

Distributed Neural Network Controller Based on Feature Enhancement with Flexible Time Prescribed Performance for Multi-UAV Swarms

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Abstract—This paper focuses on the design of a distributed neural network (DNN) controller to address the unknown disturbance and error convergence problems in multiple unmanned aerial vehicle (multi-UAV) swarms. The multi-UAV swarm is organized through a communication topology that connects individual UAVs with virtual leaders. To address the problem of unknown disturbances, a feature enhancement (FE) layer is designed for neural networks utilizing the Hadamard product and feature matrix, combined with a state predictor (SP) and a distributed learning strategy. This approach improves the approximation performance of the traditional radial basis function neural network in handling unknown disturbances within multi-UAV swarms. Additionally, to address the error convergence problem, the flexible time prescribed performance (FTPP) algorithm is integrated into the controller design, ensuring that trajectory tracking errors remain within predefined bounds over a specified time period T . Using Lyapunov theory, Barbalat's lemma, and Gronwall inequality, it is proven that the tracking errors of the multi-UAV swarm in the overall closed-loop system converge to an arbitrarily small region and remain uniformly ultimately bounded (UUB). Finally, simulation results further demonstrate the effectiveness of the proposed control algorithms.

Index Terms—Distributed neural network (DNN), multiple unmanned aerial vehicle (multi-UAV) swarms, feature enhancement (FE), flexible time prescribed performance (FTPP), uniformly ultimately bounded (UUB).

I. INTRODUCTION

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WITH the evolution of unmanned aerial vehicle (UAV) technology, an increasing number of fields are utilized UAVs as essential tools for various purposes, such as assisting in communication [1], exploring unknown environments [2], [3], industrial inspection [4], payload [5], [6], and more. Compared to individual UAV, multiple unmanned aerial vehicle (multi-UAV) swarms can enhance work efficiency and save time. However, with the increase in the number of UAVs, controlling a multi-UAV swarm becomes significantly more challenging than controlling a single UAV. These challenges in flight control typically center around two main aspects: one is the effects of unmodeled dynamics and unknown disturbances [7]–[9]; the other one is the convergence of flight trajectory errors [10]–[12]. To address these two challenges, scientists have undertaken various efforts.

Responding to the problems caused by unmodeled dynamics and unknown disturbances, the excellent approximation properties of neural networks (NNs) offer promising solutions to the problems, and several scholars have conducted relevant research in this area. To address the issues arising from unknown system dynamics, a single-layer NN was designed with a trajectory tracking controller for UAV in [7]. An adaptive law for the NN was derived using a projection operator to ensure the boundedness of the neural weights. Inspired by Hopfield NN, [8] designed a recurrent NN for the dynamic control of a UAV to handle the unknown interference problem. This network can train itself through self-iteration once the input is provided. For multi-agent systems, to ensure good tracking performance, [13] employed a radial basis function (RBF) NN to estimate the values of unknown nonlinear functions, including external disturbances and unmodeled dynamics. However, similar to the study of individual intelligences in [7], the NNs used in [13] for multi-UAV systems did not employ a distributed learning strategy; instead, each UAV in the swarm used a local learning strategy. In [14], a distributed learning strategy for RBF NNs was designed for multi-UAVs with second-order systems, which can improve the performance of NNs for approximating nonlinear terms in multi-UAV systems. Similarly, [15] utilized a distributed learning strategy to estimate uncertain dynamics in high-order nonlinear multi-agent systems, allowing all controllers to share real-time data on estimated weights.

Besides, in order to enhance the convergence performance of trajectory tracking errors, an increasing number of prescribed performance control (PPC) methods have been em-

ployed. During the tracking process, an error transformation mechanism was designed in [11] to keep the tracking error within predefined precision. Unlike conventional PPC approaches, this improved PPC method was applied to the cooperative control of UAVs, ensuring that the tracking error remains invariant. Finite-time prescribed performance functions have been used to transform the distributed tracking errors of all follower UAVs in [12]. These functions convert the distributed tracking errors into a new set of errors by incorporating user-specified transient and steady-state requirements. For underactuated surface vessels, the trajectory tracking controller designed in [16] uses the classical PPC law to ensure that all signal errors converge to a bounded set. Furthermore, in [17], a modified PPC was proposed for unknown strict-feedback systems. This modification retains the inherent simplicity of PPC while also ensuring quantitative performance guarantees. Compared to the classical prescribed performance, [14], [18], [19] introduced the tunnel prescribed performance (TPP) to achieve better tracking performance. The TPP is designed to be effective for any initial condition. This strategy not only has a simple and clean form, but it is also effective in reducing overshooting.

Based on the discussion of the above studies, it is evident that the applications of RBF NNs is becoming increasingly common, with significant efforts focused on improving RBF NNs, such as designing distributed learning strategies for multi-agent systems [14], [15]. However, in controller design using RBF NNs, the data related to unmodeled dynamics and unknown disturbances are often unknown, making it impossible to train the network with samples in advance. Therefore, the characteristics of the eigenvalues of the sample data are crucial for the approximation performance of the NNs. None of the existing works [7], [8], [13]–[15] consider the issue of feature enhancement (FE) of the sample data, which could improve the approximation capability of the NNs. At the same time, there is a significant amount of work focused on limiting systematic errors based on PPC or improved PPC [16]–[20]. However, these existing works do not significantly improve the convergence time of errors. In other words, they lack a control mechanism to adjust the time required for error convergence.

Motivated by the aforementioned works, this article focuses on addressing problems caused by the unmodeled dynamics and unknown disturbances in multi-UAV swarms, as well as the problems related to the trajectory tracking errors convergence. To address the problems of unmodeled dynamics and unknown disturbances, this article designs DNN controller based on FE with the state predictor (SP) for a multi-UAV swarm. Additionally, the FTTP is proposed to ensure that trajectory tracking errors converge within bounded regions over flexible time intervals. The main contributions of this article are outlined as follows:

- 1) In contrast to single-individual NN controllers, we focus on DNN controller design for multi-UAV swarms. Unlike the approaches in [7], [8], [13], we employ online distributed learning, enabling individual UAV to access information about their neighbours' weights updating rates during the learning process. This method enhances

the accuracy of swarm NNs learning with the unmodeled dynamics and unknown disturbances. Furthermore, instead of using state differences, referencing SP allow the NN to use state prediction errors to train neural weights, which can improve the leaning speed of NNs.

- 2) Unlike the DNN in [14], [15] and the NNs in [7], [8], [13], [21], we design and introduce the FE into the DNN. This modification enables the DNN to more effectively learn data features, addressing the issue of low accuracy commonly encountered in NNs when working with small data samples.
- 3) Unlike the existing literature [16]–[20], the proposed FTTP can ensure that trajectory tracking errors converge to predefined specified regions within a bounded and flexible time T with a faster errors convergence speed. Furthermore, according to Lyapunov theory, Barbalat's lemma and Gronwall inequality, the tracking errors of the total closed-loop system are UUB.

Notations: $\|\cdot\|$ represents the Euclidean norm of a vector, $\hat{(\cdot)}$ is the estimation value of $(\cdot)$, and $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)$. $a := b$ indicates that b is defined as a . e_1, e_2 and e_3 are unit vectors defined as $e_1 := [1, 0, 0]^T$, $e_2 := [0, 1, 0]^T$, and $e_3 := [0, 0, 1]^T$. $\lambda_{\min \vee (\max)}(\cdot)$ is the minimum eigenvalue or the maximum eigenvalue of the matrix. $\circ$ is the Hadamard product, and $\otimes$ is the Kronecker product.

II. PRELIMINARIES

A. Graph Theory

In this paper, to describe the communication topology among UAVs and the virtual leader in the swarm, we represent it as a directed graph $\mathcal{G} = \{\mathcal{O}, \mathcal{E}\}$, where $\mathcal{O} \in \{1, 2, \dots, N\}$ is the nodes set, and $\mathcal{E} \subseteq \{(i, j), i, j \in \mathcal{O}\}$ denotes the set of edges. Then, the Laplacian matrix associated with $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is an adjacency matrix, N is a positive constant representing the number of nodes in the graph. The element a_{ij} in the matrix denotes the presence of a directed edge from node i to node j . Specifically, $a_{ij} = 1$ if there is a directed edge $(i \rightarrow j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. $\mathcal{D} = \text{diag}\{d_i\} \in \mathbb{R}^{N \times N}$ is the degree matrix of $\mathcal{G}$ with $d_i = \sum_{j=1}^N a_{i,j}$.

B. NNs Based on Feature Enhancement

RBF NNs are known for their powerful function approximation capabilities, as they can accurately model complex nonlinear relationships by adjusting network parameters. However, their performance tends to degrade when data quality is poor or when the training dataset is insufficient [22]. To address this limitation, we enhance the input vectors by introducing an FE layer, making them more suitable for NN learning when working with small sample data. In the FE layer, by iterating the Hadamard product [23]–[26], introducing a weight matrix, and feeding the result into the NN, we enable the NN to better capture the underlying features and patterns in the data, as illustrated in Fig.1. The NN architecture with FE is designed as follows:

$$f_N(\mathbf{X}) = \mathbf{W}_{id}^T \varphi(\mathbf{X}) + \epsilon(\mathbf{X}). \quad (1)$$

A. Kinematic Control Law Design with FTPP

Existing works [14], [16] on PPC effectively limit error overshoot and enhance the accuracy of path tracking. Inspired by these works, a kinematic controller for the swarm is designed using the backstepping method with FTPP, ensuring fast convergence of trajectory tracking errors, defined as:

$$\mathbf{Z}_{1,i} = \sum_{j \in \mathcal{N}_i^F} a_{i,j}(\mathbf{p}_i - \mathbf{p}_j - \mathbf{p}_{i,j}) + a_{i,0}(\mathbf{p}_i - \mathbf{p}_r - \mathbf{p}_{i,r}) \quad (3)$$

where $\mathbf{p}_j$ represents the positions of the neighboring UAVs of the i -th UAV, and $\mathbf{p}_r$ denotes the position of the virtual leader. $\mathbf{p}_{i,j}$ and $\mathbf{p}_{i,r}$ are the preset spacing between the followers and the spacing between the followers and the virtual leader within the swarm, respectively.

Then, along with (2) and (3), we can have:

$$\dot{\mathbf{Z}}_{1,i} = d_i \mathbf{v}_i - \sum_{j \in \mathcal{N}_i^F} a_{i,j} \mathbf{v}_j - \sum_{j \in \mathcal{N}_i^F} a_{i,j} \dot{\mathbf{p}}_{i,j} - a_{i,0} \dot{\mathbf{p}}_r - a_{i,0} \dot{\mathbf{p}}_{i,r} \quad (4)$$

where $d_i = \sum_{j \in \mathcal{N}_i^F} a_{i,j} + a_{i,0}$, and $\mathcal{N}_i^F$ represents the set of follower UAVs associated with the i -th UAV in the swarm.

By employing FTPP to constrain path errors $\mathbf{Z}_{1,i}$, the transient and steady-state performance of multi-UAV swarm is significantly improved, as shows below:

$$-\underline{\mathbf{Z}}_{1,i} \leq \mathbf{Z}_{1,i} \leq \overline{\mathbf{Z}}_{1,i} \quad (5)$$

where $\underline{\mathbf{Z}}_{1,i}$ and $\overline{\mathbf{Z}}_{1,i}$ are time-varying boundary functions defined as:

$$\begin{cases} \overline{\mathbf{Z}}_{1,i} = (\overline{\delta}_i + \text{sign}(\mathbf{Z}_{1,i}(0)))\mathbf{q}_i - \mathbf{q}_{i,\infty} \text{sign}(\mathbf{Z}_{1,i}(0)) \\ \underline{\mathbf{Z}}_{1,i} = (\underline{\delta}_i - \text{sign}(\mathbf{Z}_{1,i}(0)))\mathbf{q}_i + \mathbf{q}_{i,\infty} \text{sign}(\mathbf{Z}_{1,i}(0)) \end{cases} \quad (6)$$

with $0 \leq \|\overline{\delta}_i\|, \|\underline{\delta}_i\| \leq 1$. $\mathbf{q}_i$ and $\mathbf{q}_{i,0}$ can be designed as:

$$\begin{cases} \mathbf{q}_i = (\mathbf{q}_{i,0} - \mathbf{q}_{i,\infty})\exp(-\mu_p(Tt/(T-t))) + \mathbf{q}_{i,\infty}, & t < T \\ \mathbf{q}_{i,0} = \mathbf{q}_{i,\infty}, & t \geq T \end{cases} \quad (7)$$

where $\|\mathbf{q}_{i,0}\| > \|\mathbf{q}_{i,\infty}\| \in \mathbb{R}^+$, $\mu_p \in \mathbb{R}^+$, and T is the flexible time of FTPP.

Then, the error $\mathbf{Z}_{1,i}$ can be equivalently transformed into an unconstrained error ζ_i using the error transformation function:

$$\mathbf{Z}_{1,i} = \frac{\overline{\mathbf{Z}}_{1,i} + \underline{\mathbf{Z}}_{1,i}}{2} \Pi_r(\zeta_i) + \frac{\overline{\mathbf{Z}}_{1,i} - \underline{\mathbf{Z}}_{1,i}}{2} \quad (8)$$

where $\Pi_r := (-\infty, \infty) \rightarrow (-1, 1)$ is a strictly transformation function, and the function $\Pi_r(\zeta_i)$ can be chosen as $\Pi_r(\zeta_i) = (\exp(\zeta_i) - \exp(-\zeta_i))/(\exp(\zeta_i) + \exp(-\zeta_i))$.

By solving the inverse function of $\Pi_r(\zeta_i)$, the unconstrained error ζ_i can be deduced as:

$$\begin{aligned} \zeta_i &= \Pi_r^{-\text{inv}}(\mathbf{Z}_{1,i}, \overline{\mathbf{Z}}_{1,i}, \underline{\mathbf{Z}}_{1,i}) \\ &= \frac{1}{2} \ln \frac{\mathbf{1} + \mathbf{\Pi}_i}{\mathbf{1} - \mathbf{\Pi}_i} \end{aligned} \quad (9)$$

where $\mathbf{\Pi}_i = (2\mathbf{Z}_{1,i} - \overline{\mathbf{Z}}_{1,i} + \underline{\mathbf{Z}}_{1,i})/(\overline{\mathbf{Z}}_{1,i} + \underline{\mathbf{Z}}_{1,i})$.

Along with (8) and (9), the time derivative of ζ_i can be calculated as:

$$\dot{\zeta}_i = \frac{\partial \zeta_i}{\partial \mathbf{\Pi}_i} \left(\frac{\partial \mathbf{\Pi}_i}{\partial \mathbf{Z}_{1,i}} \dot{\mathbf{Z}}_{1,i} + \frac{\partial \mathbf{\Pi}_i}{\partial \overline{\mathbf{Z}}_{1,i}} \dot{\overline{\mathbf{Z}}}_{1,i} + \frac{\partial \mathbf{\Pi}_i}{\partial \underline{\mathbf{Z}}_{1,i}} \dot{\underline{\mathbf{Z}}}_{1,i} \right). \quad (10)$$

Define $\zeta_i = [\zeta_{i,x}, \zeta_{i,y}, \zeta_{i,z}]^T$, $\overline{\mathbf{E}}_{i,1} = [\overline{E}_{i,x}, \overline{E}_{i,y}, \overline{E}_{i,z}]^T$, and $\underline{\mathbf{E}}_{i,1} = [\underline{E}_{i,x}, \underline{E}_{i,y}, \underline{E}_{i,z}]^T$. Then, (10) can be transformed into the following form:

$$\dot{\zeta}_i = \mathbf{H}_{\mathbf{Z}_{1,i}} \dot{\mathbf{Z}}_{1,i} + \mathbf{H}_{\overline{\mathbf{Z}}_{1,i}} \dot{\overline{\mathbf{Z}}}_{1,i} + \mathbf{H}_{\underline{\mathbf{Z}}_{1,i}} \dot{\underline{\mathbf{Z}}}_{1,i} \quad (11)$$

where $\mathbf{H}_{\mathbf{Z}_{1,i}} = \text{diag}\{\mathbf{H}_{\mathbf{Z}_{1,i}}\} \in \mathbb{R}^{3 \times 3}$ with $\mathbf{H}_{\mathbf{Z}_{1,i}} = (\partial \zeta_i / \partial \mathbf{\Pi}_i) \times (\partial \mathbf{\Pi}_i / \partial \mathbf{Z}_{1,i})$; $\mathbf{H}_{\overline{\mathbf{Z}}_{1,i}} = \text{diag}\{\mathbf{H}_{\overline{\mathbf{Z}}_{1,i}}\} \in \mathbb{R}^{3 \times 3}$ with $\mathbf{H}_{\overline{\mathbf{Z}}_{1,i}} = (\partial \zeta_i / \partial \mathbf{\Pi}_i) \times (\partial \mathbf{\Pi}_i / \partial \overline{\mathbf{Z}}_{1,i})$; $\mathbf{H}_{\underline{\mathbf{Z}}_{1,i}} = \text{diag}\{\mathbf{H}_{\underline{\mathbf{Z}}_{1,i}}\} \in \mathbb{R}^{3 \times 3}$ with $\mathbf{H}_{\underline{\mathbf{Z}}_{1,i}} = (\partial \zeta_i / \partial \mathbf{\Pi}_i) \times (\partial \mathbf{\Pi}_i / \partial \underline{\mathbf{Z}}_{1,i})$.

Substituting (4) into (11) yields:

$$\begin{aligned} \dot{\zeta}_i &= \mathbf{H}_{\mathbf{Z}_{1,i}} \left(d_i \mathbf{v}_i - \sum_{j \in \mathcal{N}_i^F} a_{i,j} \mathbf{v}_j - \sum_{j \in \mathcal{N}_i^F} a_{i,j} \dot{\mathbf{p}}_{i,j} - a_{i,0} \dot{\mathbf{p}}_r \right. \\ &\quad \left. - a_{i,0} \dot{\mathbf{p}}_{i,r} \right) + \mathbf{H}_{\overline{\mathbf{Z}}_{1,i}} \dot{\overline{\mathbf{Z}}}_{1,i} + \mathbf{H}_{\underline{\mathbf{Z}}_{1,i}} \dot{\underline{\mathbf{Z}}}_{1,i}. \end{aligned} \quad (12)$$

Define $\tilde{\mathbf{v}}_j = \hat{\mathbf{v}}_j - \mathbf{v}_j$. To stabilize the error dynamics in equation (12), we construct a FTPP-based kinematic control law α_i , which is employed to stabilize the error dynamics of the system as follows:

$$\begin{aligned} \alpha_i &= \frac{1}{d_i} \left(-\mathbf{H}_{\mathbf{Z}_{1,i}}^{-1} (\mathbf{K}_{i,1} \zeta_i + \mathbf{H}_{\overline{\mathbf{Z}}_{1,i}} \dot{\overline{\mathbf{Z}}}_{1,i} + \mathbf{H}_{\underline{\mathbf{Z}}_{1,i}} \dot{\underline{\mathbf{Z}}}_{1,i}) \right. \\ &\quad \left. + \sum_{j \in \mathcal{N}_i^F} a_{i,j} \tilde{\mathbf{v}}_j + \sum_{j \in \mathcal{N}_i^F} a_{i,j} \dot{\mathbf{p}}_{i,j} + a_{i,0} \dot{\mathbf{p}}_r + a_{i,0} \dot{\mathbf{p}}_{i,r} \right) \end{aligned} \quad (13)$$

where $\mathbf{K}_{i,1} \in \mathbb{R}^{3 \times 3}$ is a positive definite control gain matrix.

Substituting (13) into (12), $\dot{\zeta}_i$ is transformed into the following form: $\dot{\zeta}_i = \mathbf{H}_{\mathbf{Z}_{1,i}} \sum_{j \in \mathcal{N}_i^F} a_{i,j} \tilde{\mathbf{v}}_j - \mathbf{K}_{i,1} \zeta_i$.

B. Dynamic Control Law Design with DNN Based on FE and SP

For each individual UAV of the swarm, we define the velocity error as follows:

$$\mathbf{Z}_{2,i} = \mathbf{v}_i - \mathbf{v}_d \quad (14)$$

where $\mathbf{v}_d \in \mathbb{R}^3$ is the ideal linear velocity of the multi-UAV swarm.

According to (2), the derivative of $\mathbf{Z}_{2,i}$ is

$$\dot{\mathbf{Z}}_{2,i} = m_i^{-1} \mathbf{f}_i + g \mathbf{e}_3 + m_i^{-1} \mathbf{D}_i - \dot{\mathbf{v}}_d. \quad (15)$$

To stabilize $\dot{\mathbf{Z}}_{2,i}$, we design the virtual control law $\mathbf{f}_i \in \mathbb{R}^3$ of the multi-UAV swarm as follows:

$$\mathbf{f}_i = -m_i g \mathbf{e}_3 - m_i \mathbf{D}_i + m_i \dot{\mathbf{v}}_d - \mathbf{K}_{i,2} m_i \mathbf{Z}_{2,i} \quad (16)$$

where $\mathbf{K}_{i,2} \in \mathbb{R}^{3 \times 3}$ is a positive-definite design matrix for velocity.

The unmodeled dynamics and unknown disturbance $\mathbf{D}_i$ can be estimated by various methods, such as the following DNN based on FE (1):

$$\mathbf{D}_i = \mathbf{W}_{id}^T \boldsymbol{\varphi}(\mathbf{X}_i) + \boldsymbol{\epsilon}_i(\mathbf{X}_i) \quad (17)$$

where $\mathbf{X}_i = \mathbf{W}_W(\mathbf{x}_i \circ \mathbf{x}_i)$, and $\mathbf{x}_i = [\mathbf{Z}_{1,i}^T, \mathbf{Z}_{2,i}^T, \mathbf{f}_i^T, \boldsymbol{\alpha}_i^T]^T \in \mathbb{R}^{12}$ is the input of the DNN.

Then, the control law $\mathbf{f}_i$ can be re-expressed as:

$$\mathbf{f}_{i,d} = -m_i g \mathbf{e}_3 - m_i \dot{\mathbf{W}}_i^T \boldsymbol{\varphi}(\mathbf{X}_i) + m_i \dot{\mathbf{v}}_d - \mathbf{K}_{i,2} m_i \mathbf{Z}_{2,i}. \quad (18)$$

Furthermore, we decompose $\mathbf{f}_{i,d}$ into $f_{i,x}$, $f_{i,y}$, and $f_{i,z}$ along the x-axis, y-axis, and z-axis, respectively. To enhance the NN approximation and mitigate overshooting of the control signals, we incorporated an SP [21] into the DNN based on FE as:

$$\dot{\mathbf{v}}_i = \dot{\mathbf{W}}_i^T \boldsymbol{\varphi}(\mathbf{W}_W(\mathbf{x}_i \circ \mathbf{x}_i)) + m_i^{-1} \mathbf{f}_{i,d} + g \mathbf{e}_3 - \mathbf{K}_v \tilde{\mathbf{v}}_i \quad (19)$$

where $\mathbf{K}_v \in \mathbb{R}^{3 \times 3}$ is a tuning matrix used to mitigate overshooting and smoothen the control signals.

And the update law of $\dot{\mathbf{W}}_i$ is designed as:

$$\dot{\mathbf{W}}_i = -\Gamma_i \boldsymbol{\varphi}(\mathbf{W}_W(\mathbf{x}_i \circ \mathbf{x}_i)) \tilde{\mathbf{v}}_i^T - K_{W_i} \sum_{j \in \mathcal{N}_i^F} a_{i,j} (\hat{\mathbf{W}}_i - \hat{\mathbf{W}}_j) \quad (20)$$

where $\Gamma_i \in \mathbb{R}^+$ is adaptation gain, $K_{W_i} \in \mathbb{R}^+$ is the tuning parameter. An estimation of neuron weight vectors for the neighboring UAVs of the i -th UAV, denoted by $\hat{\mathbf{W}}_j$, implies UAVs of the swarm can update their neuron weight vectors using the information from their neighbors based on the communication topology.

C. Attitude Control Law Design

For each individual i -th UAV of the swarm, we define the attitude error as:

$$\mathbf{Z}_{i,\Theta} = \boldsymbol{\Theta}_i - \boldsymbol{\Theta}_d \quad (21)$$

where $\boldsymbol{\Theta}_d = [\phi_d, \theta_d, \psi_d]^T$ is the ideal attitude angle with $\phi_d := \arctan((f_{i,x} \cos \psi_d \cos \theta_d + f_{i,y} \sin \psi_d \cos \theta_d) / f_{i,z})$, $\theta_d := \arctan((f_{i,x} \cos \psi_d + f_{i,y} \sin \psi_d) / f_{i,z})$, $\psi_d := 0$.

According to (2), the derivative of $\mathbf{Z}_{i,\Theta}$ is

$$\begin{aligned} \dot{\mathbf{Z}}_{i,\Theta} &= \dot{\boldsymbol{\Theta}}_i - \dot{\boldsymbol{\Theta}}_d \\ &= \mathbf{R}_{\omega_i} \boldsymbol{\omega}_i - \dot{\boldsymbol{\Theta}}_d. \end{aligned} \quad (22)$$

Based on (2), we derive the ideal angular velocity for the i -th UAV using backstepping method method as:

$$\boldsymbol{\omega}_{i,d} = -\mathbf{K}_{i,3} \mathbf{R}_{\omega}^{-1} \mathbf{Z}_{i,\Theta} + \mathbf{R}_{\omega}^{-1} \dot{\boldsymbol{\Theta}}_d. \quad (23)$$

Then the angular velocity error of each individual UAV is defined as follows:

$$\begin{aligned} \mathbf{Z}_{i,\omega} &= \boldsymbol{\omega}_i - \boldsymbol{\omega}_{i,d} \\ &= \boldsymbol{\omega}_i + \mathbf{K}_{i,3} \mathbf{R}_{\omega}^{-1} \mathbf{Z}_{i,\Theta} - \mathbf{R}_{\omega}^{-1} \dot{\boldsymbol{\Theta}}_d \end{aligned} \quad (24)$$

where $\mathbf{K}_{i,3} \in \mathbb{R}^{3 \times 3}$ is a positive gain matrix.

With (24), the dynamics of $\mathbf{Z}_{i,\omega}$ is expressed as:

$$\dot{\mathbf{Z}}_{i,\omega} = -\mathbf{J}^{-1} \mathbf{S}(\boldsymbol{\omega}_i) \mathbf{J} \boldsymbol{\omega}_i + \mathbf{J}^{-1} \boldsymbol{\tau}_i - \dot{\boldsymbol{\omega}}_{i,d}. \quad (25)$$

Similar to the virtual control law $\mathbf{f}_i$, we design the torque input for i -th UAV of the swarm as:

$$\boldsymbol{\tau}_i = \mathbf{S}(\boldsymbol{\omega}_i) \mathbf{J} \boldsymbol{\omega}_i + \mathbf{J} \dot{\boldsymbol{\omega}}_{i,d} - \mathbf{K}_{i,4} \mathbf{J} \mathbf{Z}_{i,\omega} \quad (26)$$

where $\mathbf{K}_{i,4} \in \mathbb{R}^{3 \times 3}$ is a positive gain matrix.

IV. PERFORMANCE ANALYSIS

This section addresses the stability of multi-UAV swarm in detail. The main results are provided by the following theorem using Lyapunov stability theory, *Lemma 1* and *Lemma 2*.

Theorem 1. *Under Assumptions 1-3, consider a multi-UAV swarm with N UAVs and a virtual leader. When the DNN controller is chosen as follows: the kinematic control law with FTTP (13), the dynamic control law with DNN based on feature enhancement (16), the attitude control laws (23) and (26), the state predictor (19), and the update law of DNN based on feature enhancement (20), all signals in the closed-loop system are uniformly ultimately bounded (UUB), and the constraint (5) of FTTP is never violated.*

Proof: The dynamics of the errors in the multi-UAV swarm system can be rewritten as:

$$\begin{cases} \dot{\tilde{\mathbf{v}}}_i = \tilde{\mathbf{W}}_i^T \boldsymbol{\varphi}(\mathbf{W}_W(\mathbf{v}_i \circ \mathbf{v}_i)) - \mathbf{K}_v \tilde{\mathbf{v}}_i - m_i^{-1} \mathbf{D}_i(\cdot) \\ \dot{\tilde{\mathbf{W}}}_i = -\Gamma_i \boldsymbol{\varphi}(\mathbf{W}_W(\mathbf{v}_i \circ \mathbf{v}_i)) \tilde{\mathbf{v}}_i^T \\ \quad - K_{W_i} \sum_{j \in \mathcal{N}_i^F} a_{i,j} (\tilde{\mathbf{W}}_i - \tilde{\mathbf{W}}_j) \\ \dot{\boldsymbol{\zeta}}_i = \mathbf{H}_{Z_{1,i}} \sum_{j \in \mathcal{N}_i^F} a_{i,j} \tilde{\mathbf{v}}_j - \mathbf{K}_{i,1} \boldsymbol{\zeta}_i \\ \dot{\mathbf{Z}}_{2,i} = -\mathbf{K}_{i,2} \mathbf{Z}_{2,i} \\ \dot{\mathbf{Z}}_{i,\Theta} = -\mathbf{K}_{i,3} \mathbf{Z}_{i,\Theta} \\ \dot{\mathbf{Z}}_{i,\omega} = -\mathbf{K}_{i,4} \mathbf{Z}_{i,\omega}. \end{cases} \quad (27)$$

Define a Lyapunov function as follows:

$$V = V_1 + V_2. \quad (28)$$

Then, the Lyapunov function V is divided into two components, V_1 and V_2 , with corresponding operations performed on each. The first component, V_1 , is defined as follows:

$$V_1 = \frac{1}{2} \sum_{i \in \mathcal{N}_1^N} (\tilde{\mathbf{v}}_i^T \tilde{\mathbf{v}}_i + \frac{1}{\Gamma_i} \text{tr}(\tilde{\mathbf{W}}_i^T \tilde{\mathbf{W}}_i)). \quad (29)$$

The derivation of V_1 is obtained by combining (19) and (20) is shown below:

$$\begin{aligned} \dot{V}_1 &= \sum_{i \in \mathcal{N}_1^N} \tilde{\mathbf{v}}_i^T \dot{\tilde{\mathbf{v}}}_i + \frac{1}{\Gamma_i} \sum_{i \in \mathcal{N}_1^N} \text{tr}(\tilde{\mathbf{W}}_i^T \dot{\tilde{\mathbf{W}}}_i) \\ &\leq - \sum_{i \in \mathcal{N}_1^N} \mathbf{K}_v \tilde{\mathbf{v}}_i^T \tilde{\mathbf{v}}_i + \sum_{i \in \mathcal{N}_1^N} \tilde{\mathbf{v}}_i^T \tilde{\mathbf{W}}_i \boldsymbol{\varphi}(\mathbf{W}_W(\mathbf{v}_i \circ \mathbf{v}_i)) \\ &\quad - \sum_{i \in \mathcal{N}_1^N} \tilde{\mathbf{v}}_i^T \boldsymbol{\epsilon}_i - \sum_{i \in \mathcal{N}_1^N} \tilde{\mathbf{W}}_i^T \boldsymbol{\varphi}(\mathbf{W}_W(\mathbf{v}_i \circ \mathbf{v}_i)) \tilde{\mathbf{v}}_i^T \\ &\leq - \sum_{i \in \mathcal{N}_1^N} \mathbf{K}_v \tilde{\mathbf{v}}_i^T \tilde{\mathbf{v}}_i - \sum_{i \in \mathcal{N}_1^N} \tilde{\mathbf{v}}_i^T \boldsymbol{\epsilon}_i. \end{aligned} \quad (30)$$

Remark 1 The term $-K_{W_i} \sum_{j \in \mathcal{N}_i^F} a_{i,j} (\tilde{\mathbf{W}}_i - \tilde{\mathbf{W}}_j)$ is used to correct and reduce the weight estimation errors between UAVs in the swarm, ultimately driving the weight estimation of each UAV to converge to its ideal value $\mathbf{W}_{id}$. As time progresses, this term becomes progressively smaller, eventually approaching zero.

The following inequalities can be obtained by completing the square and applying Young's inequality:

$$-\sum_{i \in N_1^N} \mathbf{K}_v \tilde{\mathbf{v}}_i^T \tilde{\mathbf{v}}_i \leq -\lambda_{\min}(\mathbf{K}_v) \|\tilde{\mathbf{v}}\|^2 \quad (31)$$

and

$$-\sum_{i \in N_1^N} \tilde{\mathbf{v}}_i^T \boldsymbol{\epsilon}_i \leq \frac{\|\tilde{\mathbf{v}}\|^2}{2} + \frac{\|\boldsymbol{\epsilon}^*\|^2}{2} \quad (32)$$

where $\tilde{\mathbf{v}} = [\tilde{\mathbf{v}}_1^T, \tilde{\mathbf{v}}_2^T, \dots, \tilde{\mathbf{v}}_i^T]^T$, and $\boldsymbol{\epsilon}^* = [\boldsymbol{\epsilon}_1^T, \boldsymbol{\epsilon}_2^T, \dots, \boldsymbol{\epsilon}_i^T]^T$.

Thus, we get:

$$\dot{V}_1 \leq -(\lambda_{\min}(\mathbf{K}_v) - \frac{1}{2}) \|\tilde{\mathbf{v}}\|^2 + \frac{1}{2} \|\boldsymbol{\epsilon}^*\|^2. \quad (33)$$

We consider the Lyapunov function V_2 as:

$$V_2 = \frac{1}{2} \sum_{i \in N_1^N} (\zeta_i^T \zeta_i + \mathbf{Z}_{2,i}^T \mathbf{Z}_{2,i} + \mathbf{Z}_{i,\Theta}^T \mathbf{Z}_{i,\Theta} + \mathbf{Z}_{i,\omega}^T \mathbf{Z}_{i,\omega}). \quad (34)$$

Using (12), (13), (15), (16), (22), (23), (25) and (26), the derivation of V_2 can be obtained as follows:

$$\begin{aligned} \dot{V}_2 &= \sum_{i \in N_1^N} (\zeta_i^T \dot{\zeta}_i + \mathbf{Z}_{2,i}^T \dot{\mathbf{Z}}_{2,i} + \mathbf{Z}_{i,\Theta}^T \dot{\mathbf{Z}}_{i,\Theta} + \mathbf{Z}_{i,\omega}^T \dot{\mathbf{Z}}_{i,\omega}) \\ &= -\mathbf{K}_{i,1} \sum_{i \in N_1^N} \zeta_i^T \zeta_i - \mathbf{K}_{i,2} \sum_{i \in N_1^N} \mathbf{Z}_{2,i}^T \mathbf{Z}_{2,i} \\ &\quad - \mathbf{K}_{i,3} \sum_{i \in N_1^N} \mathbf{Z}_{i,\Theta}^T \mathbf{Z}_{i,\Theta} - \mathbf{K}_{i,4} \sum_{i \in N_1^N} \mathbf{Z}_{i,\omega}^T \mathbf{Z}_{i,\omega} \\ &\quad + \sum_{i \in N_1^N} \zeta_i^T \mathbf{H}_{Z_1,i} \sum_{j \in N_i^F} a_{i,j} \tilde{\mathbf{v}}_j. \end{aligned} \quad (35)$$

Applying Young's inequality and completing the square to our specific terms, we get:

$$\begin{cases} -\mathbf{K}_{i,1} \sum_{i \in N_1^N} \zeta_i^T \zeta_i \leq -\lambda_{\min}(\mathbf{K}_1) \|\zeta\|^2 \\ -\mathbf{K}_{i,2} \sum_{i \in N_1^N} \mathbf{Z}_{2,i}^T \mathbf{Z}_{2,i} \leq -\lambda_{\min}(\mathbf{K}_2) \|\mathbf{Z}_2\|^2 \\ -\mathbf{K}_{i,3} \sum_{i \in N_1^N} \mathbf{Z}_{i,\Theta}^T \mathbf{Z}_{i,\Theta} \leq -\lambda_{\min}(\mathbf{K}_3) \|\mathbf{Z}_\Theta\|^2 \\ -\mathbf{K}_{i,4} \sum_{i \in N_1^N} \mathbf{Z}_{i,\omega}^T \mathbf{Z}_{i,\omega} \leq -\lambda_{\min}(\mathbf{K}_4) \|\mathbf{Z}_\omega\|^2 \\ \sum_{i \in N_1^N} \zeta_i^T \mathbf{H}_{Z_1,i} \sum_{j \in N_i^F} a_{i,j} \tilde{\mathbf{v}}_j = \zeta^T \mathbf{H}_{Z_1} (\mathcal{A} \otimes \mathbf{I}_3) \tilde{\mathbf{v}} \\ \leq \frac{\lambda_{\max}(\mathbf{H}_{Z_1}) \lambda_{\max}(\mathcal{A})}{2} (\|\zeta\|^2 + \|\tilde{\mathbf{v}}\|^2) \end{cases} \quad (36)$$

where

$$\begin{cases} \mathbf{K}_1 = \text{diag}\{\mathbf{K}_{1,1}, \mathbf{K}_{2,1}, \dots, \mathbf{K}_{i,1}\} \\ \mathbf{K}_2 = \text{diag}\{\mathbf{K}_{1,2}, \mathbf{K}_{2,2}, \dots, \mathbf{K}_{i,2}\} \\ \mathbf{K}_3 = \text{diag}\{\mathbf{K}_{1,3}, \mathbf{K}_{2,3}, \dots, \mathbf{K}_{i,3}\} \\ \mathbf{K}_4 = \text{diag}\{\mathbf{K}_{1,4}, \mathbf{K}_{2,4}, \dots, \mathbf{K}_{i,4}\} \\ \mathbf{H}_{Z_1} = \text{diag}\{\mathbf{H}_{Z_{1,1}}, \mathbf{H}_{Z_{2,1}}, \dots, \mathbf{H}_{Z_{i,1}}\} \\ \zeta = [\zeta_1^T, \zeta_2^T, \dots, \zeta_i^T]^T \\ \mathbf{Z}_2 = [\mathbf{Z}_{2,1}^T, \mathbf{Z}_{2,2}^T, \dots, \mathbf{Z}_{2,i}^T]^T \\ \mathbf{Z}_\Theta = [\mathbf{Z}_{1,\Theta}^T, \mathbf{Z}_{2,\Theta}^T, \dots, \mathbf{Z}_{i,\Theta}^T]^T \\ \mathbf{Z}_\omega = [\mathbf{Z}_{1,\omega}^T, \mathbf{Z}_{2,\omega}^T, \dots, \mathbf{Z}_{i,\omega}^T]^T. \end{cases}$$

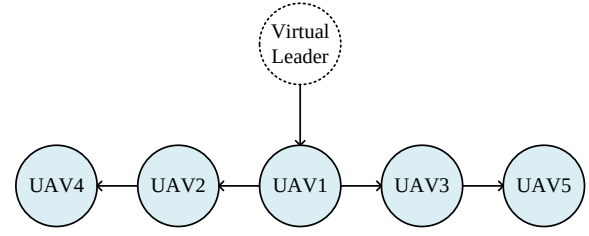


Fig. 4. The communication topology within the multi-UAV swarm ($N = 5$) with a virtual leader.

Then, we can have:

$$\begin{aligned} \dot{V}_2 &\leq -\lambda_{\min}(\mathbf{K}_1) \|\zeta\|^2 - \lambda_{\min}(\mathbf{K}_2) \|\mathbf{Z}_2\|^2 \\ &\quad - \lambda_{\min}(\mathbf{K}_3) \|\mathbf{Z}_\Theta\|^2 - \lambda_{\min}(\mathbf{K}_4) \|\mathbf{Z}_\omega\|^2 \\ &\quad + \frac{\lambda_{\max}(\mathbf{H}_{Z_1}) \lambda_{\max}(\mathcal{A})}{2} (\|\zeta\|^2 + \|\tilde{\mathbf{v}}\|^2). \end{aligned} \quad (37)$$

By differentiating V with applying (28), (33) and (37), the following expression is obtained:

$$\begin{aligned} \dot{V} &\leq -[\lambda_{\min}(\mathbf{K}_v) - \frac{1 + \lambda_{\max}(\mathbf{H}_{Z_1}) \lambda_{\max}(\mathcal{A})}{2}] \|\tilde{\mathbf{v}}\|^2 \\ &\quad - [\lambda_{\min}(\mathbf{K}_1) - \frac{\lambda_{\max}(\mathbf{H}_{Z_1}) \lambda_{\max}(\mathcal{A})}{2}] \|\zeta\|^2 \\ &\quad - \lambda_{\min}(\mathbf{K}_2) \|\mathbf{Z}_2\|^2 - \lambda_{\min}(\mathbf{K}_3) \|\mathbf{Z}_\Theta\|^2 \\ &\quad - \lambda_{\min}(\mathbf{K}_4) \|\mathbf{Z}_\omega\|^2 + \frac{1}{2} \|\boldsymbol{\epsilon}^*\|^2. \end{aligned} \quad (38)$$

Letting

$$\begin{cases} \beta_1 = \lambda_{\min}(\mathbf{K}_v) - \frac{1 + \lambda_{\max}(\mathbf{H}_{Z_1}) \lambda_{\max}(\mathcal{A})}{2} > 0 \\ \beta_2 = \lambda_{\min}(\mathbf{K}_1) - \frac{\lambda_{\max}(\mathbf{H}_{Z_1}) \lambda_{\max}(\mathcal{A})}{2} > 0 \\ \beta_3 = \lambda_{\min}(\mathbf{K}_2) > 0 \\ \beta_4 = \lambda_{\min}(\mathbf{K}_3) > 0 \\ \beta_5 = \lambda_{\min}(\mathbf{K}_4) > 0 \\ \varpi^* = \frac{1}{2} \|\boldsymbol{\epsilon}^*\|^2. \end{cases} \quad (39)$$

This implies that $\dot{V} \leq -hV + \varpi^*$ with $h = \min\{2\beta_1, 2\beta_2, 2\beta_3, 2\beta_4, 2\beta_5\}$ and ϖ^* is bounded by $\lim_{t \rightarrow \infty} \|\boldsymbol{\epsilon}(\mathbf{X})\| = 0$. Therefore, we can conclude that all error signals within the closed-loop system are UUB.

With Lemma 1, $\dot{V}_1$ is uniformly continuous and we can get $\dot{V}_1 \rightarrow 0$ as $t \rightarrow \infty$. This means that $\|\tilde{\mathbf{W}}_i\| \rightarrow 0$, $\tilde{\mathbf{W}}_i \rightarrow \mathbf{W}_{id}$ as $t \rightarrow \infty$. The same reasoning can be used to obtain $\|\tilde{\mathbf{v}}_i\| \rightarrow 0$, $\tilde{\mathbf{v}}_i \rightarrow \mathbf{v}_i$ as $t \rightarrow \infty$. With Lemma 2, we can get $V \leq \frac{\varpi^*}{h}$. Finally, the proof of Theorem 1 is complete. ■

V. SIMULATIONS AND COMPARISONS

In this section, based the the objectives outlined in part II-C, we demonstrate the effectiveness of the proposed control laws through simulation results. In this simulation, we use five UAVs, labeled as UAV1, UAV2, UAV3, UAV4, UAV5, to track a virtual leader with the communication topology shown in Fig. 4. UAV1 follows the same path as the virtual leader, while UAVs 1 through 5 maintain their relative distances and formation via the communication topology.

TABLE I
THE SIMULATION PARAMETERS FOR THE MULTI-UAV SWARM.

Parameters	Symbols and Values
Errors Coefficients	$K_{1,1}=\text{diag}\{5, 5, 5\}$; $K_{1,2}=\text{diag}\{250, 200, 200\}$; $K_{2,1}=\text{diag}\{5, 5, 5\}$; $K_{2,2}=\text{diag}\{250, 200, 200\}$; $K_{3,1}=\text{diag}\{5, 5, 5\}$; $K_{3,2}=\text{diag}\{250, 200, 200\}$; $K_{4,1}=\text{diag}\{5, 5, 5\}$; $K_{4,2}=\text{diag}\{250, 200, 200\}$; $K_{5,1}=\text{diag}\{5, 5, 5\}$; $K_{5,2}=\text{diag}\{250, 200, 200\}$.
UAV Parameters	$m = 0.2(\text{kg})$; $g = 9.81$.
DNNs & State predictor	$K_{1,v}=\text{diag}\{500, 500, 500\}$; $K_{2,v}=\text{diag}\{500, 500, 500\}$; $K_{3,v}=\text{diag}\{500, 500, 500\}$; $K_{4,v}=\text{diag}\{500, 500, 500\}$; $K_{5,v}=\text{diag}\{500, 500, 500\}$; $\Gamma_i = 10000$.
Disturbances	$D_i := \{f_{xx} = -2 \sin(v_{i,x}) + 0.5 \sin(0.01t)$; $f_{xy} = -2 \sin(v_{i,y}) + 0.5 \cos(0.01t)$; $f_{xz} = -\sin(v_{i,z}) + 0.5 \sin(0.01t)\}$.

The UAV in the swarm is modeled as shown in Equation (2). The reference trajectory p_r of virtual leader is designed as $p_{r,x} = 1.5 \sin(0.4t - 0.5)(\text{m})$; $p_{r,y} = 0.5 \sin(0.8t)(\text{m})$; $p_{r,z} = 1.5(\text{m})$. Other simulation parameters are shown in Table I. The hardware used for the simulations consists of an 11th-generation Intel® Core™ i7-1165G7 processor with a base frequency of 2.80 GHz, 16.0 GB of system memory, and the Windows 11 operating system. The MATLAB/Simulink version utilized is 2022b.

A. DNN based on FE with SP

To achieve the distributed learning objective, as depicted in Fig. 4, we design DNN with FE and SP. These features allow each individual UAV to access information regarding the weight update rates of its neighbors during the learning process. Compared to the RBF NNs in [7], [8], [13]–[15], our proposed method enhances both the learning speed and accuracy of the NN. Figs. 5 to 9 illustrate the approximation effectiveness of various NN methods in handling the unmodeled dynamics and unknown disturbances D_i for each UAV in the swarm, respectively. It is evident that the DNN based on FE and SP (DFEN) outperforms other methods, including RBF NN (RBF) with SP, and NN with FE (FEN) and SP, in terms of approximation performance.

B. FTTP

To achieve the geometric objective and the FTTP objective, as shown in Fig. 2, we design FTTP to ensure the trajectory tracking errors converge to predefined specified regions with a flexible time T . From Fig. 10 to Fig. 14, it is evident that, compared to PPC in [16], the FTTP results in faster convergence of tracking errors. Additionally, T can be adjusted within a bounded range to control the rate of errors convergence.

By utilizing a DNN based on FE and SP, along with FTTP, we design a controller for the multi-UAV swarm to effectively track the trajectory of a virtual leader, achieving excellent tracking performance as shown in Fig. 15 and Fig. 16. From Fig. 17 to Fig. 20, we observe that the linear velocity, angular velocity, attitude angle, and control input of each individual UAV in the swarm remain within a reasonable ranges.

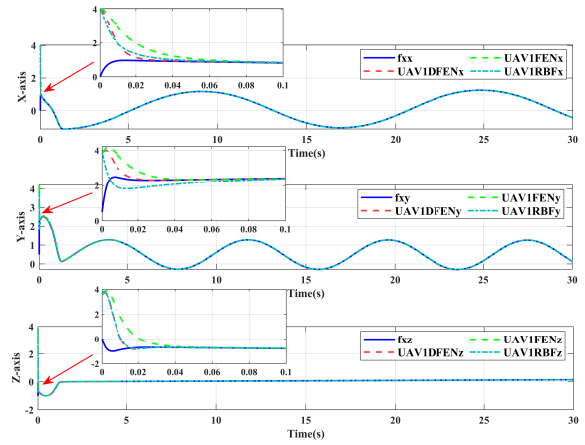


Fig. 5. Based on the communication topology, the effectiveness of different NNs in approximating the disturbances D_1 for UAV1.

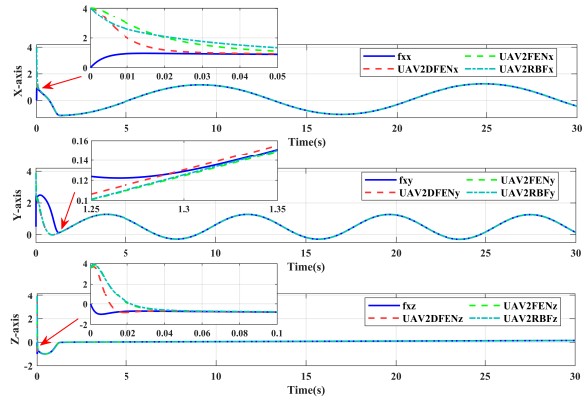


Fig. 6. Based on the communication topology, the effectiveness of different NNs in approximating the disturbances D_2 for UAV2.

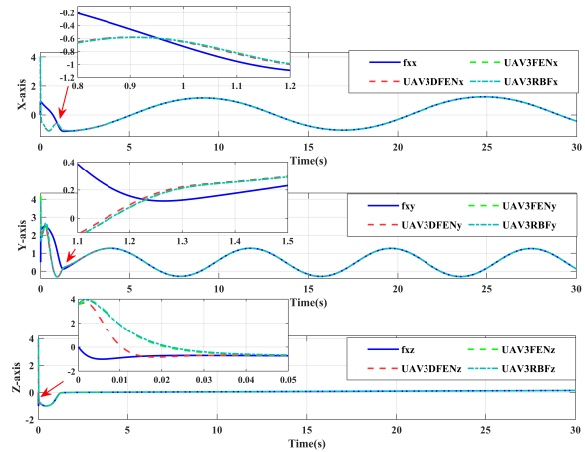


Fig. 7. Based on the communication topology, the effectiveness of different NNs in approximating the disturbances D_3 for UAV3.

VI. CONCLUSION

In this paper, a DNN controller based on FE and SP with FTTP is designed for a multi-UAV swarm, addressing the trajectory tracking problem in the presence of unmodeled dynamics and unknown disturbances while achieving superior approximation performance. To enhance the learning speed

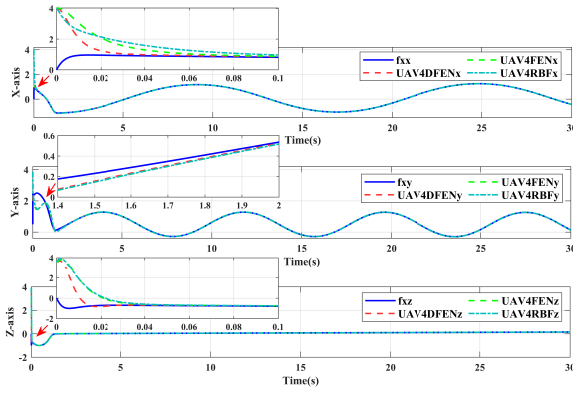


Fig. 8. Based on the communication topology, the effectiveness of different NNs in approximating the disturbances D_4 for UAV4.

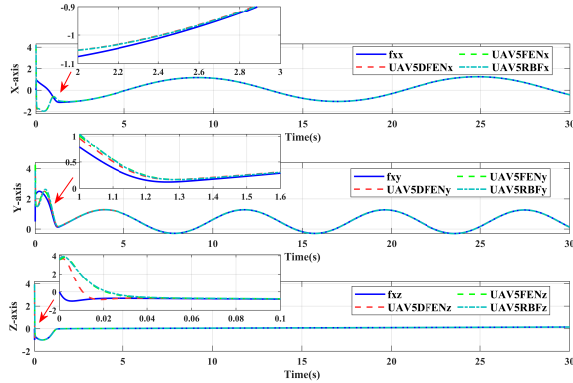


Fig. 9. Based on the communication topology, the effectiveness of different NNs in approximating the disturbances D_5 for UAV5.

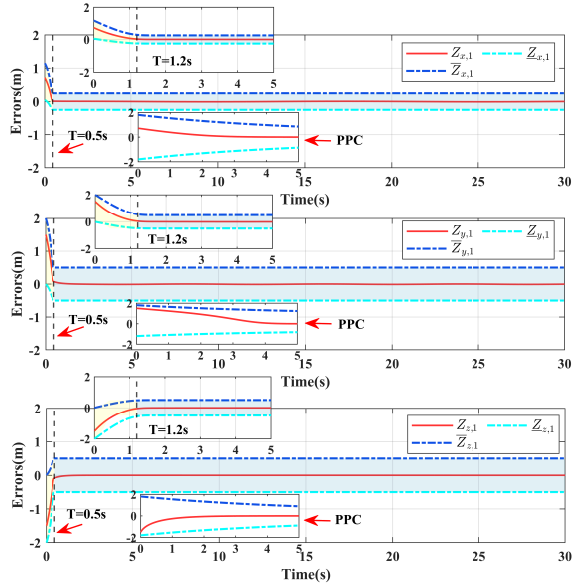


Fig. 10. With the communication topology, the trajectory errors for UAV1 under the proposed FTTP constraint at 0.5s and 1.2s, and the standard PPC constraint proposed as proposed in [16].

and accuracy of the DNN, we incorporate the prediction errors from the SP and the weight update rates of neighboring UAVs, based on the communication topology, to update each

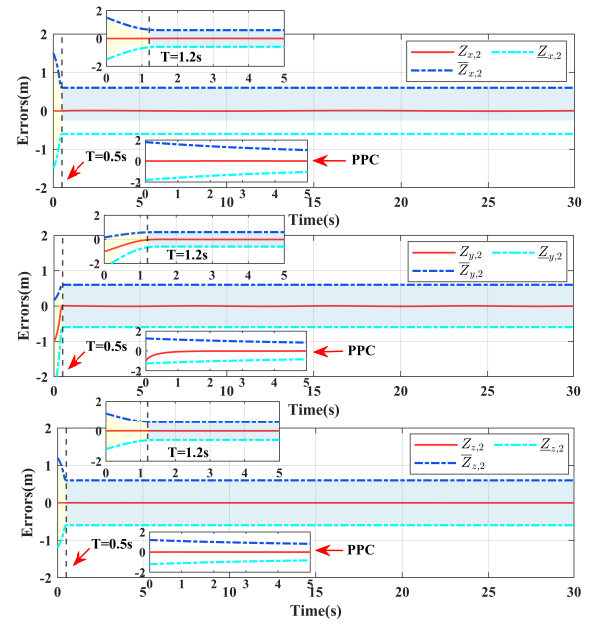


Fig. 11. With the communication topology, the trajectory errors for UAV2 under the proposed FTTP constraint at 0.5s and 1.2s, and the standard PPC constraint proposed as proposed in [16].

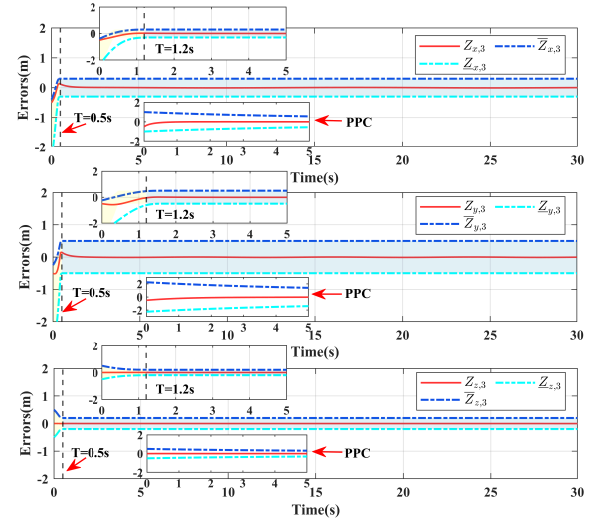


Fig. 12. With the communication topology, the trajectory errors for UAV3 under the proposed FTTP constraint at 0.5s and 1.2s, and the standard PPC constraint proposed as proposed in [16].

UAV's neuron weights during the learning process. Compared with traditional RBF NNs and local learning strategies, our proposed method achieves better approximation results with FE. Additionally, to ensure the trajectory tracking errors converge to predefined specified regions within a bounded and the flexible time T , FTTP is proposed to limit the trajectory tracking errors of the multi-UAV swarm. Compared with PPC, FTTP converges the errors more quickly. All error signals of the closed-loop system are UUB, as proven through Lyapunove stability theory, Barbalat's lemma, and Gronwall inequality. Simulation results demonstrate the effectiveness and advantages of our control approach. In future work, we will explore switching topologies for the multi-UAV swarm and incorporate

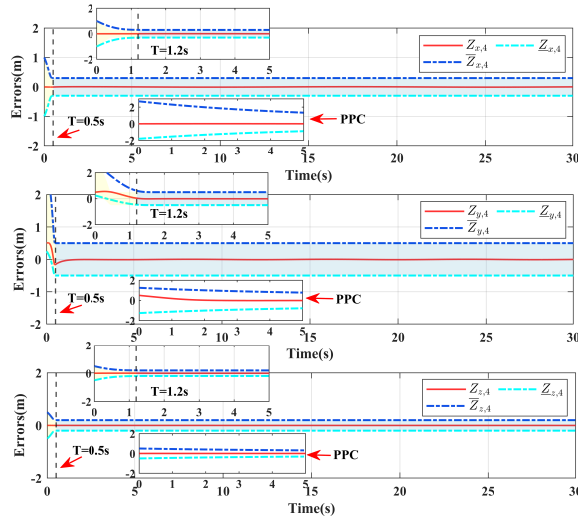


Fig. 13. With the communication topology, the trajectory errors for UAV4 under the proposed FTTP constraint at 0.5s and 1.2s, and the standard PPC constraint proposed as proposed in [16].

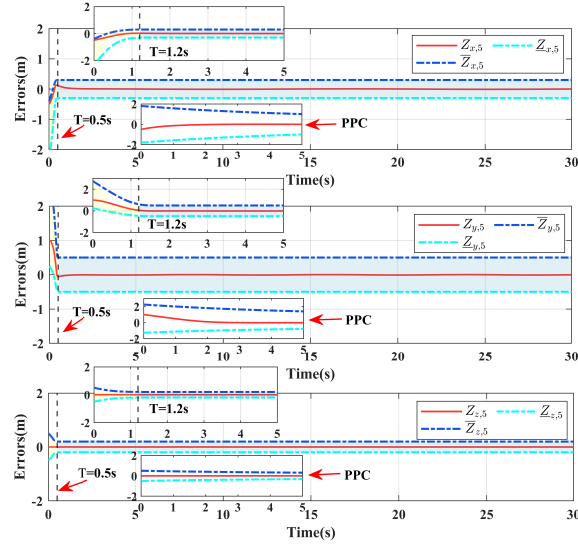


Fig. 14. With the communication topology, the trajectory errors for UAV5 under the proposed FTTP constraint at 0.5s and 1.2s, and the standard PPC constraint proposed as proposed in [16].

obstacle avoidance strategies. The controller developed in this paper will undergo experimental validation on a real-world platform, with an experimental setup designed to enhance the hardware computational capabilities of UAVs. This will enable practical testing of the controller's performance under real-world conditions.

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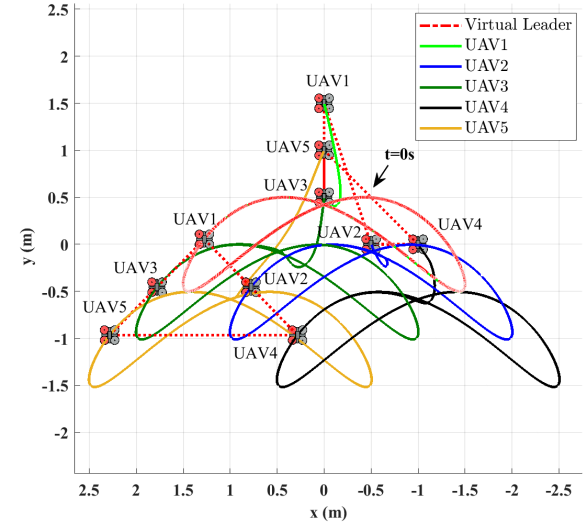


Fig. 15. The actual 2D top-view image showing the trajectories of the multi-UAV swarm.

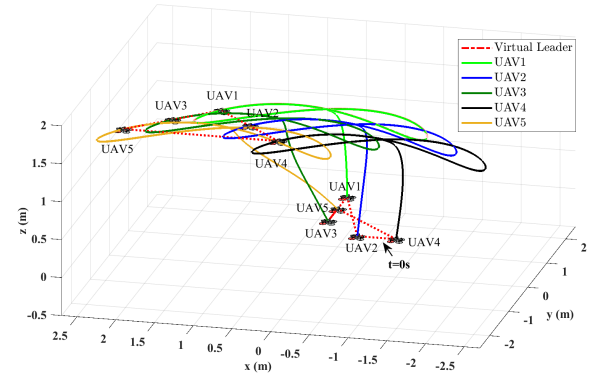


Fig. 16. The actual 3D trajectories of the multi-UAV swarm.

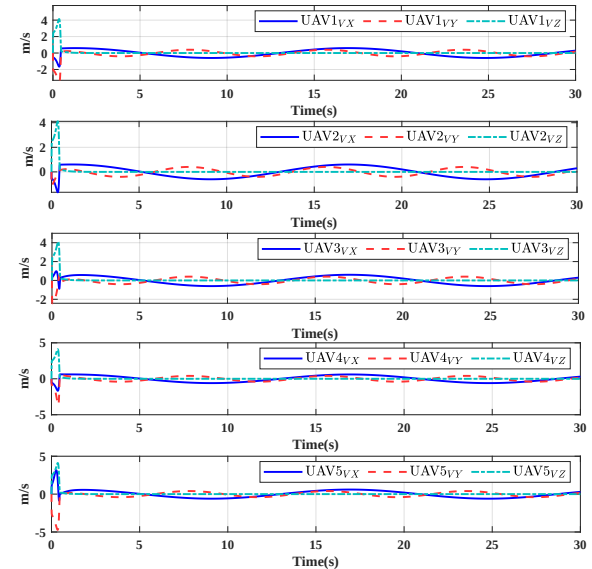


Fig. 17. The linear velocity of each individual UAV in the swarm.

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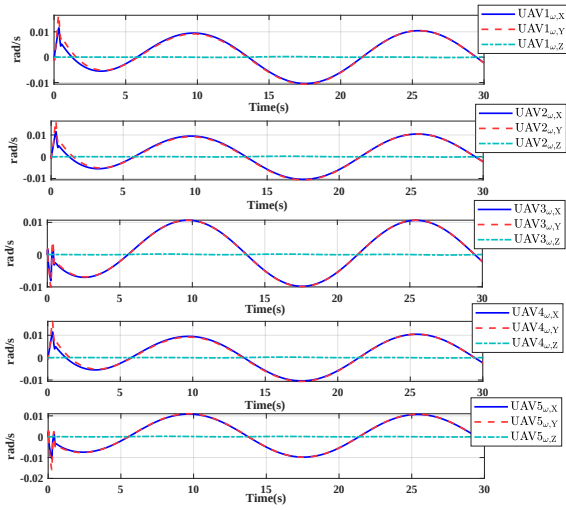


Fig. 18. The angular velocity of each individual UAV in the swarm.

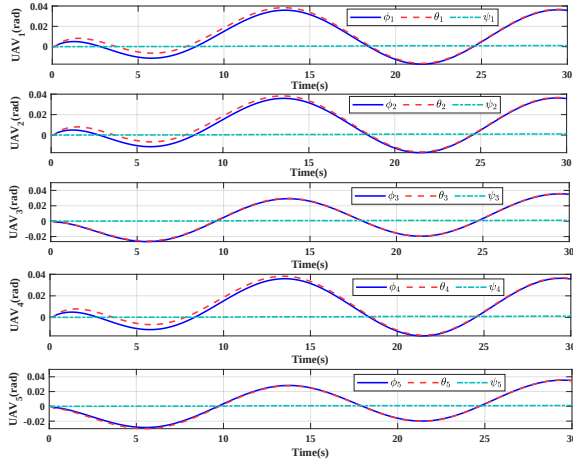


Fig. 19. The attitude angle of each individual UAV in the swarm.

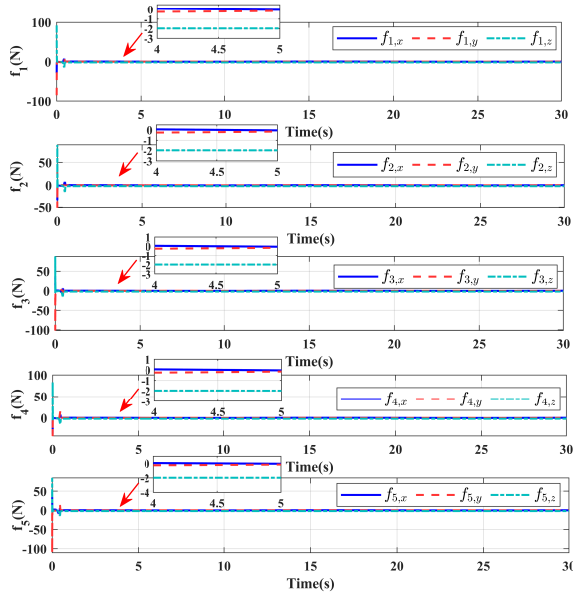


Fig. 20. The control input of each individual UAV in the swarm.

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