



Delay-Dependent Stability Analysis in Haptic Rendering

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Abstract

Nowadays haptic devices have lots of applications in virtual reality systems. While using a haptic device, one of the main requirements is the stable behavior of the system. An unstable behavior of a haptic device may damage itself and even may hurt its operator. Stability of haptic devices in the presence of inevitable time delay in addition to a suitable zero-order hold is studied in the presented paper, using two different methods. Both presented methods are based on Lyapunov-Krazuvskii functional. In the first method, a model transform is performed to determine the stability boundary, while the second approach is based on Free Weighing Matrices (FWMs). Delay-dependent stability criteria are determined by solving Linear Matrix Inequalities (LMIs). Results of these two methods are compared with each other and verified by simulations as well as experiments on a KUKA Light Weight Robot 4 (LWR4). It is concluded that using free weighing matrices leads to more unknown parameters and needs more calculation, but its results are less conservative.

Keywords Haptic devices · Stability · Linear matrix inequality

1 Introduction

A haptic system consists of a haptic interface which is in contact with an operator. The haptic interface commonly includes several elements, such as haptic device, controller, virtual environment (VE) and visual display [1]. The task of a haptic device is to create the sense of touching of virtual objects for the operator. Haptic devices are widely used in applications such as bone surgery [2], virtual prototyping [3], teleoperation [4], surgery training for medical students [5], and human-robot interaction [6]. When touching a real object, the contact condition and transformed forces between a human operator and a large number of objects can be modeled by a spring and damper. Therefore, virtual environment is modeled as a spring and damper with coefficients of K and B respectively, in discrete domain.

Resultant force of this spring and damper in k^{th} sample is calculated as follows:

$$(K + B \frac{z-1}{Tz})X(k) = KX(k) + B \frac{X(k) - X(k-1)}{T} \quad (1)$$

where $X(k)$ and $X(k-1)$ represents the position of the haptic device in samples of k and $k-1$, respectively. The second term in Eq. 1 is the average velocity between samples of k and $k-1$; so its continuous equivalent is $Bse^{-sT/2}X(s)$. As a result, continuous equivalent of Eq. 1 is $(K + Bse^{-sT/2})X(s)$. The higher the values of K and B , the better the transparency of the haptic device [7]. But in the case of large values of K and B , the haptic device may become unstable and can damage itself or even hurt the operator. In other words, simulating stiff objects is the most challenging problem from stability viewpoint.

In this paper stability analysis of a haptic device while simulating a stiff virtual object is presented using Linear Matrix Inequalities. Two different methods, which are based on defining a Lyapunov-Krazuvskii functional are utilized. Results are compared and verified by simulation and experiments.

System description and literature review are presented in Sections 2 and 3 respectively. Section 4 discusses about model transform and free weighting matrices, also LMIs which should be solved to determine the stability boundary. These two methods are compared and some simulations and

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experiments are reported to validate the presented theory in Sections 5 and 7, respectively. Finally the paper ends up with conclusion.

2 System Description

Operators are in contact with haptic devices. Hogan has shown that a human operator can actively move her/his hand and generate energy in frequencies less than 10 Hz [8]. Instability in haptic devices occurs is frequencies more than several hundred Hertz [9]. In these frequencies the operator cannot actively insert energy to the haptic device and acts like a passive element (due to mass and damping of the hand) and absorbs energy from the haptic device, which is in favor of the stability of the system. It is stated in [10, 9] that taking into account the effect of the operator makes the haptic device more stable. As a result, the "worst-case scenario" in stability analysis would be the case where the operator's hand is omitted [11, 12].

A haptic device may be nonlinear and may have several degrees of freedom. However, when simulating a virtual object for the operator, its configuration and location is almost constant and may have only small movements around a steady point. Thus, the dynamic of the haptic device can be linearized around its operating point, and can be represented as an equivalent mass and viscous friction, like a single degree of freedom system. With this assumption, a haptic device is modeled by a mass of m which moves with viscous friction of b and Coulomb friction of c as schematically depicted in Fig. 1.

As a design requirement of the haptic system, the stiffness of the robot has to be much higher than the maximum value of the virtual environment stiffness to be

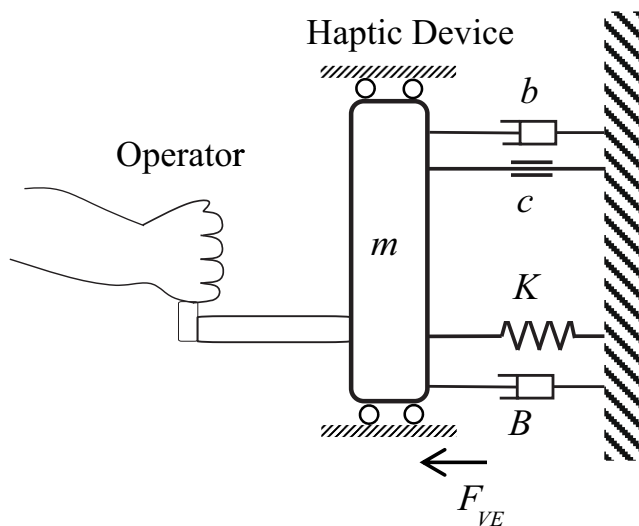


Fig. 1 Schematic view of the haptic device

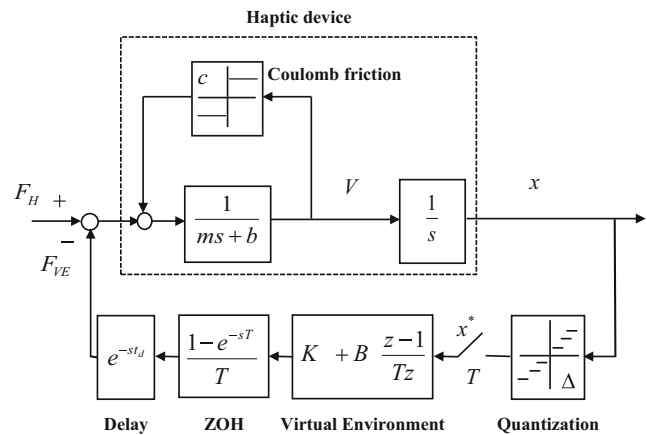


Fig. 2 1-DOF nonlinear model of the haptic device

simulated (K). The robot is equipped with a position sensor, but no velocity sensor or velocity observer is utilized. The required velocity signal is calculated from position signal, using backward difference method. Based on the explained specifications, the nonlinear model of the haptic device is depicted in Fig. 2. In this figure resolution of position sensor is denoted by Δ , and x^* is sampled position with sampling time of T .

Coulomb friction dissipates energy from system and makes the haptic device more stable. Due to quantization and discretization in position sensor, sensed position can be different from real one. Depending on whether the sensed position is more or less than the real one, applied torques by actuators can be more or less than desired one. Due to this difference between applied and desired torques, energy may leaks to the system [13]. In references such as [9, 13], and [14] it is shown that the dissipated energy by Coulomb friction is higher than added energy to the system by quantization and discretization in sensor, in well-known haptic devices. Based on this assumption, two nonlinear effects of Coulomb friction and quantization are omitted, which leads to a simpler and linear model of the haptic device as shown in Fig. 3.

Inevitably time delay exists in any control loop, due to communication, computation, sensor delay or motor

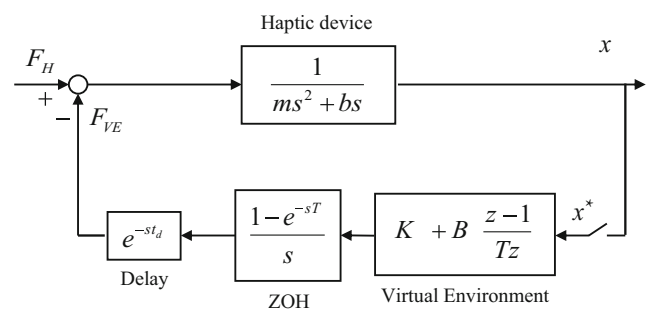


Fig. 3 1-DOF nonlinear model of the haptic device

control. Since the model of the haptic device is linear, all time delays can be added up to a single time delay of t_d [12]. Based on presented simplifications, a block diagram of the haptic device is presented in Fig. 4. Due to discrete nature of the controller, a zero-order hold sampler, with sampling time of T is also included in the control loop. In this figure x is the position of the haptic device, F_H is the force of the operator's hand and assumed to be zero, F_{VE} is the force of the virtual environment and calculated as follows:

$$F_{VE}(s) = (K + Bse^{-sT/2})e^{-st_d} \frac{1 - e^{-sT}}{sT} X(s). \quad (2)$$

3 Literature Review

Passivity is more conservative and stringent criterion than stability. A passive system cannot generate energy and can only dissipate it. As mentioned before, the operator's hand cannot insert energy to a haptic system while it is near the stability boundary. In fact, it only dissipates energy from the haptic device and is a passive element. Combination of two passive systems is a passive system too [15]. It means that if the haptic device is passive by itself, then its integration with human operator is passive too. Thus, in analyzing the passivity of haptic systems, the operator is omitted and only the haptic device is studied to be passive. Colgate *et al.* derived a passivity criterion for a haptic device with mass of m , viscous friction of b and sampling time of T , which is simulating a discrete spring of K and damper of B as follows:

$$KT/2 + |B| = b \quad (3)$$

With the aim of decreasing the differences between stability and passivity boundaries, passivity observer/ passivity controller has been designed to dissipate the net generated energy in each sampling time by a virtual damper [16]. For reducing the vibrations in the haptic system, the generated energy in a time interval (not in each sampling time) has been dissipated by the virtual damper [17].

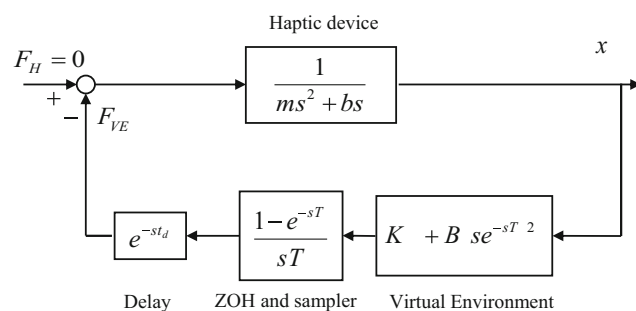


Fig. 4 Linear model of the haptic device

Stability analysis of haptic devices was first studied by Minsky *et al.* [18]. They modeled the haptic device as a linear mass and damper, considered the VE as a continuous spring with stiffness and neglected the operator, since it has positive effect on the stability. In the case of time delay equal to a sampling time, they found a linear formulation for stability which is valid only for small values of virtual stiffness, as follows:

$$K = \frac{b}{T} \quad (4)$$

By neglecting the effect of the operator, a linear formulation for stability boundary was determined in [10] and improved in [11] as following equation:

$$K = \frac{b + B}{T/2 + t_d} \quad (5)$$

where t_d is time delay. This formulation is valid only for small values of t_d and B

Experimental results show that increasing viscous friction of the robot (b), either electrically [19, 20], or magnetically [21], and also small values of virtual damping (B) has positive effect on the boundary of stability [22, 23], which is in agreement with Eq. 5. In the case of large values of B and t_d , numerical methods have been used to determine the boundary of stability [11, 12].

Recently an analytical method based on frequency analysis was presented by Mashayekhi *et al.*, which can predict the boundary of stability by following equations:

$$\begin{cases} K = T\omega^2 \times \frac{m\omega\cos((T+t_d)\omega) + b\sin((T+t_d)\omega)}{\sin(T\omega)} \\ B = T\omega \times \frac{m\omega\sin((T/2+t_d)\omega) - b\cos((T/2+t_d)\omega)}{\sin(T\omega)}, \end{cases} \quad (6)$$

where ω starts from zero and goes to:

$$\omega_{max} = \frac{1}{T+t_d} \times \frac{-(p_2 - aq_1) - \sqrt{(p_2 - aq_1)^2 - 4p_3(p_1 - a)}}{2(p_1 - a)}, \quad (7)$$

where in this equation $a = b(T + t_d)$ and $p_1 = -0.4087$, $p_2 = 1.325$, $p_3 = 0.07507$ and $q_1 = -\pi/2$ [24]. In contrast to previous analytical formulations which were valid only for small values of time delay and virtual damping, Eq. 6 is valid for small and even large values of them.

To study the stability problem in time-domain systems, Lyapunov-Krasovskii functional and Razumikhin function methods are commonly used [25]. They were presented by mathematicians Krasovskii and Razumikhin in 1950s. Before achievement of the technology of fast computation by computers, it was too difficult to construct Lyapunov-Krasovskii functional and Razumikhin functions. After development of the computers, numerical methods are used to solve Raccati equations in Linear Matrix Inequalities (LMIs), and these two methods are now widely used [26–28]. Two different criteria exist in this field: delay-independent criterion and delay-dependent criterion. In a delay-independent criterion, stability condition is valid for

all time delays. It can be easily concluded that the result is too conservative in this case, specially when time delay is too small. In delay-dependent criterion, the stability depends on time delay and the resultant stability boundaries are less conservative.

4 Novel Stability Criterion

A 1-DOF model of the haptic device is illustrated in Fig. 4. With assumption of fast sampling rate and using Taylor series expansion, it could be shown that ZOH and sampler can be approximated by a delay of $T/2$. Using this assumption, the force of virtual environment (F_{VE}) is calculated as:

$$F_{VE} = Kx(t - t_d - T/2) + B\dot{x}(t - t_d - T). \quad (8)$$

Using Newton's second law, the acceleration of the robot is:

$$\ddot{x} = -\frac{b}{m}\dot{x} - \frac{F_{VE}}{m} \quad (9)$$

By defining state variables of $x_1 = x$ and $x_2 = \dot{x}$, state-space equations can be written as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{b}{m}x_1 - \frac{K}{m}x_1(t - t_d - \frac{T}{2}) - \frac{B}{m}x_2(t - t_d - T). \end{cases} \quad (10)$$

Defining a new vector of $X = [x_1, x_2]^T$, the state space equation of the haptic device can be re-written as follows:

$$\dot{X}(t) = \sum_{i=0}^2 A_i X(t - h_i), \quad (11)$$

wherein A_0 , A_1 and A_2 matrices are defined as:

$$A_0 = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 \\ -\frac{K}{m} & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{B}{m} \end{bmatrix} \quad (12)$$

and h_0 , h_1 and h_2 are defined as:

$$h_0 = 0, h_1 = T/2 + t_d, h_2 = T + t_d \quad (13)$$

4.1 Model Transform Approach

By defining a new parameter of $Y(t) = \dot{X}(t)$, Eq. 11 can be re-written as:

$$-Y(t) + \sum_{i=0}^2 A_i X(t - h_i) = 0. \quad (14)$$

A Lyapunov-Krasovskii functional is suggested as follows [29]:

$$\begin{aligned} V(t) &= V_1(t) + V_2(t) \\ &= [X^T(t) \ Y^T(t)] E P \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} \\ &+ \sum_{i=0}^2 \int_{-h}^0 \int_{t+\theta}^t Y^T(s) R_i Y(s) ds d\theta R_i > 0, \end{aligned} \quad (15)$$

where E and P matrices are defined as:

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \quad (16)$$

with:

$$P_1 = P_1^T > 0, R_i = R_i^T > 0. \quad (17)$$

In view of definition of V_1 , V_2 , $R_i > 0$ and $P_i > 0$, it is clear that V_1 , V_2 and V are positive definite functions.

R_1, R_2, P_1, P_2, P_3 are 2×2 matrices which should be determined by LMI toolbox in MATLAB software. It should be noted that V_1 and V_2 functions are both scalar quantities. According to definition of P matrix, V_1 can be re-written as:

$$V_1(t) = X^T(t) P_1 X(t). \quad (18)$$

Time derivation of the first part of the Lyapunov function is:

$$\dot{V}_1(t) = 2 [X^T(t) \ Y^T(t)] P^T \begin{bmatrix} \dot{X}(t) \\ 0 \end{bmatrix} \quad (19)$$

Substituting Eqs. 14 and 16, 19 is simplified and reformed as:

$$\begin{aligned} \dot{V}_1(t) &= 2 [X^T(t) \ Y^T(t)] \begin{bmatrix} P_2^T \left(\sum_{i=0}^2 A_i \right) P_1 - P_2^T & \\ P_3^T \left(\sum_{i=0}^2 A_i \right) & -P_3^T \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \\ &+ 2 [X^T(t) \ Y^T(t)] P^T \begin{bmatrix} 0 \\ -\sum_{i=1}^2 A_i \int_{t-h_i}^t Y(s) ds \end{bmatrix} \end{aligned} \quad (20)$$

The first term in Eq. 20 is a scalar quantity, which can be simplified as:

$$\begin{aligned} &2 [X^T(t) \ Y^T(t)] \begin{bmatrix} P_2^T \left(\sum_{i=0}^2 A_i \right) P_1 - P_2^T & \\ P_3^T \left(\sum_{i=0}^2 A_i \right) & -P_3^T \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \\ &= [X^T(t) \ Y^T(t)] \begin{bmatrix} P_2^T \left(\sum_{i=0}^2 A_i \right) P_1 - P_2^T & \\ P_3^T \left(\sum_{i=0}^2 A_i \right) & -P_3^T \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \\ &+ \begin{bmatrix} X^T(t) \\ Y^T(t) \end{bmatrix}^T \begin{bmatrix} P_2^T \left(\sum_{i=0}^2 A_i \right) P_1 - P_2^T & \\ P_3^T \left(\sum_{i=0}^2 A_i \right) & -P_3^T \end{bmatrix}^T [X \ Y]^T \end{aligned}$$

After simplifications and substituting in Eq. 20, time derivation of the first part of Lyapunov function becomes:

$$\begin{aligned} \dot{V}_1(t) &= [X^T(t) \ Y^T(t)] \Omega \begin{bmatrix} X \\ Y \end{bmatrix} \\ &- 2 [X^T(t) \ Y^T(t)] P^T \begin{bmatrix} 0 \\ \sum_{i=1}^2 A_i \int_{t-h_i}^t Y(s) ds \end{bmatrix}, \end{aligned} \quad (21)$$

where Ω is defined as:

$$\Omega = \begin{bmatrix} P_2^T A_{012} + P_2 A_{012}^T & P_1 - P_2^T + P_3 A_{012}^T \\ P_1 - P_2 + P_3^T A_{012} & -P_3 - P_3^T \end{bmatrix} \quad (22)$$

A_{012} and A_{012}^T are:

$$A_{012} = \sum_{i=0}^2 A_i = A_1 + A_2 + A_3$$

$$A_{012}^T = \sum_{i=0}^2 A_i^T = A_1^T + A_2^T + A_3^T \quad (23)$$

From the other side, time derivation of the second term is:

$$\dot{V}_2 = \sum_{i=1}^2 \left(h_i Y^T(t) R_i Y(t) - \int_{t-h_i}^t Y^T(s) R_i Y(s) ds \right) \quad (24)$$

Then, time derivation of the total Lyapunov function becomes:

$$\begin{aligned} \dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) = \begin{bmatrix} X^T(t) & Y^T(t) \end{bmatrix} \Psi \begin{bmatrix} X \\ Y \end{bmatrix} \\ &+ 2 \begin{bmatrix} X^T(t) & Y^T(t) \end{bmatrix} P^T \begin{bmatrix} 0 \\ -\sum_{i=1}^2 A_i \int_{t-h_i}^t Y(s) ds \end{bmatrix} \\ &- \sum_{i=1}^2 \int_{t-h_i}^t Y^T(s) R_i Y(s) ds \end{aligned} \quad (25)$$

where for simplification, new function of Ψ is defined as:

$$\Psi = \begin{bmatrix} A_{012} P_2^T + P_2 A_{012}^T & P_1 - P_2^T + P_3 A_{012}^T \\ P_1 - P_2 + P_3^T A_{012} & -P_3 - P_3^T + \sum_{i=1}^2 h_i R_i \end{bmatrix} \quad (26)$$

Note that Ψ contains linear combinations of unknown matrices. Using this definition, time derivation of Lyapunov function is calculated as:

$$\begin{aligned} \dot{V}(t) &= \begin{bmatrix} X^T(t) & Y^T(t) \end{bmatrix} \Psi \begin{bmatrix} X \\ Y \end{bmatrix} \\ &+ 2 \begin{bmatrix} X^T(t) & Y^T(t) \end{bmatrix} P^T \begin{bmatrix} 0 \\ -\sum_{i=1}^2 A_i \int_{t-h_i}^t Y(s) ds \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} P_2 A_{012}^T + P_2 A_{012} & P_1 - P_2^T + P_3 A_{012}^T & h_1 P_2^T A_1 & h_2 P_2^T A_2 \\ P_1 - P_2^T + P_3^T A_{012} & -P_3 - P_3^T + h_1 R_1 + h_2 R_2 & h_1 P_3^T A_1 & h_2 P_3^T A_2 \\ h_1 A_1^T P_2 & h_1 A_1^T P_3 & -h_1 R_1 & 0 \\ h_2 A_2^T P_2 & h_2 A_2^T P_3 & 0 & -h_2 R_2 \end{bmatrix}_{8 \times 8} < 0. \quad (33)$$

Then the stability condition of the haptic device, presented by Eq. 11 is that positive matrices of P_1, R_1, R_2 and arbitrary matrices of P_2 and P_3 are found so that Eq. 33 is valid. For this purpose LMI toolbox of MATLAB software is used to find these matrices.

$$- \sum_{i=1}^2 \int_{t-h_i}^t Y^T(s) R_i Y(s) ds \quad (27)$$

In this equation, integral terms should be removed. Knowing that $\forall ab \subseteq R^n, \forall R = R^T > 0$, we have [25]:

$$-2a^T b \leq a^T R a + b^T R^{-1} b \quad (28)$$

Then a and b are chosen as follows:

$$\begin{cases} a^T = \begin{bmatrix} X^T(t) & Y^T(t) \end{bmatrix} P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix}, \\ b = Y(s). \end{cases} \quad (29)$$

Using this definition and after integrating from both sides of Eq. 28, following equation is reached:

$$\begin{aligned} &-2 \int_{t-h_i}^t \begin{bmatrix} X^T(t) & Y^T(t) \end{bmatrix} P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix} Y(s) ds \leq \\ &h_i \begin{bmatrix} X^T(t) & Y^T(t) \end{bmatrix} P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_i^{-1} \begin{bmatrix} 0 & A_i \end{bmatrix} P \begin{bmatrix} X^T(t) & Y^T(t) \end{bmatrix} \\ &+ \int_{t-h_i}^t Y^T(s) R_i^{-1} Y(s) ds. \end{aligned} \quad (30)$$

Thus integral terms of Eq. 27 are omitted and stability condition of the haptic device is determined as follows:

$$\dot{V} = \begin{bmatrix} X \\ Y \end{bmatrix}^T \left(\Psi + \sum_{i=1}^2 h_i P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_i^{-1} \begin{bmatrix} 0 & A_i^T \end{bmatrix} P \right) \begin{bmatrix} X \\ Y \end{bmatrix} < 0. \quad (31)$$

For validating (31), below equation should be valid:

$$\Psi + \sum_{i=1}^2 h_i P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_i^{-1} \begin{bmatrix} 0 & A_i^T \end{bmatrix} P < 0. \quad (32)$$

It is observed that Ψ matrix is a linear combination of unknown terms. But the second term in Eq. 32 is multiple of three unknown matrices, thus it is nonlinear. Using Schur complement, stability condition is determined as (33):

In view of the presented theory, the following steps should be fulfilled to find the boundary of stability in a haptic system:

1. The numerical values of m, b, T and t_d are identified.

- For each value of virtual damping (B), an initial guess for virtual stiffness (K) is chosen.
- Substituting numerical values of all parameters into Eqs. 12 and 13, numerical values of A_0 , A_1 and A_2 matrices and h_1 , h_2 are determined.
- LMI Toolbox of MATLAB is utilized to find P_1 , P_2 , P_3 , R_1 and R_2 matrices, so that Eqs. 17 and 33 are valid.
- If these matrices are found under mentioned assumption, then the operating point of (B , K) is stable. If not, this point is unstable.
- In the case of stability (instability), value of K for next iteration is increased (decreased), and steps 4 and 5 are repeated until the boundary of the stability is found. The boundary of stability is defined as a stable point which its difference with a nearest unstable point is smaller than a small and fixed value.
- B is changed, so that the whole boundary is reached.

Using numerical values of $T = 1 \text{ ms}$, $b = 1 \text{ Ns/m}$ and $m = 1 \text{ Kg}$, Eq. 33 is solved to find the boundary of stability in different time delays. For better presentation of the results, dimensionless parameters are used as listed in Table 1.

Following the mentioned steps, dimensionless virtual stiffness is plotted versus dimensionless virtual damping for small and large time delays, in Figs. 5 and 6, respectively.

From Figs. 5 and 6 it is concluded that increasing time delay has negative effect on the boundary of stability and makes the system unstable.

4.2 Free Weighting Matrix Approach (FWM)

A Lyapunov-Krasovskii functional is defined as follows [30]:

$$\begin{aligned}
 V(t) = & X^T(t)PX(t) + \int_{t-h_1}^t X^T(s)Q_1X(s)ds \\
 & + \int_{t-h_2}^t X^T(s)Q_2X(s)ds \\
 & + \int_{-h_1}^0 \int_{t+\theta}^t \dot{X}^T(s)W_1\dot{X}(s)dsd\theta
 \end{aligned}$$

Table 1 Dimensionless parameters

Parameter	Variable	Dimensionless variable
Sampling time	T	
Mass	m	
Delay	t_d	$d = t_d/T$
Virtual stiffness	K	$\alpha = KT^2/m$
Virtual damping	B	$\beta = BT/m$
Physical damping	b	$\delta = bT/m$

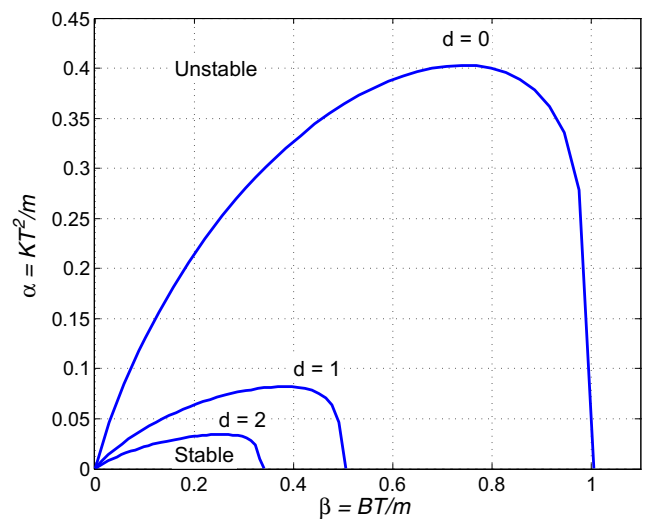


Fig. 5 Dimensionless virtual stiffness versus dimensionless virtual damping, for small values of time delay, determined from model transform

$$\begin{aligned}
 & + \int_{-h_2}^0 \int_{t+\theta}^t \dot{X}^T(s)W_2\dot{X}(s)dsd\theta \\
 & + \int_{-h_2}^{-h_1} \int_{t+\theta}^t \dot{X}^T(s)W_3\dot{X}(s)dsd\theta
 \end{aligned} \quad (34)$$

If there exist positive definite matrices Q_1 , Q_2 and P , and positive semi-definite matrices W_i , X_{ii} , Y_{ii} and Z_{ii} for each $i = 1, 2, 3$, and any arbitrary matrices N_i , S_i , T_i , X_{ij} , Y_{ij} and Z_{ij} with $1 \leq i < j \leq 3$ such that the following LIMs hold [30], then the system presented by Eq. 11, with

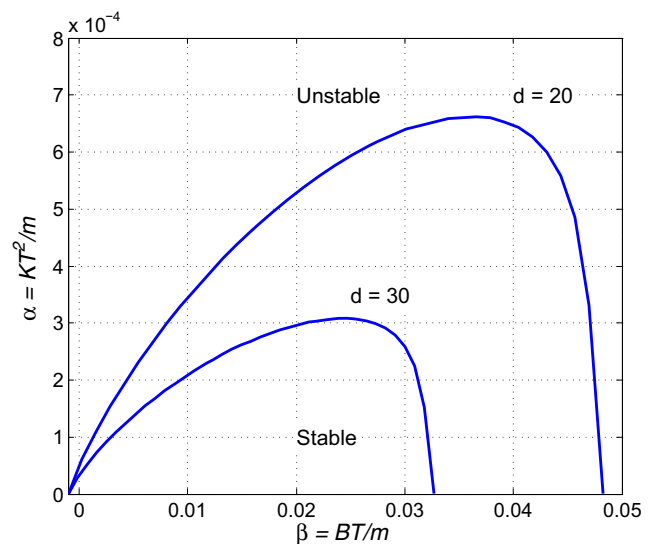


Fig. 6 Dimensionless virtual stiffness versus dimensionless virtual damping, for large values of time delay, determined from model transform

Lyapunov function of Eq. 34 is asymptotically stable:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ \Phi_{12}^T & \Phi_{22} & \Phi_{23} \\ \Phi_{13}^T & \Phi_{23}^T & \Phi_{33} \end{bmatrix} < 0, \quad (35)$$

$$\Psi_1 = \begin{bmatrix} X_{11} & X_{12} & X_{13} & N_1 \\ X_{12}^T & X_{22} & X_{23} & N_2 \\ X_{13}^T & X_{23}^T & X_{33} & N_3 \\ N_1^T & N_2^T & N_3^T & W_1 \end{bmatrix} \geq \quad (36)$$

$$\Psi_2 = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & S_1 \\ Y_{12}^T & Y_{22} & Y_{23} & S_2 \\ Y_{13}^T & Y_{23}^T & Y_{33} & S_3 \\ S_1^T & S_2^T & S_3^T & W_2 \end{bmatrix} \geq \quad (37)$$

$$\Psi_3 = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & -T_1 \\ Z_{12}^T & Z_{22} & Z_{23} & -T_2 \\ Z_{13}^T & Z_{23}^T & Z_{33} & -T_3 \\ -T_1^T & -T_2^T & -T_3^T & W_3 \end{bmatrix} \geq \quad (38)$$

where:

$$\begin{aligned} \Phi_{11} &= PA_0 + A_0^T P + Q_1 + Q_2 + N_1 + N_1^T + S_1 \\ &+ S_1^T + A_0^T H A_0 + h_1 X_{11} + h_2 Y_{11} + (h_2 - h_1) Z_{11} \\ \Phi_{12} &= PA_1 - N_1 + N_2^T + S_2^T - T_1 + A_0^T H A_1 \\ &+ h_1 X_{12} + h_2 Y_{12} + (h_2 - h_1) Z_{12} \\ \Phi_{13} &= PA_2 + N_3^T + S_3^T - S_1 + T_1 + A_0^T H A_2 \\ &+ h_1 X_{13} + h_2 Y_{13} + (h_2 - h_1) Z_{13} \\ \Phi_{22} &= -Q_1 - N_2 - N_2^T - T_2 - T_2^T + A_1^T H A_1 \\ &+ h_1 X_{22} + h_2 Y_{22} + (h_2 - h_1) Z_{22} \\ \Phi_{23} &= -N_3^T - S_2 + T_2 - T_3^T + A_1^T H A_2 \\ &+ h_1 X_{23} + h_2 Y_{23} + (h_2 - h_1) Z_{23} \\ \Phi_{33} &= -Q_2 - S_3 - S_3^T + T_3 + T_3^T + A_2^T H A_2 \\ &+ h_1 X_{33} + h_2 Y_{33} + (h_2 - h_1) Z_{33} \end{aligned}$$

$$H = h_1 W_1 + h_2 W_2 + (h_2 - h_1) W_3 \quad (39)$$

It is clear that with assumptions of $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $W_1 \geq 0$, $W_2 \geq 0$, $W_3 \geq 0$, Lyapunov function of $V(t)$ is a positive definite function ($V > 0$). Now is it proved that based on defined matrices, time derivation of this Lyapunov function is negative ($\dot{V} < 0$). It means that $X(t)$ and $\dot{X}(t)$ will converge to zero, and the haptic device is asymptotically stable. For this purpose, time derivation of V is calculated as:

$$\begin{aligned} \dot{V}(t) &= 2X^T(t)P[A_0X(t) + A_1X(t-h_1) + A_2X(t-h_2)] \\ &+ X^T(t)Q_1X(t) - X^T(t-h_1)Q_1X(t-h_1) \\ &+ X^T(t)Q_2X(t) - X^T(t-h_2)Q_2X(t-h_2) \\ &+ (h_2 - h_1)\dot{X}^T(t)W_3\dot{X}(t) - \int_{t-h_2}^{t-h_1} \dot{X}^T(s)W_3\dot{X}(s)ds \end{aligned}$$

$$\begin{aligned} &+ h_1\dot{X}^T(t)W_1\dot{X}(t) - \int_{t-h_1}^t \dot{X}^T(s)W_1\dot{X}(s)ds \\ &+ h_2\dot{X}^T(t)W_2\dot{X}(t) - \int_{t-h_2}^t \dot{X}^T(s)W_2\dot{X}(s)ds \end{aligned} \quad (40)$$

From the other side, below equations are always valid:

$$\begin{aligned} X(t) - X(t-h_1) - \int_{t-h_1}^t \dot{X}(s)ds &= 0, \\ X(t) - X(t-h_2) - \int_{t-h_2}^t \dot{X}(s)ds &= 0, \\ X(t-h_2) - X(t-h_1) - \int_{t-h_2}^{t-h_1} \dot{X}(s)ds &= 0. \end{aligned} \quad (41)$$

Thus for each matrix of N_i ($i = 1, 2, 3$) Eq. 42 is valid.

$$\begin{aligned} &2[X^T(t)N_1 + X^T(t-h_1)N_2 + X^T(t-h_2)N_3] \\ &\times [X(t) - X(t-h_1) - \int_{t-h_1}^t \dot{X}(s)ds] = 0. \end{aligned} \quad (42)$$

Similar equations like Eq. 42 are used, while using T_i and S_i instead of N_i , without writing to shortening the paper.

From the other side, for any arbitrary matrices of $X_{ii} = X_{ii}^T \geq 0$, $Y_{ii} = Y_{ii}^T \geq 0$ and $Z_{ii} = Z_{ii}^T \geq 0$ ($i = 1, 2, 3$) and $X_{ij}Y_{ij}$ and Z_{ij} ($1 \leq i < j \leq 3$) Eq. 43 is valid too:

$$\begin{bmatrix} X(t) \\ X(t-h_1) \\ X(t-h_2) \end{bmatrix}^T \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12}^T & \Gamma_{22} & \Gamma_{23} \\ \Gamma_{13}^T & \Gamma_{23}^T & \Gamma_{33} \end{bmatrix} \begin{bmatrix} X(t) \\ X(t-h_1) \\ X(t-h_2) \end{bmatrix} = 0, \quad (43)$$

where each element is defined as follows (each elements equals to zero):

$$\begin{aligned} \Gamma_{ij} &= h_1(X_{ij} - X_{ij}) + h_2(Y_{ij} - Y_{ij}) \\ &+ (h_1 - h_2)(Z_{ij} - Z_{ij}), \quad (1 \leq i < j \leq 3). \end{aligned} \quad (44)$$

According to $\int_{t-r}^t f(t)ds = rf(t)$ and adding left hand-side terms of Eqs. 42–43 to $\dot{V}$, it is simplified as follows:

$$\begin{aligned} \dot{V}(t) &= \xi_1^T(t)\Phi\xi_1(t) \\ &- \int_{t-h_1}^t \xi_2^T(ts)\Psi_1\xi_2(ts)ds \\ &- \int_{t-h_2}^t \xi_2^T(ts)\Psi_2\xi_2(ts)ds \\ &- \int_{t-h_2}^{t-h_1} \xi_2^T(ts)\Psi_3\xi_2(ts)ds \end{aligned} \quad (45)$$

where $\xi_1(t)$ and $\xi_2(ts)$ are defined as following equations:

$$\begin{aligned} \xi_1(t) &= [X^T(t), X^T(t-h_1), X^T(t-h_2)]^T \\ \xi_2(ts) &= [\xi_1^T(t), \dot{X}^T(s)]^T \end{aligned} \quad (46)$$

and $\Phi, \Psi_1, \Psi_2, \Psi_3$ matrices were defined before by Eqs. 35–38.

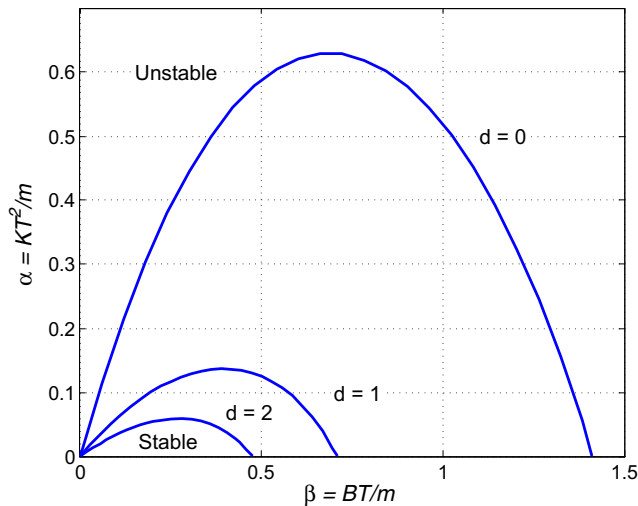


Fig. 7 Dimensionless virtual stiffness versus dimensionless virtual damping, for small values of time delay, determined from FWM method

It can be seen that with assumption of $\Phi < 0$ and $\Psi_i \geq 0$, ($i = 1, 2, 3$) time derivation of the Lyapunov function is negative. Thus stability condition of the system described by Eq. 11 is that $\Phi < 0$ and $\Psi_i \geq 0$, ($i = 1, 2, 3$) matrices define by Eqs. 35 to 38 must be found by linear matrix inequality toolbox of MATLAB software.

The steps for plotting the stability boundary are similar to the steps of the earlier method. The only difference is that in step 4, Φ and Ψ_i ($i = 1, 2, 3$) matrices should be found instead of P_1, P_2, P_3, R_1 and R_2 matrices.

Using this method, dimensionless virtual stiffness is plotted versus dimensionless virtual damping for small and large time delays, in Figs. 7 and 8, respectively.

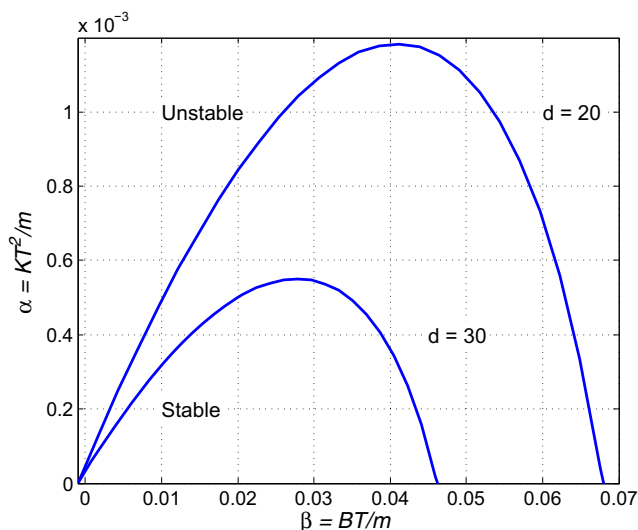


Fig. 8 Dimensionless virtual stiffness versus dimensionless virtual damping, for large values of time delay, determined from FWM method

In the next sections these two methods are compared with simulation and experiments.

5 Simulations

Several simulations were performed to check the validity of presented approaches in determining the stability boundary, and to compare the methods. For this purpose a Simulink model similar to the model illustrated in Fig. 3 was created in MATLAB software. The parameters of Freedom 6 haptic device were given as the system parameters, including the effective mass of $m = 0.250 \text{ Kg}$, effective viscous friction of $b = 0.01 \text{ Ns/m}$ and sampling time of $T = 1 \text{ ms}$ [9] and results are presented in dimensionless format. For each value of B , an initial guess for K was chosen and simulation was performed. If the haptic system was stable, in next iteration a larger value for K was chosen; and vice versa. Iterations were repeated until the stability boundary was identified with a desired accuracy.

Figure 9 compares simulation result with theoretical results of *model transform* and *FWM* methods for small time delays of $d = [0, 2]$. It can be observed that both theoretical stability boundaries are inside simulation boundaries. Particularly, the stability boundary of *model transform* result is more conservative than *FWM* method.

In most real applications, small values of virtual damping are used. In this case a zoomed-view of this figure is depicted in Fig. 10. It can be seen that for small values of virtual damping, the error between the results of *FWM* and simulations becomes smaller, while *model transform* results are a little conservative.

For bigger time delays, these two methods are compared with simulations in Figs. 11 and 12, for large and small values of virtual damping respectively.

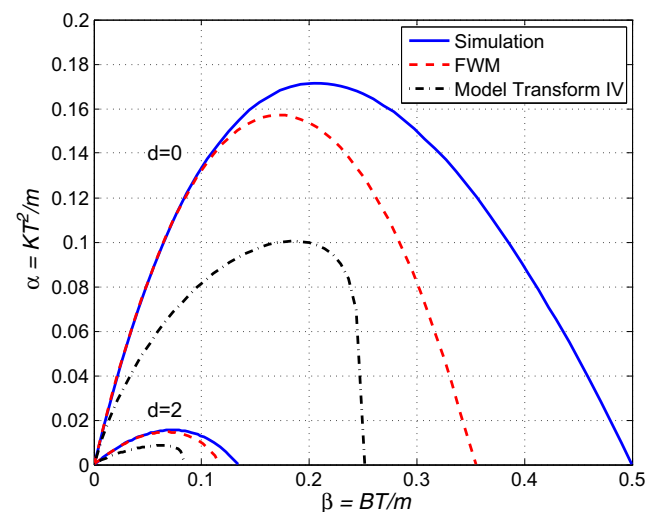


Fig. 9 Dimensionless virtual stiffness versus dimensionless virtual damping, for small values of time delay

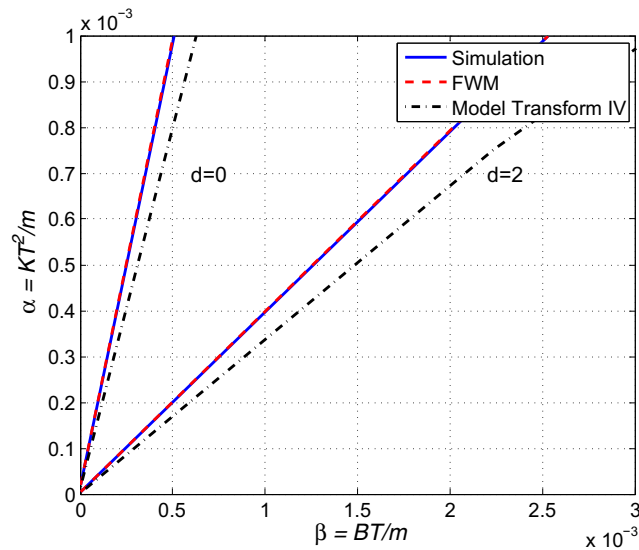


Fig. 10 Dimensionless virtual stiffness versus dimensionless virtual damping, for small values of time delay

From Figs. 11 and 12 it is concluded that conservatisms of these two methods is not related to the time delay. In fact, for both small and large values of time delay, results of *FWM* method are a little and results of *model transform* method are more conservative.

6 Extension to Multi DOF Haptic Device

Assuming a serial multi degree of freedom haptic device, with n joints and without redundancy *i.e.* $n \leq 6$, its nonlinear dynamic can be written as follows:

$$M(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau_{ext} \quad (47)$$

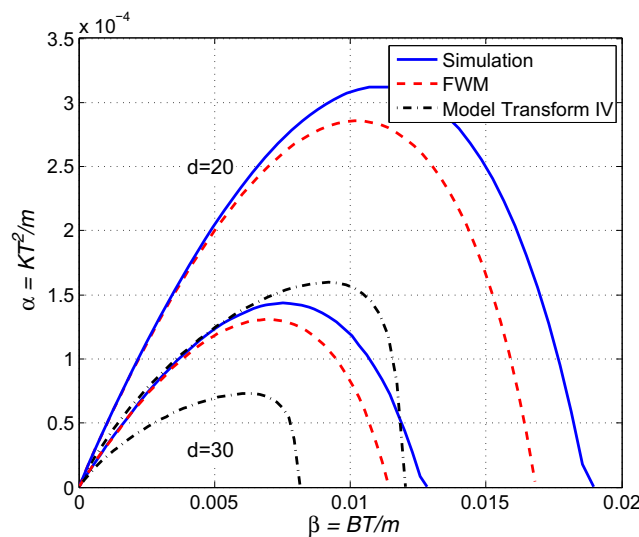


Fig. 11 Dimensionless virtual stiffness versus dimensionless virtual damping, for large values of time delay

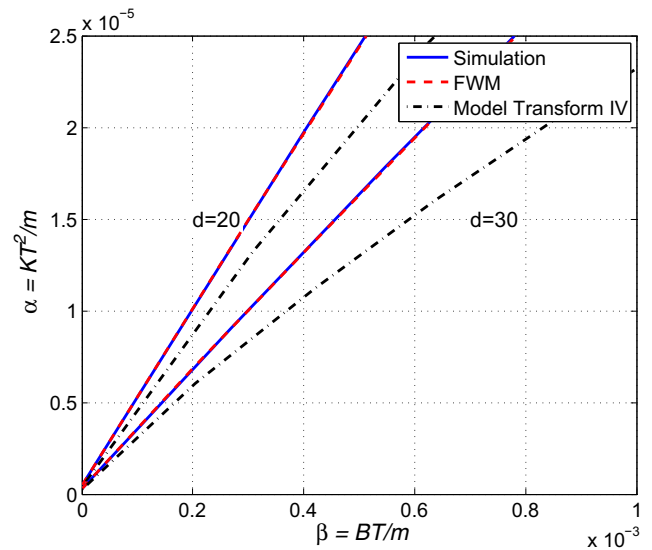


Fig. 12 Dimensionless virtual stiffness versus dimensionless virtual damping, for large values of time delay

where $q \in R^n$ is the vector of joint variables, $M(q)$ is the mass/inertia matrix, $C(q, \dot{q})$ is the vector of centrifugal/Coriolis torques, $G(q)$ is the gravitational vector and τ_{ext} is external applied torque (from virtual environment or human operator). In haptic applications, gravity torques are calculated based on joint variables and compensated by actuators of the haptic device, so $G(q)$ could be neglected.

When simulating a stiff virtual object, the penetration depth in virtual environment is small (almost less than one or two millimeters), which leads to small variations in the joints variables. Thus its nonlinear dynamic, can be linearized in the operating point, and written in state-space form as following equation:

$$\dot{X}^*(t) = \sum_{i=0}^2 A_i^* X^*(t - h_i). \quad (48)$$

In this case A_i^* and X^* are obviously different from 1-DOF case and have size of $(2n) \times (2n)$ and $2n \times 1$ respectively.

Following the mentioned steps in Section 4.1, stability boundaries can be determined, but with more calculations. In fact using *model transform* approach [29], matrices of R_1 , R_2 and P_1 (defined in Eq. 17) are symmetric with dimension of $2n \times 2n$, while P_2 and P_3 are arbitrary matrices with the same dimension. Then $6n^2 + 3n$ unknown elements for all symmetric matrices and $8n^2$ elements for all arbitrary matrices have to be determined which means a total number of $14n^2 + 3n$ unknown elements.

Using *model transform* method [30], 15 symmetric matrices with dimension of $2n \times 2n$ and 18 arbitrary matrices with the same dimension should be determined. It leads to total number of $102n^2 + 15n$ unknown parameter.

It can be easily concluded that when using a multi degree of freedom model for the haptic device, cause to increase

the unknown parameters rapidly significantly increases with the number of degrees of freedoms (n). Thus in the case of multi DOF dynamic, calculating the boundary of stability is time consuming.

When simulating a virtual object, due to small variations in joints variables, effective mass and effective damping can be calculated around the operating point, based on equivalent kinetic energy and equivalent dissipated energy, respectively.

Suppose that the end effector of the robot is moving with velocity of $\dot{\vec{s}}$, in direction of $\vec{s}$, and around a fixed known point in space (P). Using inverse kinematics, joint space variables (q) can be easily determined, and also joint space velocity is calculated as $\dot{q} = J^{-1}\dot{\vec{s}}$, where J is Jacobian matrix and $q = [\theta_1, \theta_1, \dots, \theta_n]^T$ is vector of joint variables. In each configuration, total kinetic energy can be calculated as $\frac{1}{2}\dot{q}^T M(q)\dot{q}$, where $M(q)$ is mass/inertia matrix in that configuration [31]. Replacing this total kinetic energy with kinetic energy of an effective mass of m , which is moving with velocity of $\dot{\vec{s}}$, in location of P , then effective mass can be calculated as follows:

$$m = \frac{\dot{q}^T M(q) \dot{q}}{\dot{s}^2} \quad (49)$$

The effective mass is independent of velocity and depends only on the position/orientation of the end effector and robot configuration.

From the other side, dissipated power in i^{th} joint is $b_i \dot{\theta}_i^2$, where $\dot{\theta}_i$ and b_i are angular velocity and physical damping of i^{th} joint, respectively. Then effective linear damping (b) can be calculated from the following equation:

$$\sum_{i=1}^n b_i \dot{\theta}_i^2 = b \dot{s}^2 \quad (50)$$

where the left term is summation of total dissipations in all joints and the right term is its equivalent dissipation.

Based on this concept, in most of haptic applications, the multi DOF dynamic is simplified to a 1 DOF dynamic with an equivalent mass and damper; then stability boundaries are determined using different approaches [10–12, 24, 32].

7 Experiments

A KUKA Light Weight Robot 4 was utilized to perform experiments for validating the theoretical results. This robot has seven degrees of freedom and is used widely in impedance control applications [33]. Nonlinear effects such as Coulomb friction and quantization are negligible due to high resolution in sensors. This robot is a very efficient and portable robot, weighting only 16 kg, with a payload of 7 kg. Motors, gears and sensors are accommodated inside an aluminum housing, as well as the necessary control and power electronic circuits. The redundant DoF provides the

arm with additional flexibility, and can be used in different ways, based on the task at the hand. It can be exploited to avoid obstacle, to increase the manipulability, or in general to obtain more favorable motions for the desired task.

Technical details and specifications for this device can be found in [34]. The control of the KUKA arm has been implemented by taking advantage of the FRI (*Fast Research Interface*) library, available at [35], which provides a simple user interface for communicating with the robot. The library runs on a remote PC, that is connected to the KRC (*KUKA Robot Controller*) via Ethernet.

During the experiments the first joint of this robot was utilized for simulating a virtual environment, while all other joints were fixed with large controller gains Fig. 13.

For each value of B , an initial guess for K was chosen. The robot started to move from an initial position to see the behavior of its oscillations. The process of finding the boundary of the stability is similar to the method used in the simulations.

The sampling time was set to be $2ms$ and different time delays of $t_d = [20, 50]ms$ were selected. The obtained results are presented in Fig. 14 and compared with theoretical results of *FWM* and *model transform* methods.

From this figure it is clear that while using a small time delay (e.g. $t_d = 20ms$), the stability curve is very large and only a small area of it is visible in this figure. It should be mentioned that reaching to higher stiffness in the experiments, cause severe vibrations in the robot

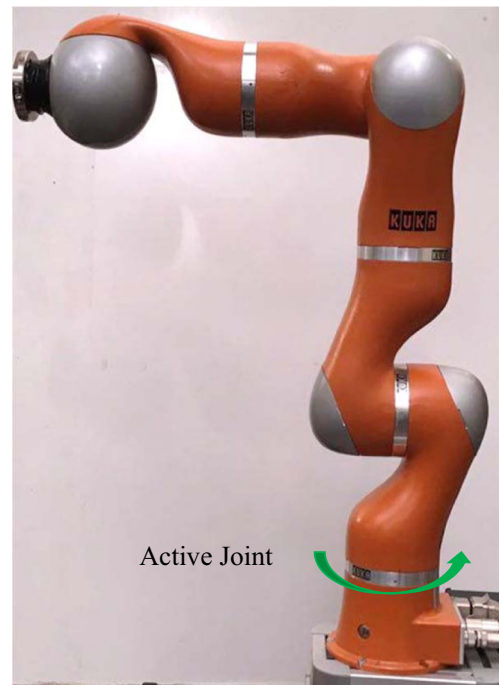


Fig. 13 Experimental and theoretical stability boundary

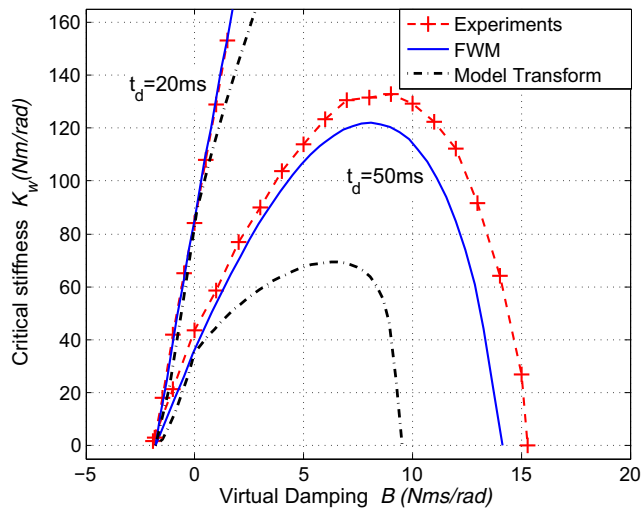


Fig. 14 Experimental and theoretical stability boundary

and may damage it; so the maximum stiffness was limited to 160Nm/rad in the experiment for the sake of the safety. In this small area, both *FWM* and *Model Transform* methods can predict the boundary of stability with good precision. In fact for small values of virtual damping, both presented methods can predict the boundary of stability with good precision, which is consistent with simulation results depicted in Figs. 10 and 12.

In the case of larger time delay (e.g. $t_d = 50\text{ms}$), the whole stability curve can be seen. In this case, experimental results are consistent with *FWM* method but somehow different from *Model Transform*. Then it is concluded that *FWM* has better precision for predicting the boundary of stability, for small and large values of time delay and virtual damping, in comparison to *Model Transform*.

This robot is also used to determine the hand impedance and its effect on the boundary of stability, as depicted in Fig. 15 [36].

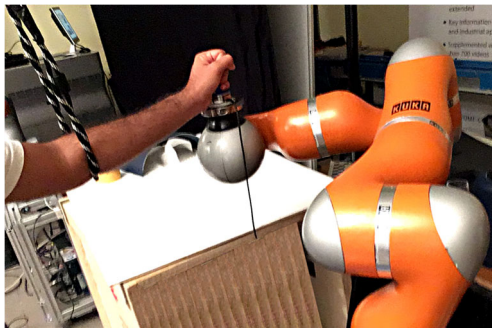


Fig. 15 Experimental setup for human hand identification and stability analysis, using KUKA Lightweight 4 robot

8 Conclusion

The stability problem of a haptic device, while simulating a stiff object was studied. Two different Lyapunov-Krazuvskii functionals were utilized in theoretical study. The first method (*model transform*) has five unknown matrices with dimensions of 2×2 , where three of them were symmetric and the rest two were non-symmetric. Then total number of 17 elements of the matrices should be determined. Whereas in the second method, 117 elements should be determined. Thus using the *FWM* method, needs more calculations and therefore needs more time to determine the boundary of stability. However, using *FWM* leads to more accurate results. The LMIs were solved by LMI toolbox of MATLAB and compared with simulation and experimental results on a KAKA Light Weight Robot.

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