choosing $\mathbf{a}_o$ as [1]

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_c + \mathbf{K}_{Do}\Delta\boldsymbol{\omega}_{ce} + \mathbf{K}_{Po}\mathbf{R}_e^e \boldsymbol{\epsilon}_{ce} \quad (24)$$

where $\mathbf{K}_{Do}$, $\mathbf{K}_{Po}$ are suitable positive definite matrix gains, $\mathbf{R}_e$ is the rotation matrix of the end-effector frame and ${}^e\boldsymbol{\epsilon}_{ce}$ is the vector part of the unit quaternion expressing the mutual orientation

between the reference and the end-effector frame when referred to the base frame. The interested reader is referred to [3] for details on the use of the unit quaternion for robot control.

Typically, a position and orientation trajectory for a desired frame d is specified in terms of $\mathbf{p}_d$ and $\mathbf{R}_d$. In the case of end-effector contact with the environment, it is worth introducing a mechanical impedance between the desired and the reference frame so as to keep limited the values of the interaction force and moment $\mathbf{h} = [\mathbf{f}^T \quad \boldsymbol{\mu}^T]^T$. Hence, the commanded motion for the reference frame c can be computed via the impedance equations so as to achieve a suitable compliant behaviour for the end-effector [14].

In this fashion, the motion control loop is actually separated from the impedance control loop. In particular, the motion control loop can be purposefully made stiff so as enhance rejection to disturbance and modelling inaccuracies but, rather than ensuring tracking of the desired end-effector position and orientation, it shall ensure tracking of a reference position and orientation resulting from the impedance control action. In other words, the desired position and orientation together with the measured contact force and moment are input to the impedance equation which, via a suitable integration, generates the position and orientation to be used as a reference for the motion control action.

As far as the reference frame is concerned, the position $\mathbf{p}_c$ can be computed via the translational impedance equation

$$\mathbf{M}_p \Delta \ddot{\mathbf{p}}_{dc} + \mathbf{D}_p \Delta \dot{\mathbf{p}}_{dc} + \mathbf{K}_p \Delta \mathbf{p}_{dc} = \mathbf{f}, \quad (25)$$

where $\Delta \mathbf{p}_{dc} = \mathbf{p}_d - \mathbf{p}_c$, and $\mathbf{M}_p$, $\mathbf{D}_p$ and $\mathbf{K}_p$ are positive definite matrices representing the mass, damping, and stiffness characterizing the impedance.

In the classical 6-DOF approaches, the orientation of the reference frame $\mathbf{R}_c$ is computed via an impedance equation formally equal to (25), where a minimal representation of the end-effector orientation (i.e., in terms of three Euler angles) is used. However, as shown in [2], the rotational stiffness may become not well defined even for small orientation displacements and cannot be specified in a consistent way with the task geometry. This problem can be solved by resorting to a class of geometrically meaningful representations of the mutual orientation between the desired frame and the reference frame. In detail, a geometrically consistent impedance equation for the rotational part is [2]

$$\mathbf{M}_o \Delta {}^c \dot{\boldsymbol{\omega}}_{dc} + \mathbf{D}_o \Delta {}^c \boldsymbol{\omega}_{dc} + \mathbf{K}'_o {}^c \boldsymbol{\epsilon}_{dc} = {}^c \boldsymbol{\mu}, \quad (26)$$

where ${}^c \boldsymbol{\epsilon}_{dc}$ is the vector part of the unit quaternion that can be extracted from ${}^c \mathbf{R}_d = \mathbf{R}_c^T \mathbf{R}_d$. The rotational stiffness matrix in (26) is $\mathbf{K}'_o = 2\mathbf{E}^T(\eta_{dc}, {}^c \boldsymbol{\epsilon}_{dc}) \mathbf{K}_o$, with $\mathbf{E}(\eta_{dc}, {}^c \boldsymbol{\epsilon}_{dc}) = \eta_{dc} \mathbf{I} - \mathbf{S}({}^c \boldsymbol{\epsilon}_{dc})$, where $\mathbf{S}(\cdot)$ the matrix operator performing the cross product between two (3×1) vectors.

A. Case Study

A case study aimed at testing the performance of the proposed impedance control for the Tricept robot has been developed. To the purpose, the MATLAB/Simulink environment has been used. The kinematic and dynamic parameters of the structure have been computed via CAD estimates on the basis of the links geometry. An end effector tool has been considered, composed by a rigid stick ending with a circular disk. The end-effector frame has its origin at the center of the disk and its approach axis normal to the disk surface and pointing outwards.

The dynamic model of the manipulator used in the simulation is the exact numerical model; instead, the dynamic compensation terms in the inverse dynamics control law (20) are computed on the basis of the approximate symbolic model. The control is implemented digitally with a 1 ms sampling time.

In the first phase of the commanded task, the end-effector desired position is required to make a straight line motion with a position displacement $\Delta \mathbf{p}_e = [0.1 \ 0.3 \ -0.35]^T$ m along the axes of the base frame, while the end-effector frame is required to rotate of an angle $\Delta \theta_e = 0.9425$ rad about the y -axis of the base frame. Then, a vertical position displacement $\Delta \mathbf{p}_e = [0 \ 0 \ -0.27]^T$ m is commanded with constant orientation. Finally, after a 1 s lapse, the z -axis position is taken back of 0.27 m. The trajectory along the path is generated according to an interpolating spline polynomial with null initial and final velocities and accelerations, and a total duration of 7 s.

The gains of the inner loop control actions in (23),(24) have been set to $\mathbf{K}_{Pp} = 4000\mathbf{I}_3$, $\mathbf{K}_{Po} = 10000\mathbf{I}_3$, $\mathbf{K}_{Dp} = 250\mathbf{I}_3$, $\mathbf{K}_{Do} = 200\mathbf{I}_3$.

The environment is constituted by a flat surface with translational stiffness of 5000 N/m, which obstructs the end-effector path along the z -axis (Fig. 3). Hence, the disk comes in contact with the surface with unknown mutual position and orientation.

The parameters of the translational part of the impedance equation (25) have been set to $\mathbf{M}_p = 0.2\mathbf{I}_3$, $\mathbf{D}_p = 100\mathbf{I}_3$, $\mathbf{K}_p = 200\mathbf{I}_3$, while the parameters of the rotational part of the impedance equation (26) have been set to $\mathbf{M}_o = 0.2\mathbf{I}_3$, $\mathbf{D}_o = \mathbf{I}_3$, $\mathbf{K}_o = 3\mathbf{I}_3$. Notice that the stiffness matrices have been chosen so as to ensure a compliant behaviour at the end effector (limited values of contact force and moment) during the constrained motion, while the damping matrices have been chosen so as to guarantee a well-damped behaviour.

The results in Figs. 4,5 show the effectiveness of the proposed impedance controller. After the contact, the component of the position error between the end-effector frame and the desired frame $\Delta \mathbf{p}_{de} = \mathbf{p}_d - \mathbf{p}_e$ along the z -axis significantly deviates from zero, as expected, while small errors can be seen also for the components along the x - and the y -axis due to contact friction. As for the orientation error, the second component of the orientation displacement between the end-effector frame and the

desired frame ϵ_{de}^e significantly deviates from zero since the end-effector frame has to rotate after the contact in order to comply with the surface.

Also, a prevailing component of the contact force can be observed along the z -axis after the contact. As for the contact moment, a prevailing component about the y -axis is present, as expected. It can be recognized that all the above quantities keep limited values during the interaction. Hence, a compliant behaviour is successfully achieved.

V. CONCLUSION

In this paper kinematic and dynamic modelling of the Tricept parallel robot has been considered. The adoption of simplifying assumptions in deriving the dynamic model has led to a symbolic form of the equations of motion of reduced complexity. Then, a geometrically consistent six-DOF impedance has been designed to manage the interaction with the environment. Simulations have been carried out to test the effectiveness of the proposed impedance control strategy.

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TABLE CAPTION

Tab. I. Average and maximum percent error on joint torques.

FIGURE CAPTIONS

Fig. 1. The Tricept robot.

Fig. 2. Sketch of the parallel kinematic structure.

Fig. 3. The Tricept robot executing an interaction task.

Fig. 4. Position and orientation errors.

Fig. 5. Contact force and moment.

duration [s]	percent error	
	average	maximum
1.0	4.2	31.7
2.5	4.0	17.3
5.0	3.9	12.2
10.0	3.9	11.0

TABLE I

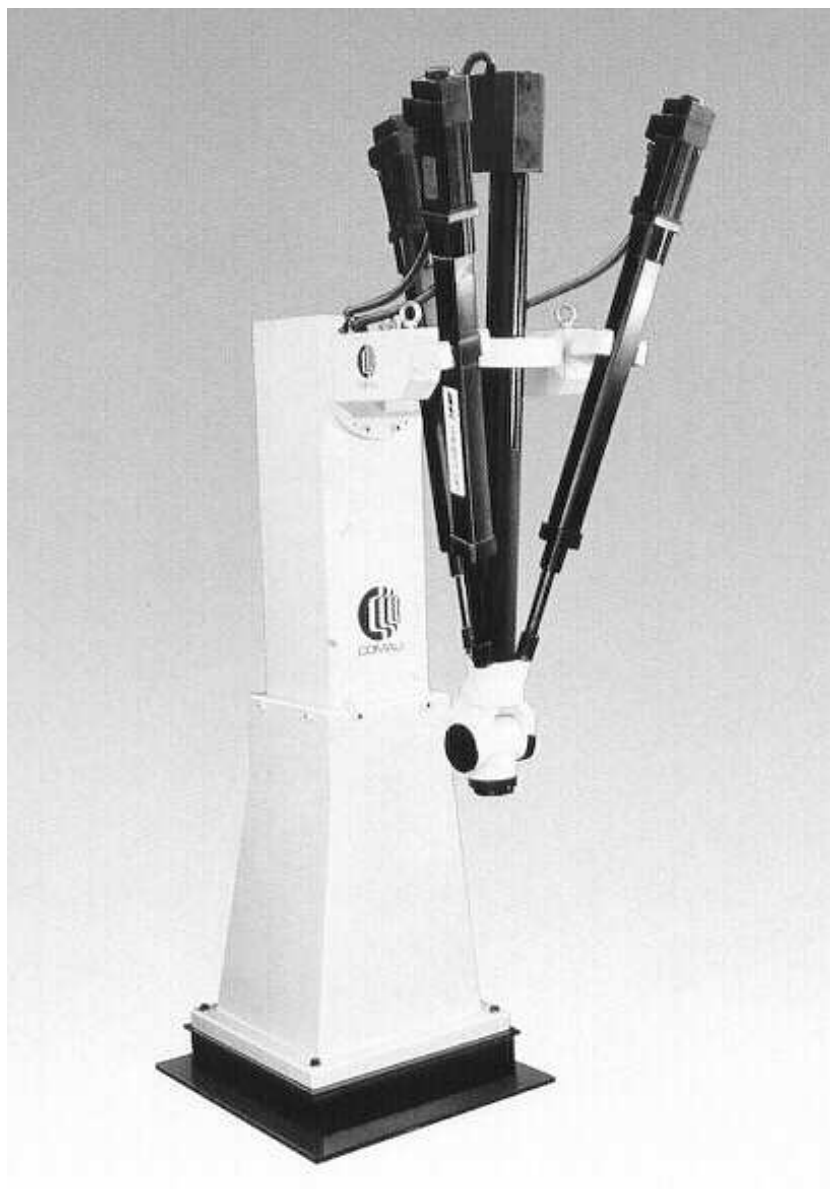


Fig. 1.

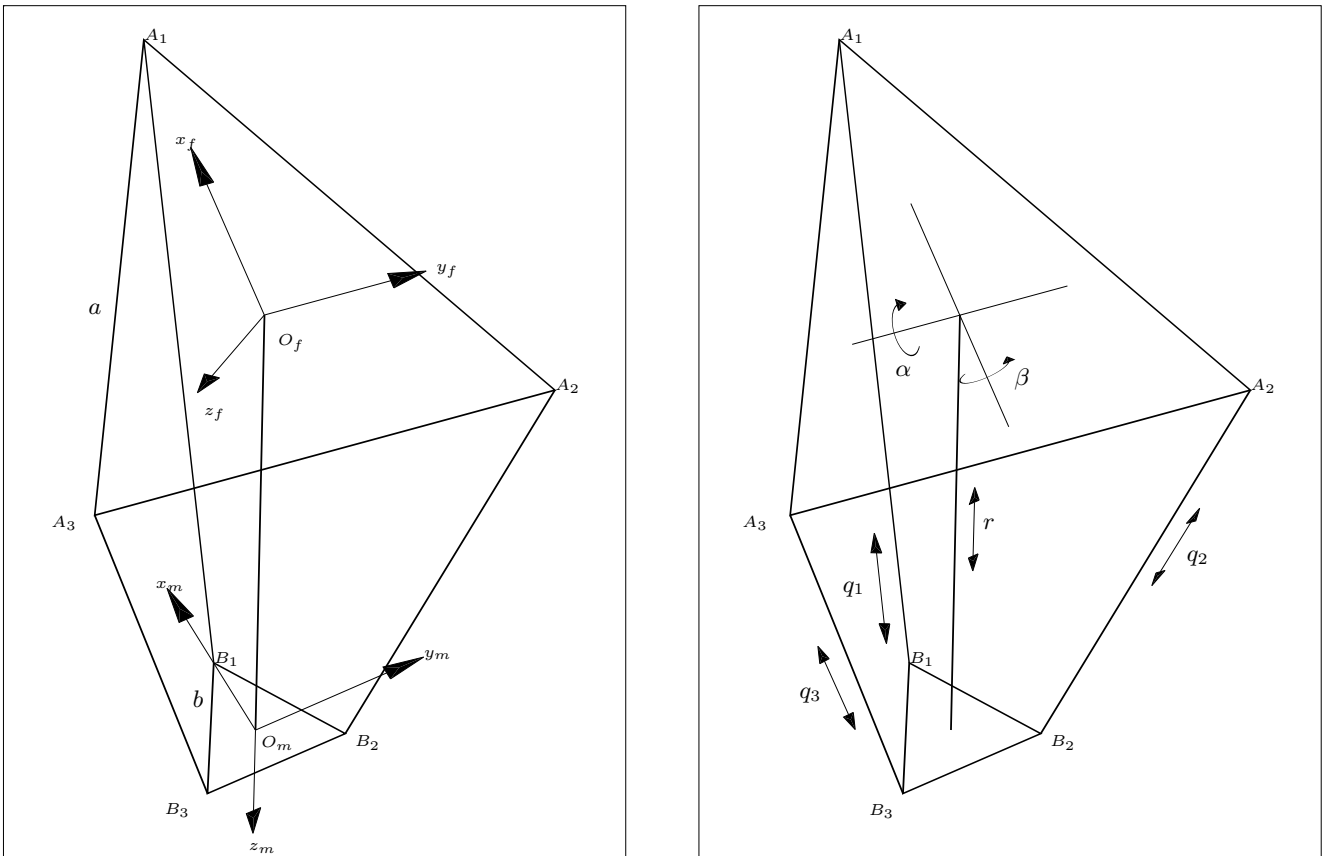


Fig. 2.

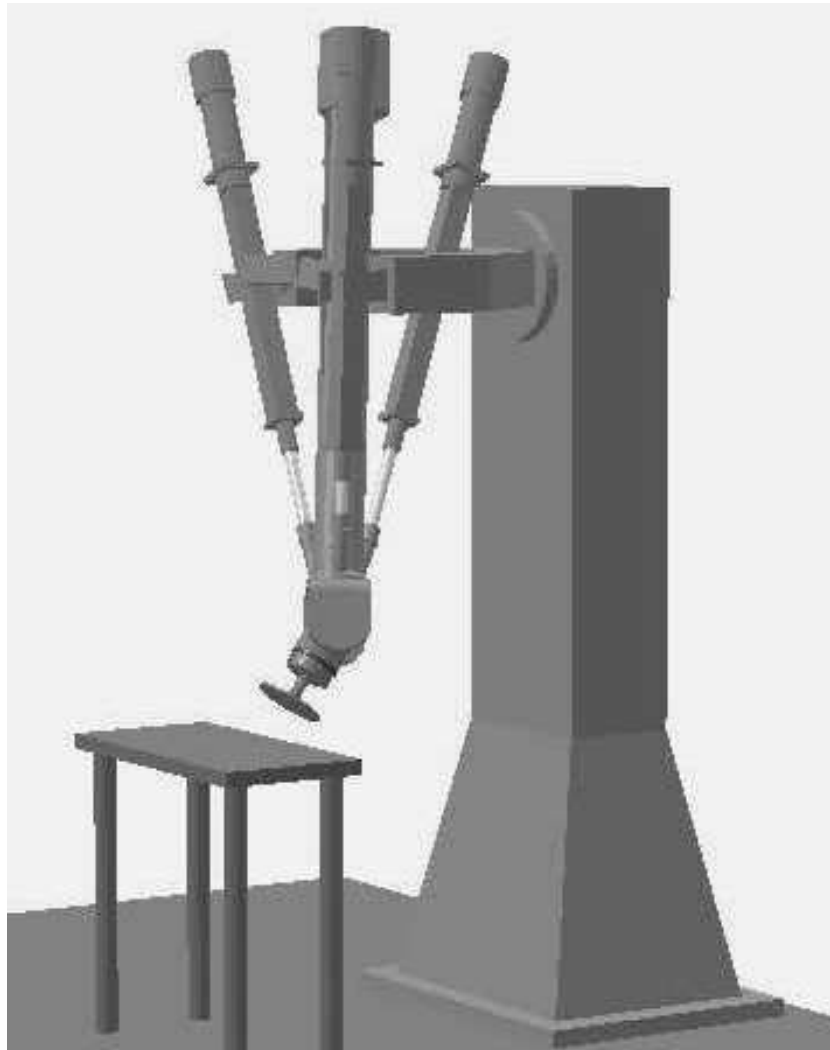


Fig. 3.

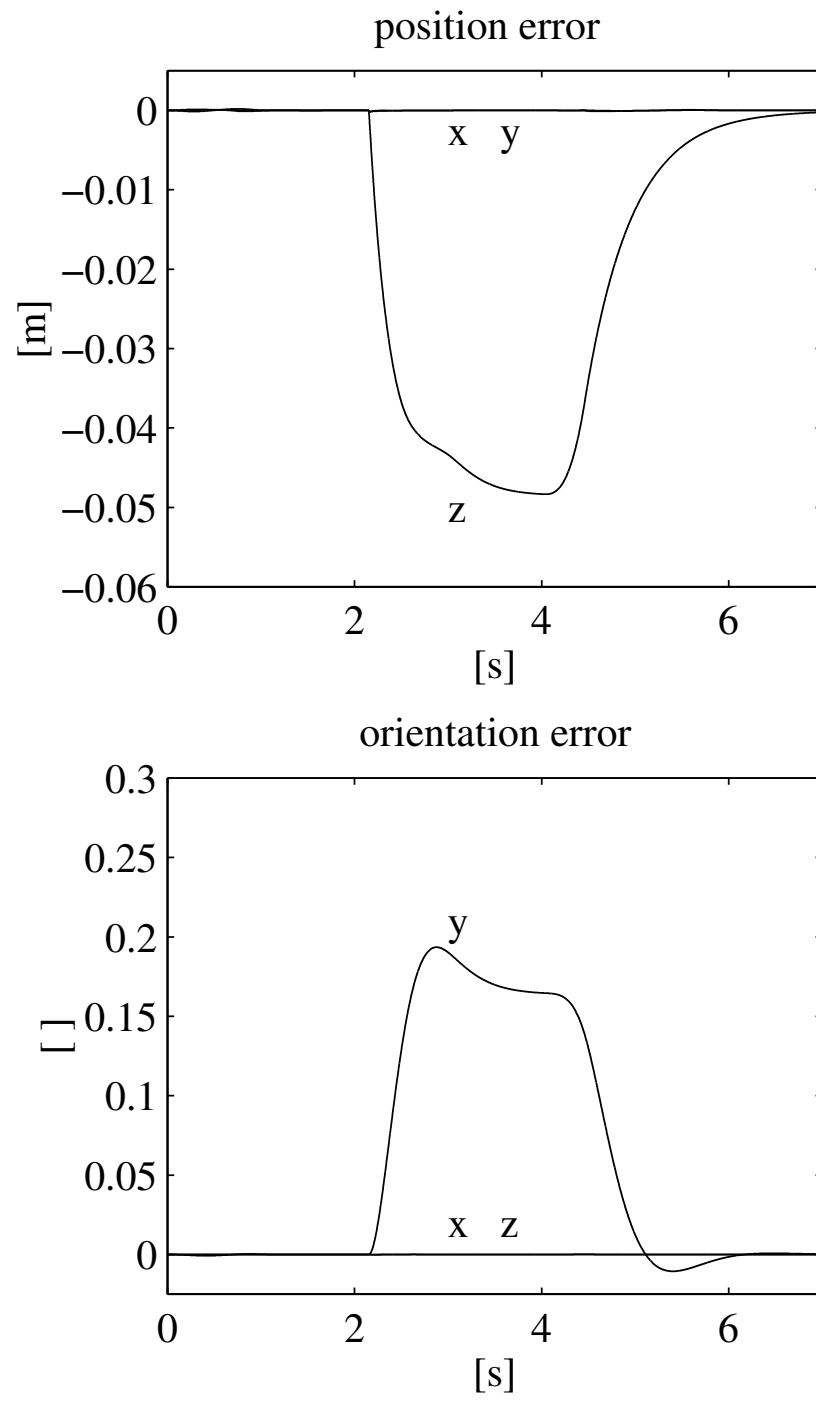


Fig. 4.

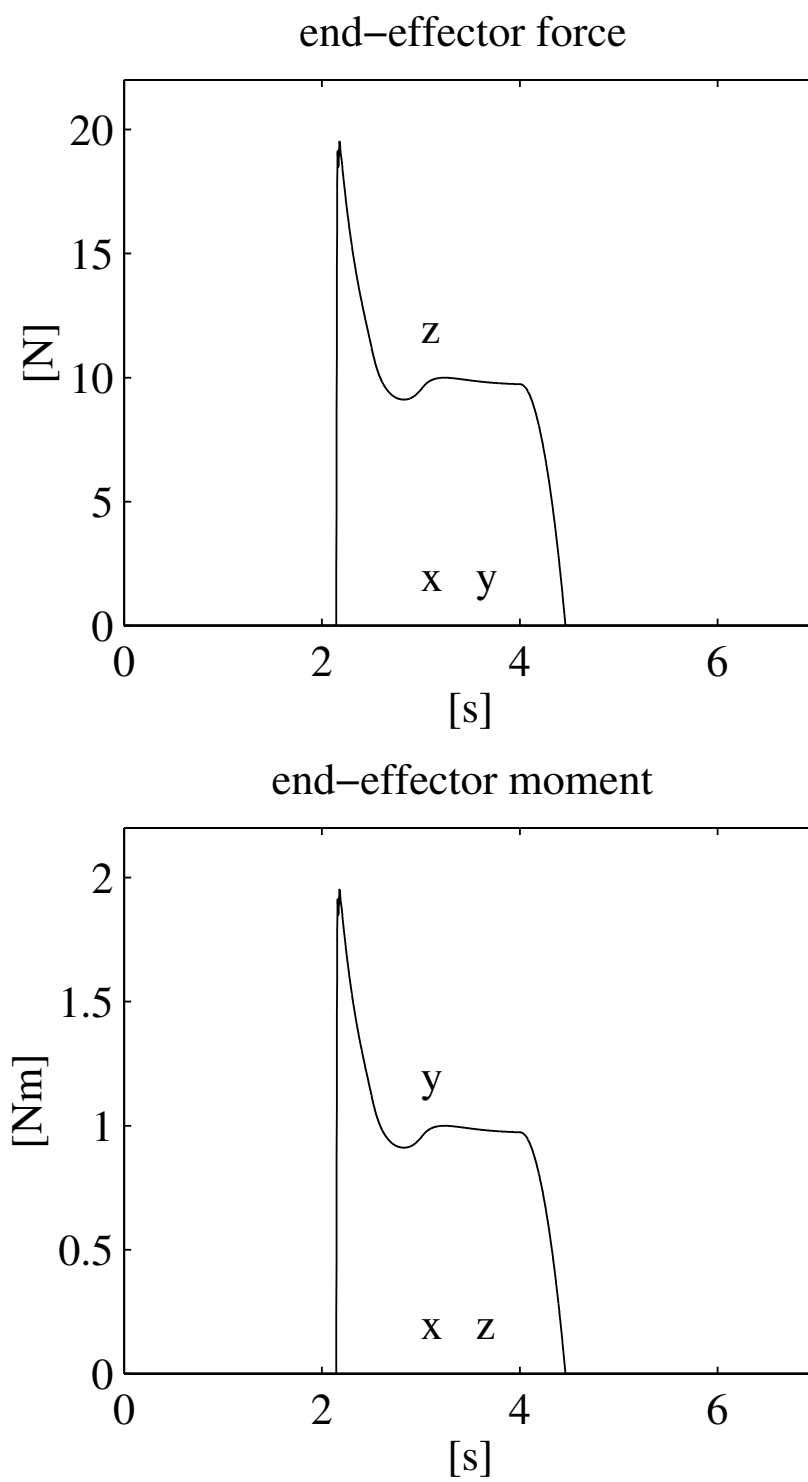


Fig. 5.