

AN ADAPTIVE FORCE/POSITION REGULATOR FOR ROBOT MANIPULATORS

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SUMMARY

An adaptive force/position regulator for robot manipulators in contact with an elastically compliant surface is proposed in this paper. The controller consists of a PD action on the position loop, a PI action on the force loop, together with estimated gravity compensation and desired contact force feedforward. The closed-loop system is shown to have a unique equilibrium state and asymptotic stability in the neighbourhood of this state is proven via the classical Lyapunov method. To cope with imperfect gravity compensation, the controller is made adaptive with respect to a suitable set of parameters in the gravity term. Numerical case studies are developed for a three-joint elbow manipulator.

KEY WORDS Robots Force/position regulation PID control Stability Lyapunov methods Adaptive control

1. INTRODUCTION

A fundamental requirement for the success of a manipulation task is the capability to handle interaction between the manipulator and the environment. Typically the quantity that describes the status of interaction most effectively is the contact force at the manipulator end-effector. High values of contact force are generally undesirable since they may stress both the manipulator and the environment. The use of a purely motion control strategy for controlling the interaction usually gives degraded performance and can even cause instability.¹

If force sensor information is not available for control purposes, the interaction can be controlled indirectly by assigning a suitable dynamic behaviour between position and force variables at the contact (e.g. impedance control).^{2,3}

On the other hand, if contact force measurements are suitably embedded into the controller, both position and force variables can be controlled even if it is not possible to simultaneously impose arbitrary values of position and force along each task direction. The most widely adopted solution to this problem is the so-called hybrid control strategy^{4–7} in which position and force are controlled along orthogonal directions through the use of selection matrices operating on both reference and feedback quantities. Stability of hybrid control was investigated by Wen and Murphy,⁸ while an adaptive hybrid controller was proposed by Lozano and Brogliato.⁹ The problem of force/position control with force sensory feedback was also studied for the general case of constrained motion tasks.^{10,11}

A conceptually different approach to force/position control of robot manipulators is the parallel control strategy.¹² As opposed to the hybrid control strategy, both force and position

variables are handled along the same task space direction without any selection mechanism. The effectiveness of the scheme is ensured by the dominance of the force control loop over the position control loop along the task directions where interaction occurs. This makes the scheme suitable to manage contacts with unstructured environment and unplanned collisions, which are known to represent a drawback for hybrid controllers. Extensive description of the parallel approach and performance of a control scheme with full dynamic compensation in the case of contact with an elastically compliant frictionless surface can be found in Reference 13.

A regulator based on the parallel control concept was recently proposed by Chiaverini and Siciliano¹⁴ which comprises position PD action + gravity compensation + desired force feed-forward + force PI action. Both asymptotic and exponential stability can be shown using the Lyapunov direct method; the result is local in that boundedness of the initial state is required.¹⁵ For given force and position setpoints along the constrained task directions the force error is asymptotically driven to zero at the expense of a steady state position error.

In this paper the assumption of perfect gravity compensation is relaxed. First it is shown that a different equilibrium state exists and is unique; a position error occurs also along the unconstrained task directions. Then the closed-loop system is shown to converge asymptotically to this state via the classical Lyapunov method. In order to overcome the drawback of unavoidably imperfect estimation of the gravity term, a parameter adaptation law is added to the PID force/position controller; this is made possible by a suitable linear parametrization of the gravity term. The stability analysis leads to the derivation of conditions to be satisfied by the PD position feedback gains and the PI force feedback gains.

Simulation results are presented for a three-joint elbow manipulator; the dynamic model parameters are those of the first three joints of the industrial robot COMAU SMART 6.10R. A numerical case study with an unknown payload mass confirms the good performance of the PID force/position controller with gravity parameter adaptation in comparison with the non-adaptive controller.

2. MODELLING

The dynamic model of a robot manipulator constrained by the environment can be effectively written in the operational (Cartesian) space⁵ in the form

$$\mathbf{B}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{g}(\mathbf{x}) = \mathbf{u} - \mathbf{f} \quad (1)$$

where $\mathbf{x}$ is the $m \times 1$ vector of end-effector location, $\mathbf{B}$ is the $m \times m$ inertia matrix, $\mathbf{C}\dot{\mathbf{x}}$ is the $m \times 1$ vector of Coriolis and centrifugal generalized forces, $\mathbf{g}$ is the $m \times 1$ vector of gravitational generalized forces, $\mathbf{u}$ is the $m \times 1$ vector of driving generalized forces and $\mathbf{f}$ is the $m \times 1$ vector of contact generalized forces exerted by the manipulator on the environment; all operational space quantities are expressed in a common reference frame.

When m is equal to the number of joints, n , and the manipulator moves in a singularity-free region of the workspace, the vector of operational variables constitutes a set of Lagrangian generalized co-ordinates and $\mathbf{B}$ assumes the meaning of a true inertia matrix. In contrast, in the case of kinematically redundant manipulators ($m < n$), $\mathbf{B}$ is only a pseudo inertia matrix.⁵

The $n \times 1$ vector $\boldsymbol{\tau}$ of joint actuating generalized forces is computed as

$$\boldsymbol{\tau} = \mathbf{J}^T \mathbf{u} \quad (2)$$

where $\mathbf{J}$ is the $m \times n$ manipulator Jacobian matrix.

The following notable properties of the dynamic model (1) can be established.

- (i) The matrix $\mathbf{B}$ is symmetric and positive definite. If λ_m (λ_M) denotes the minimum (maximum) eigenvalue of $\mathbf{B}$, then

$$0 < \lambda_m \mathbf{I} \leq \mathbf{B} \leq \lambda_M \mathbf{I} \quad (3)$$

where $\mathbf{I}$ is the $m \times m$ identity matrix; in the case of all joints being revolute, $\lambda_M < \infty$.¹⁶

- (ii) There exists a choice of matrix $\mathbf{C}$ such that the matrix

$$\mathbf{S}(\mathbf{x}, \dot{\mathbf{x}}) = \dot{\mathbf{B}}(\mathbf{x}) - 2\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}}) \quad (4)$$

is skew-symmetric.¹⁷ This also implies that

$$\dot{\mathbf{B}}(\mathbf{x}) = \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{C}^T(\mathbf{x}, \dot{\mathbf{x}}) \quad (5)$$

Further, the matrix $\mathbf{C}$ is upper-bounded in $\mathbf{x}$ and linear in $\dot{\mathbf{x}}$;¹⁸ hence a constant $k_C > 0$ exists such that

$$\|\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\| < k_C \|\dot{\mathbf{x}}\| \quad (6)$$

- (iii) The gravity force vector $\mathbf{g}$ can be thought of as being given by

$$\mathbf{g}(\mathbf{x}) = \partial U(\mathbf{x}) / \partial \mathbf{x} \quad (7)$$

where $U(\mathbf{x})$ is the gravitational energy in the operational space, which is bounded for any $\mathbf{x}$. A helpful property of the gravity term is that $\mathbf{g}$ has bounded partial derivatives,¹⁸ i.e.

$$\|\partial \mathbf{g}(\mathbf{x}) / \partial \mathbf{x}\| \leq \delta \quad (8)$$

for some $\delta > 0$. This also implies that

$$\|\mathbf{g}(\mathbf{x}_1) - \mathbf{g}(\mathbf{x}_2)\| \leq \delta \|\mathbf{x}_1 - \mathbf{x}_2\| \quad (9)$$

- (iv) The robot dynamics (1) can be cast in a convenient form which is linear with respect to a set of suitable constant parameters such as masses, moments of inertia, etc.^{19,20} With specific reference to the gravity term, it is quite easy to show that the vector $\mathbf{g}$ in (1) can be written in the form

$$\mathbf{g}(\mathbf{x}) = \mathbf{G}(\mathbf{x})\boldsymbol{\theta} \quad (10)$$

where $\mathbf{G}$ is a $3 \times p$ matrix, a function of end-effector position $\mathbf{x}$, and $\boldsymbol{\theta}$ is a $p \times 1$ vector of manipulator and load constant parameters.

For the purpose of the present work, attention is restricted to the case of non-redundant, non-singular manipulators with $m = n = 3$, i.e. only translational motion and linear force components are considered. Then $\mathbf{x}$ denotes the end-effector position and $\mathbf{f}$ denotes the end-effector linear forces at the contact.

Accurate modelling of the contact between the manipulator and the environment is usually difficult to obtain in analytic form owing to the complexity of the physical phenomena involved during the interaction. It is then reasonable to resort to a simple but significant model relying on the robustness of the control system in order to absorb the effects of inaccurate modelling.

Upon these premises the case of an environment constituted by a rigid, frictionless and elastically compliant plane is analysed.¹²⁻¹⁴ The model of the contact force takes on the simple

form

$$\mathbf{f} = \mathbf{K}(\mathbf{x} - \mathbf{x}_0) \quad (11)$$

where $\mathbf{x}$ is the position of the contact point, $\mathbf{x}_0$ is a point of the plane at rest and $\mathbf{K}$ is the 3×3 constant symmetric *stiffness matrix*²¹ that establishes a linear mapping between $\mathbf{x} - \mathbf{x}_0$ and $\mathbf{f}$; notice that equation (11) holds only when the manipulator is in contact with the environment and all quantities are expressed in a common reference frame. Moreover, observe the following.

- (i) The contact force is orthogonal to the plane for any vector $\mathbf{x} - \mathbf{x}_0$; then a base of $\mathcal{R}(\mathbf{K})$, where $\mathcal{R}(\mathbf{K})$ denotes the range space of matrix $\mathbf{K}$, is the unit vector $\mathbf{n}$ orthogonal to the plane and $\text{rank}(\mathbf{K}) = 1 < 3$.
- (ii) All vectors $\mathbf{x} - \mathbf{x}_0$ lying on the plane do not contribute to the contact force; then a base of $\mathcal{N}(\mathbf{K})$, where $\mathcal{N}(\mathbf{K})$ denotes the null space of matrix $\mathbf{K}$, is a pair of linearly independent unit vectors $(\mathbf{p}_1, \mathbf{p}_2)$ tangential to the plane.
- (iii) Because of the symmetry of $\mathbf{K}$, $\mathcal{R}(\mathbf{K}) \equiv \mathcal{R}(\mathbf{K}^T)$ and a convenient choice for $(\mathbf{p}_1, \mathbf{p}_2)$ is such that the columns of the matrix

$$\mathbf{R} = (\mathbf{p}_1 \ \mathbf{p}_2 \ \mathbf{n}) \quad (12)$$

form a set of orthonormal vectors constituting a base of $\mathbb{R}^3$.

According to the above remarks, the matrix $\mathbf{K}$ can be decomposed as

$$\mathbf{K} = \mathbf{R} \text{diag}(0, 0, k) \mathbf{R}^T = k \mathbf{n} \mathbf{n}^T \quad (13)$$

where $\mathbf{R}$, defined in (12), is the rotation matrix from the frame attached to the plane to the reference frame; then $k > 0$ is the stiffness coefficient characterizing the contact, which acts along the direction orthogonal to the plane.

3. PID FORCE/POSITION REGULATOR

A typical regulation task for a robot manipulator in contact with the environment can be prescribed in terms of a position setpoint $\mathbf{x}_d$ and a force setpoint $\mathbf{f}_d$. It can be recognized that in general it is not possible to simultaneously achieve both setpoints.

An effective strategy to comply with the task geometry is the parallel control approach,^{12,13} which offers good robustness to inaccurate contact modelling. The key feature is to have a force control loop working in parallel with a position control loop along each task space direction. The logical conflict between the two loops is managed by imposing dominance of the force control action over the position one.

A force/position regulator in the parallel control framework for system (1) was proposed¹⁴ based on position PD action + gravity compensation + desired force feedforward + force PI action, i.e.

$$\mathbf{u} = k_P \Delta \mathbf{x} - k_D \dot{\Delta \mathbf{x}} + \mathbf{g}(\mathbf{x}) + \mathbf{f}_d + k_F \Delta \mathbf{f} + k_I \int_0^t \Delta \mathbf{f} \, d\sigma \quad (14)$$

where $\Delta \mathbf{x} = \mathbf{x}_d - \mathbf{x}$ is the position error, $\Delta \mathbf{f} = \mathbf{f}_d - \mathbf{f}$ is the force error and $k_P, k_D, k_F, k_I > 0$ are suitable feedback gains.

It is important to remark that no exact knowledge of the stiffness coefficient k is required by control (14), although it is expected that the actual value of environment stiffness affects the performance of the closed-loop system. Notice that, differently from Chiaverini and

Sciavicco,¹² no full dynamic model compensation is required, since the controller is formed by linear actions together with gravity compensation only.

Looking at the properties of elastic contact model (11), the only possibility of obtaining a null force error is to assign a setpoint $\mathbf{f}_d \in \mathcal{R}(\mathbf{K})$. On the other hand, this is consistent with the fact that the considered environment can generate reaction forces only along the direction normal to the plane. If no information about the geometry of the environment is available, the null vector can be assigned to $\mathbf{f}_d$, which is anyhow in the range space of $\mathbf{K}$. Thus in the remainder it is assumed that $\mathbf{f}_d \in \mathcal{R}(\mathbf{K})$. Analogously, it can be recognized that null position errors can be obtained only on the plane of contact, while the component of $\mathbf{x}$ along the normal direction has to accommodate the force requirement specified by $\mathbf{f}_d$; thus $\mathbf{x}_d$ can be freely reached only in $\mathcal{N}(\mathbf{K})$.

As demonstrated by Chiaverini and Siciliano,¹⁴ the unique equilibrium state $\{\mathbf{x}_\infty, \mathbf{f}_\infty\}$ for system (2) under control (14) is

$$\mathbf{x}_\infty = \frac{1}{k} \mathbf{n} \mathbf{n}^T (\mathbf{f}_d + k \mathbf{x}_0) + (\mathbf{I} - \mathbf{n} \mathbf{n}^T) \mathbf{x}_d \quad (15)$$

$$\mathbf{f}_\infty = k \mathbf{n} \mathbf{n}^T (\mathbf{x}_\infty - \mathbf{x}_0) = \mathbf{f}_d \quad (16)$$

An example of construction of the equilibrium state in a two-dimensional case is illustrated in Figure 1.

In order to study the stability of system (2), (14) around $\{\mathbf{x}_\infty, \mathbf{f}_\infty\}$, define

$$\mathbf{e} = \mathbf{x}_\infty - \mathbf{x} \quad (17)$$

which by virtue of (15), (11) and (13) can also be written as

$$\mathbf{e} = (\mathbf{I} - \mathbf{n} \mathbf{n}^T) \Delta \mathbf{x} + \frac{1}{k} \mathbf{n} \mathbf{n}^T \Delta \mathbf{f} = \Delta \mathbf{x} + \frac{1}{k_P} \mathbf{d} \quad (18)$$

where

$$\mathbf{d} = \frac{k_P}{k} \mathbf{n} \mathbf{n}^T [\mathbf{f}_d + k(\mathbf{x}_0 - \mathbf{x}_d)] \quad (19)$$

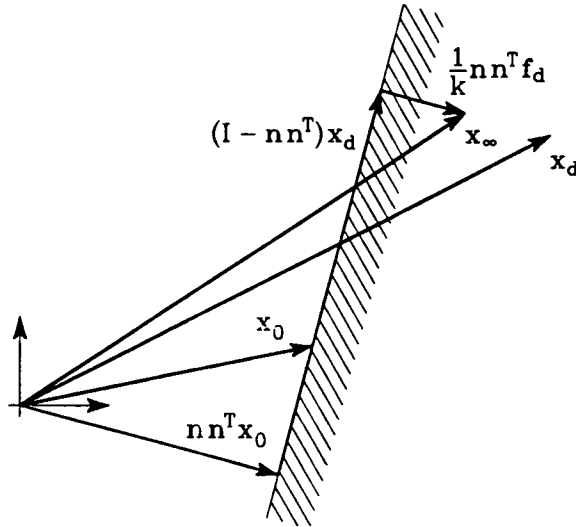


Figure 1. Construction of the equilibrium point in a two-dimensional case

is a constant vector accounting for the effects of the environment contact force and the desired force setpoint, both in $\mathcal{R}(\mathbf{K})$. For later use, notice that

$$\mathbf{n}^T \mathbf{e} = \frac{1}{k} \mathbf{n}^T \Delta \mathbf{f} \quad (20)$$

From (17) we have

$$\dot{\mathbf{e}} = -\dot{\mathbf{x}} \quad (21)$$

Further, define

$$s = \frac{1}{k} \mathbf{n}^T \left(\int_0^t \Delta \mathbf{f} \, d\sigma - \frac{1}{k_1} \mathbf{d} \right) \quad (22)$$

In view of the assumptions made above, the vector in parentheses lies on $\mathcal{R}(\mathbf{K})$; thus, projecting this vector along the direction normal to the contact plane allows us to extract the only non-null scalar component which is relevant for the present analysis. Deriving (22) with respect to time and accounting for (20) yields

$$\dot{s} = \mathbf{n}^T \mathbf{e} \quad (23)$$

At this point consider the 7×1 augmented state vector

$$\mathbf{z} = (\dot{\mathbf{x}} \quad \mathbf{e} \quad s)^T \quad (24)$$

The augmented system described by (1), (21) and (23) under control (14) can be written in the standard compact homogeneous form

$$\dot{\mathbf{z}} = \mathbf{F} \mathbf{z} \quad (25)$$

where

$$\mathbf{F} = \begin{pmatrix} -\mathbf{B}^{-1}(\mathbf{C} + k_D \mathbf{I}) & \mathbf{B}^{-1}(k_P \mathbf{I} + k_F k_{nn}^T) & k_1 k \mathbf{B}^{-1} \mathbf{n} \\ -\mathbf{I} & \mathbf{O} & \mathbf{0} \\ \mathbf{0}^T & \mathbf{n}^T & 0 \end{pmatrix} \quad (26)$$

with $k_F = 1 + k_F$; $\mathbf{O}$ denotes the null matrix and $\mathbf{0}$ the null vector of proper dimensions. Notice that some handy reductions, using the expression for $\mathbf{K}$ in (13) and the definition of s in (22), have been performed to derive (26).

The following result can be stated.

Theorem 1

If the assumption

$$\|\mathbf{e}(0)\| \leq \Phi < \infty \quad (27)$$

holds, then there exists a choice of feedback gains k_P , k_D , k_F and k_1 that makes the origin of the state space $\mathbf{z} = \mathbf{0}$ for system (25), (26) asymptotically stable. ■

Proof. See Reference 15. ■

Further, under slightly more restrictive conditions for the feedback gains it is possible to prove exponential stability for system (25), (26).¹⁵

4. IMPERFECT GRAVITY COMPENSATION

The nice feature of the above force/position regulator is the computational simplicity of control law (14); this is formed by a PID linear term and a non-linear term that compensates for gravity. In the following the requirement for perfect compensation is relaxed. It will be shown that the system remains asymptotically stable, but around a different equilibrium position. To overcome this inconvenience, an adaptive version of the regulator will be proposed in the next section.

Consider the modified control law

$$\mathbf{u} = k_P \Delta \mathbf{x} - k_D \dot{\Delta \mathbf{x}} + \hat{\mathbf{g}}(\mathbf{x}) + \mathbf{f}_d + k_F \Delta \mathbf{f} + k_I \int_0^t \Delta \mathbf{f} d\sigma \quad (28)$$

where $\hat{\mathbf{g}}$ denotes the best available estimate of the gravity term $\mathbf{g}$.

By analogy with (7), the estimate of the gravity force is

$$\hat{\mathbf{g}}(\mathbf{x}) = \partial \hat{U}(\mathbf{x}) / \partial \mathbf{x} \quad (29)$$

and a property similar to (8) can be assumed for $\hat{\mathbf{g}}$, i.e.

$$\| \partial \hat{\mathbf{g}}(\mathbf{x}) / \partial \mathbf{x} \| \leq \delta \quad (30)$$

for some $\delta > 0$. This also implies that

$$\| \hat{\mathbf{g}}(\mathbf{x}_1) - \hat{\mathbf{g}}(\mathbf{x}_2) \| \leq \delta \| \mathbf{x}_1 - \mathbf{x}_2 \| \quad (31)$$

In the following the new equilibrium state for system (1) under control (28) is found. In view of imperfect gravity compensation, the system at equilibrium satisfies the equations

$$(\mathbf{I} - \mathbf{n}\mathbf{n}^T) [\mathbf{g}(\mathbf{x}) - \hat{\mathbf{g}}(\mathbf{x})] = k_P (\mathbf{I} - \mathbf{n}\mathbf{n}^T) (\mathbf{x}_d - \mathbf{x}) \quad (32)$$

$$\mathbf{n}^T \mathbf{x} = \frac{1}{k} \mathbf{n}^T (\mathbf{f}_d + k \mathbf{x}_0) \quad (33)$$

Note that in the case $\hat{\mathbf{g}} = \mathbf{g}$, equations (32) and (33) have the unique solution given by (15) and (16). According to the analysis of Section 2, the matrix $\mathbf{I} - \mathbf{n}\mathbf{n}^T$ projects a vector on to the plane of contact characterized by $(\mathbf{p}_1, \mathbf{p}_2)$. Let $\mathbf{g}_p$ and $\mathbf{x}_p$ respectively denote the 2×1 vectors given by the components of $\mathbf{R}^T \mathbf{g}$ and $\mathbf{R}^T \mathbf{x}$ on the contact plane, with $\mathbf{R}$ defined as in (12). Also, let x_n and f_{dn} respectively denote the scalar component of $\mathbf{R}^T \mathbf{x}$ and $\mathbf{R}^T \mathbf{f}_d$ along the normal to the contact plane. As a consequence, equations (32) and (33) can be reduced to the three (two + one) equations

$$\mathbf{g}_p(\mathbf{x}_p, x_n) - \hat{\mathbf{g}}_p(\mathbf{x}_p, x_n) = k_P (\mathbf{x}_{dp} - \mathbf{x}_p) \quad (34)$$

$$x_n = \frac{1}{k} (f_{dn} + k x_{0n}) \quad (35)$$

Note that from the third equation x_n is a constant and thus determines the position of the contact plane at the equilibrium.

At this point, following the analysis of Tomei,¹⁸ consider the function

$$P(\mathbf{x}_p) = U_p(\mathbf{x}_p) - \hat{U}_p(\mathbf{x}_p) + \frac{1}{2} k_P \mathbf{x}_p^T \mathbf{x}_p - k_P \mathbf{x}_{dp}^T \mathbf{x}_p \quad (36)$$

where U_p is the gravitational energy evaluated with x_n as in (35). Since U_p and $\hat{U}_p$ are bounded and $\frac{1}{2} k_P \mathbf{x}_p^T \mathbf{x}_p$ is a convex function, P has an absolute minimum for a finite $\mathbf{x}_p$; since $\partial P / \partial \mathbf{x}_p = 0$ gives (34), system (34), (35) admits at least one solution and so does the original system (32),

(33). Let $\hat{\mathbf{x}}_\infty$ denote such a solution; then equations (32) and (33) evaluated for $\mathbf{x} = \hat{\mathbf{x}}_\infty$ become

$$(\mathbf{I} - \mathbf{n}\mathbf{n}^T) [\mathbf{g}(\hat{\mathbf{x}}_\infty) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty)] = k_P(\mathbf{I} - \mathbf{n}\mathbf{n}^T)(\mathbf{x}_d - \hat{\mathbf{x}}_\infty) \quad (37)$$

$$\mathbf{n}^T \hat{\mathbf{x}}_\infty = \frac{1}{k} \mathbf{n}^T (\mathbf{f}_d + k\mathbf{x}_0)$$

Subtracting (37) and (38) from (32) and (33) yields

$$(\mathbf{I} - \mathbf{n}\mathbf{n}^T) \{ [\mathbf{g}(\mathbf{x}) - \mathbf{g}(\hat{\mathbf{x}}_\infty)] - [\hat{\mathbf{g}}(\mathbf{x}) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty)] \} = k_P(\mathbf{I} - \mathbf{n}\mathbf{n}^T)(\hat{\mathbf{x}}_\infty - \mathbf{x}) \quad (39)$$

$$\mathbf{n}^T(\hat{\mathbf{x}}_\infty - \mathbf{x}) = 0 \quad (40)$$

Plugging (40) into (39) gives

$$(\mathbf{I} - \mathbf{n}\mathbf{n}^T) \{ [\mathbf{g}(\mathbf{x}) - \mathbf{g}(\hat{\mathbf{x}}_\infty)] - [\hat{\mathbf{g}}(\mathbf{x}) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty)] \} = k_P(\hat{\mathbf{x}}_\infty - \mathbf{x}) \quad (41)$$

By exploiting (9) and (31) and choosing

$$k_P > \delta + \delta \quad (42)$$

the following chain of inequalities holds for any $\mathbf{x} \neq \hat{\mathbf{x}}_\infty$:

$$\begin{aligned} & \| (\mathbf{I} - \mathbf{n}\mathbf{n}^T) \{ [\mathbf{g}(\mathbf{x}) - \mathbf{g}(\hat{\mathbf{x}}_\infty)] - [\hat{\mathbf{g}}(\mathbf{x}) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty)] \} \| \\ & \leq \| (\mathbf{I} - \mathbf{n}\mathbf{n}^T) [\mathbf{g}(\mathbf{x}) - \mathbf{g}(\hat{\mathbf{x}}_\infty)] \| + \| (\mathbf{I} - \mathbf{n}\mathbf{n}^T) [\hat{\mathbf{g}}(\mathbf{x}) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty)] \| \\ & \leq \| \mathbf{g}(\mathbf{x}) - \mathbf{g}(\hat{\mathbf{x}}_\infty) \| + \| \hat{\mathbf{g}}(\mathbf{x}) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty) \| \\ & < \delta \| \mathbf{x} - \hat{\mathbf{x}}_\infty \| + \delta \| \mathbf{x} - \hat{\mathbf{x}}_\infty \| \\ & \leq k_P \| \mathbf{x} - \hat{\mathbf{x}}_\infty \| \end{aligned} \quad (43)$$

implying that $\mathbf{x} = \hat{\mathbf{x}}_\infty$ is the unique solution to (39) and thus also to (32) and (33). In sum, the unique equilibrium state $\{\hat{\mathbf{x}}_\infty, \mathbf{f}_\infty\}$ for system (2) under control (28) with condition (42) is

$$\hat{\mathbf{x}}_\infty = \frac{1}{k} \mathbf{n}\mathbf{n}^T (\mathbf{f}_d + k\mathbf{x}_0) + (\mathbf{I} - \mathbf{n}\mathbf{n}^T) \left(\mathbf{x}_d - \frac{1}{k_P} [\mathbf{g}(\hat{\mathbf{x}}_\infty) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty)] \right) \quad (44)$$

$$\mathbf{f}_\infty = k\mathbf{n}\mathbf{n}^T(\hat{\mathbf{x}}_\infty - \mathbf{x}_0) = \mathbf{f}_d \quad (45)$$

Therefore in the case of imperfect gravity compensation the force setpoint is still attained while a different end-effector equilibrium position is reached compared with the case of perfect gravity compensation. More specifically, a comparison between (15) and (44) reveals that the components of $\mathbf{x}$ along the constrained task direction ($\mathbf{n}$) coincide — see the first terms on the right-hand side of (15) and (44) — while the imperfect gravity compensation affects the components of $\mathbf{x}$ along the unconstrained task directions ($\mathbf{p}_1, \mathbf{p}_2$), which thus differ — see the second terms on the right-hand side of (15) and (44).

Similarly to the steps prior to Theorem 1 of the previous section, the new error vector is given by

$$\hat{\mathbf{e}} = \Delta \mathbf{x} + \frac{1}{k_P} \hat{\mathbf{d}} - \frac{1}{k_P} [\mathbf{g}(\hat{\mathbf{x}}_\infty) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty)] \quad (46)$$

with

$$\hat{\mathbf{d}} = \frac{k_P}{k} \mathbf{n}\mathbf{n}^T \left(\mathbf{f}_d + k(\mathbf{x}_0 - \mathbf{x}_d) + \frac{k}{k_P} [\mathbf{g}(\hat{\mathbf{x}}_\infty) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty)] \right) \quad (47)$$

Further,

$$\mathbf{n}^T \dot{\mathbf{e}} = \frac{1}{k} \mathbf{n}^T \Delta \mathbf{f} \quad (48)$$

Notice that both $\Delta \mathbf{f}$ and $\dot{\mathbf{d}}$ belong to $\mathcal{R}(\mathbf{K})$; thus, similarly to (22), we have

$$\mathbf{s} = \frac{1}{k} \mathbf{n}^T \left(\int_0^t \Delta \mathbf{f} d\sigma - \frac{1}{k_1} \dot{\mathbf{d}} \right) \quad (49)$$

In view of the imperfect compensation $\mathbf{g}(\mathbf{x}) \neq \hat{\mathbf{g}}(\mathbf{x})$, the closed-loop system is no longer homogeneous as in (25) but is given by

$$\dot{\hat{\mathbf{z}}} = \mathbf{F}\hat{\mathbf{z}} + \boldsymbol{\mu} \quad (50)$$

with $\mathbf{F}$ as in (26), $\hat{\mathbf{z}} = (\dot{\mathbf{x}} \ \mathbf{e} \ \mathbf{s})^T$ and

$$\boldsymbol{\mu} = \begin{pmatrix} \mathbf{B}^{-1} [\hat{\mathbf{g}}(\mathbf{x}) - \hat{\mathbf{g}}(\mathbf{x}_\infty) - \mathbf{g}(\mathbf{x}) + \mathbf{g}(\mathbf{x}_\infty)] \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \quad (51)$$

Notice that $\hat{\mathbf{z}} = \mathbf{0}$ is a solution to (50). Thus the following result can be stated.

Theorem 2

If the assumption

$$\|\hat{\mathbf{e}}(0)\| \leq \hat{\Phi} < \infty \quad (52)$$

holds, then there exists a choice of feedback gains k_P , k_D , k_F and k_1 that makes the origin of the state space $\hat{\mathbf{z}} = \mathbf{0}$ for system (50), (26), (51) asymptotically stable. ■

Proof. Consider the Lyapunov function candidate

$$V(\hat{\mathbf{z}}) = \frac{1}{2} \hat{\mathbf{z}}^T \mathbf{P} \hat{\mathbf{z}} \quad (53)$$

where

$$\mathbf{P} = \begin{pmatrix} \mathbf{B} & -\rho \mathbf{B} & \mathbf{0} \\ -\rho \mathbf{B} & (k_P + \rho k_D) \mathbf{I} + k_F k_{nn}^T & k_1 k_{nn} \\ \mathbf{0}^T & k_1 k_{nn}^T & \rho k_1 k \end{pmatrix} \quad (54)$$

with $\rho > 0$. Computing the time derivative of V along the trajectories of system (50), (26), (51) gives

$$\dot{V}(\hat{\mathbf{z}}) = \hat{\mathbf{z}}^T (\mathbf{P}\mathbf{F} + \frac{1}{2} \dot{\mathbf{P}}) \hat{\mathbf{z}} + \hat{\mathbf{z}}^T \mathbf{P} \boldsymbol{\mu} \quad (55)$$

and after some algebraic manipulation

$$\dot{V} = -\dot{\mathbf{x}}^T (k_D \mathbf{I} - \rho \mathbf{B}) \dot{\mathbf{x}} - \rho \mathbf{e}^T \mathbf{C}^T \dot{\mathbf{x}} - \mathbf{e}^T [\rho k_P \mathbf{I} + (\rho k_F - k_1) k_{nn}^T] \mathbf{e} + \hat{\mathbf{z}}^T \mathbf{P} \boldsymbol{\mu} \quad (56)$$

in which the skew symmetry of matrix (4) and equation (5) have been conveniently exploited. The conditions that make V positive definite coincide with those derived for the perfect gravity compensation case,¹⁵ i.e.

$$k_P + \rho k_D > \rho^2 \lambda_M \quad (57)$$

$$\rho k_F > k_1 \quad (58)$$

Regarding $\dot{V}$ in (55), the quadratic term is the same as the one guaranteeing asymptotic stability in the perfect gravity compensation case, being now the equilibrium state $\hat{\mathbf{z}}$ in lieu of $\mathbf{z}$. Thus it is sufficient to consider the second term in (55); this is given by

$$\hat{\mathbf{z}}^T \mathbf{P} \boldsymbol{\mu} = (\dot{\mathbf{x}}^T - \rho \hat{\mathbf{e}}^T) [\hat{\mathbf{g}}(\mathbf{x}) - \hat{\mathbf{g}}(\hat{\mathbf{x}}_\infty) - \mathbf{g}(\mathbf{x}) + \mathbf{g}(\hat{\mathbf{x}}_\infty)] \quad (59)$$

In view of (9) and (31), the following inequality holds

$$\hat{\mathbf{z}}^T \mathbf{P} \boldsymbol{\mu} \leq (\|\dot{\mathbf{x}}\| + \rho \|\hat{\mathbf{e}}\|) \|\hat{\mathbf{e}}\|(\delta + \delta) \quad (60)$$

Following a similar derivation to that of Theorem 1¹⁵ and exploiting (52) and (60) allows us to find an upper bound for the function in (56) as

$$\begin{aligned} \dot{V}(\hat{\mathbf{z}}) \leq & -(k_D - \rho \lambda_M - \rho k_C \hat{\Phi}) \|\dot{\mathbf{x}}\|^2 + (\delta + \delta) \|\dot{\mathbf{x}}\| \|\hat{\mathbf{e}}\| \\ & - \rho [k_P - (\delta + \delta)] \|\hat{\mathbf{e}}\|^2 - k(\rho k_F - k_I) \hat{s}^2 \end{aligned} \quad (61)$$

This reveals that there exists a choice of k_P, k_D, k_F, k_I and ρ that makes $\dot{V}$ negative semi-definite, i.e.

$$k_D > \rho(\lambda_M + k_C \hat{\Phi}) \quad (62)$$

$$k_P > (\delta + \delta) \left(1 + \frac{\delta + \delta}{4\rho[k_D - \rho(\lambda_M + k_C \hat{\Phi})]} \right) \quad (63)$$

together with (58) again. Observing that condition (62) implies (57), it can be recognized that the conditions to be satisfied are (58), (62) and (63). To conclude asymptotic stability, the case $\dot{V} = 0$ must be analysed. It is easy to see that $\dot{V} = 0$ if and only if $\dot{\mathbf{x}} = \mathbf{0}$, $\hat{\mathbf{e}} = \mathbf{0}$ and $\hat{s} = 0$; from the first three equations of (50) and (26) it can be seen that $\tilde{\mathbf{x}} = \mathbf{0}$ if and only if $\hat{s} = 0$. Thus the only system trajectory satisfying $\dot{V} = 0$ is $\hat{\mathbf{z}} = \mathbf{0}$; owing to the LaSalle invariance set theorem, this equilibrium state is asymptotically stable. ■

Notice that the constant ρ is not used by control law (28) and thus is a degree of freedom available to satisfy conditions (58), (62) and (63). Compared with the stability analysis for control (14) presented in Reference 15, which leads to conditions on k_D, k_F and k_I only, now k_P is also involved. On the other hand, from the above equilibrium analysis k_P has to satisfy (42), which is then implied by (63). This fact is reasonable, since the effect of imperfect gravity compensation has to be properly counteracted by increasing k_P ; the larger k_P is, the closer $\hat{\mathbf{x}}_\infty$ is to $\mathbf{x}_\infty$.

5. GRAVITY PARAMETER ADAPTATION

In view of (10), the control law (28) can be rewritten as

$$\mathbf{u} = k_P \Delta \mathbf{x} - k_D \dot{\mathbf{x}} + \mathbf{G}(\mathbf{x}) \hat{\boldsymbol{\theta}} + \mathbf{f}_d + k_F \Delta \mathbf{f} + k_I \int_0^t \Delta \mathbf{f} d\sigma \quad (64)$$

where $\hat{\boldsymbol{\theta}}$ is the vector of estimated parameters.

In this situation the equations of the closed-loop system become

$$\dot{\mathbf{z}} = \mathbf{F} \mathbf{z} + \boldsymbol{\mu} \quad (65)$$

with $\mathbf{z}$ as in (24), $\mathbf{F}$ as in (26) and

$$\boldsymbol{\mu} = \begin{pmatrix} -\mathbf{B}^{-1} \mathbf{G}(\mathbf{x}) \tilde{\boldsymbol{\theta}} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \quad (66)$$

where $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$ is the parameter error vector.

In addition to control law (64), the parameter estimate vector is determined according to the update law

$$\dot{\hat{\theta}} = -\frac{1}{\nu} \mathbf{G}^T(\mathbf{x}) \left[\dot{\mathbf{x}} - \beta \left(\Delta \mathbf{x} + \frac{k_I}{k_P} \int_0^t \Delta \mathbf{f} \, d\sigma \right) \right] \quad (67)$$

with $\nu > 0$.

The following result can be stated.

Theorem 3

If the assumption

$$\{\|\mathbf{e}(0)\| \leq \Psi_1 < \infty, |s(0)| \leq \Psi_2 < \infty\} \quad (68)$$

holds, then there exists a choice of feedback gains k_P , k_D , k_F and k_I that makes the origin of the state space $\mathbf{z} = \mathbf{0}$ for system (65), (26), (66), (67) asymptotically stable and the parameter estimate vector $\hat{\theta}$ bounded ■

Proof. Consider the Lyapunov function candidate

$$W(\mathbf{z}, \hat{\theta}) = \frac{1}{2} \mathbf{z}^T \mathbf{Q} \mathbf{z} + \frac{1}{2} \nu \hat{\theta}^T \hat{\theta} \quad (69)$$

where

$$\mathbf{Q} = \begin{pmatrix} \mathbf{B} & -\beta \mathbf{B} & -\gamma \mathbf{B} \mathbf{n} \\ -\beta \mathbf{B} & (k_P + \beta k_D) \mathbf{I} + k_F k \mathbf{n} \mathbf{n}^T & (k_I k + \gamma k_D) \mathbf{n} \\ -\gamma \mathbf{n}^T \mathbf{B} & (k_I k + \gamma k_D) \mathbf{n}^T & \beta k_I k + \gamma (k_P + k_F k) \end{pmatrix} \quad (70)$$

with $\beta, \gamma > 0$. Computing the time derivative of W along the trajectories of system (65), (26), (66) gives

$$\dot{W}(\mathbf{z}, \hat{\theta}) = \mathbf{z}^T (\mathbf{Q} \mathbf{F} + \frac{1}{2} \dot{\mathbf{Q}}) \mathbf{z} + \mathbf{z}^T \mathbf{Q} \boldsymbol{\mu} + \nu \hat{\theta}^T \dot{\hat{\theta}} \quad (71)$$

and after some algebraic manipulation

$$\begin{aligned} \dot{W}(\mathbf{z}, \hat{\theta}) = & -\dot{\mathbf{x}}^T (k_D \mathbf{I} - \beta \mathbf{B}) \dot{\mathbf{x}} - \mathbf{e}^T [\beta k_P \mathbf{I} + (\beta k_F k - k_I k - \gamma k_D) \mathbf{n} \mathbf{n}^T] \mathbf{e} - \gamma k_I k s^2 \\ & - \gamma \dot{\mathbf{x}}^T \mathbf{B} \mathbf{n} \mathbf{n}^T \mathbf{e} - (\beta \mathbf{e}^T + \gamma \mathbf{s} \mathbf{n}^T) \mathbf{C}^T \dot{\mathbf{x}} - (\dot{\mathbf{x}}^T - \beta \mathbf{e}^T - \gamma \mathbf{s} \mathbf{n}^T) \mathbf{G} \hat{\theta} + \nu \hat{\theta}^T \dot{\hat{\theta}} \end{aligned} \quad (72)$$

where the skew symmetry of matrix (4) and equations (5) and (10) have been exploited. The function candidate (69), (70) accounting for (3), (23) can be lower-bounded as

$$\begin{aligned} W(\mathbf{z}, \hat{\theta}) \geq & \frac{1}{2} (\dot{\mathbf{x}}^T \mathbf{B} \mathbf{e}^T) \begin{pmatrix} (1/2\lambda_M) \mathbf{I} & -\beta \mathbf{I} \\ -\beta \mathbf{I} & (k_P + \beta k_D) \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{B} \dot{\mathbf{x}} \\ \mathbf{e} \end{pmatrix} \\ & + \frac{1}{2} (\dot{\mathbf{x}}^T \mathbf{B} \mathbf{s} \mathbf{n}^T) \begin{pmatrix} (1/2\lambda_M) \mathbf{I} & -\gamma \mathbf{I} \\ -\gamma \mathbf{I} & \gamma (k_P + k_F k) \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{B} \dot{\mathbf{x}} \\ \mathbf{s} \mathbf{n} \end{pmatrix} \\ & + \frac{1}{2} (\dot{s} \ s) \begin{pmatrix} k_F k & k_I k + \gamma k_D \\ k_I k + \gamma k_D & \beta k_I k \end{pmatrix} \begin{pmatrix} \dot{s} \\ s \end{pmatrix} + \frac{1}{2} \nu \hat{\theta}^T \dot{\hat{\theta}} \end{aligned} \quad (73)$$

Regarding $\dot{W}$, the choice

$$-\dot{\hat{\theta}} = \dot{\hat{\theta}} = -\frac{1}{\nu} \mathbf{G}^T (\dot{\mathbf{x}} - \beta \mathbf{e} - \gamma \mathbf{s} \mathbf{n}) \quad (74)$$

allows us to cancel the terms in (72) depending on $\tilde{\theta}$. It is crucial to remark that in view of (18) and (22), the choice

$$\gamma = \beta k_1 k / k_P \quad (75)$$

makes the update law (74) equal to (67). By virtue of assumption (68) and equation (6), the following inequality holds:

$$-(\beta e^T + \gamma s n^T) C^T \dot{x} \leq (\beta \Psi_1 + \gamma \Psi_2) k_C \|\dot{x}\|^2 \quad (76)$$

Thus the function (72) can be upper-bounded as

$$\begin{aligned} \dot{W}(z) \leq & -[k_D - \beta \lambda_M - (\beta \Psi_1 + \gamma \Psi_2) k_C] \|\dot{x}\|^2 - \gamma \lambda_M \|\dot{x}\| \|e\| \\ & - \beta k_P \|e\|^2 - \gamma k_1 k s^2 - (\beta k_f k - k_1 k - \gamma k_D) s^2 \end{aligned} \quad (77)$$

Equations (73) and (77) reveal that there exists a choice of k_P , k_D , k_F , k_1 and β , with γ as in (75), that makes W positive definite and $\dot{W}$ negative semidefinite provided that

$$k_D > \beta [k_C (\Psi_1 + \Psi_2 k_1 k / k_P) + \lambda_M (1 + \lambda_M (k_1 k)^2 / k_P^2)] \quad (78)$$

$$k_P > \beta (2\beta \lambda_M - k_D) \quad (79)$$

$$k_P + k_f k > 2\beta k_1 k \lambda_M / k_P \quad (80)$$

$$\beta k_f > k_1 (1 + \beta k_D / k_P)^2 \quad (81)$$

In particular, condition (81) admits real solutions for β if and only if

$$k_f / k_1 > 4k_D / k_P \quad (82)$$

The negative semidefiniteness of $\dot{W}$ implies that $\tilde{\theta}$ is bounded and thus $\hat{\theta}$ is also bounded. As above, the case $\dot{W} = 0$ needs to be analysed. It can be seen that $\dot{W} = 0$ if and only if $z = 0$; from (65), (26) and (66) it can be seen that $\dot{z} = 0$ if and only if $\mu = 0$, i.e. $G\tilde{\theta} = 0$. Thus the only system trajectory satisfying $\dot{W} = 0$ is $z = 0$; owing to the LaSalle invariance set theorem, this equilibrium state is asymptotically stable. ■

Notice that the constant β is not used in control law (64) but affects the dynamics of the adaptation mechanism in (67). The other constant γ is keenly used to render the adaptation law (67) a function of physically measurable quantities already used in control law (64), i.e. position, velocity and force measurements. Further, observe that the parameter error vector is not guaranteed to converge to zero and a residual error may exist at steady state, depending on the structure of matrix G . This feature is just inherited from classical adaptive motion control schemes.¹⁶

6. CASE STUDY

The proposed force/position controllers were tested in a case study on the industrial robot COMAU SMART 6.10R. Only the first three joints were considered, constituting a typical elbow manipulator geometry with zero shoulder offsets. The complete dynamic model and numerical data of robot parameters are reported in Reference 22. A tip payload of mass $m_1 = 10$ kg and zero inertia with its centre of mass 0.2 m from the manipulator tip was added; this was assumed to be the only relevant parameter θ in (10). Simulations were run in MATLAB on a PC-486/33.

A step motion from $x = (1.100 \ 0 \ 0)^T$ (m) to the setpoint $x_d = (1.120 \ 0 \ 0)^T$ (m) was commanded to the manipulator tip. The setpoint $f_d = 0$ was assigned for the tip force. The

geometry of the contact plane is characterized by $\mathbf{n} = (1 \ 0 \ 0)^T$ and $\mathbf{x}_0 = (1.115 \ 0 \ 0)^T$ (m); the stiffness coefficient is $k = 10^5 \text{ N m}^{-1}$. It can be recognized that an unexpected impact occurs along the direction normal to the plane at a distance of 0.005 m from the target point $\mathbf{x}_d$.

Four sets of simulations were carried out at a sampling time of 2 ms with the following controllers:

- (a) control law (14) without force action ($k_F = k_I = 0$)
- (b) control law (14) with force action;
- (c) control law (28) with non-compensated load mass ($\hat{m}_1 = 0$)
- (d) control law (64) with initially non-compensated load mass ($\hat{m}_1(0) = 0$) and adaptive law (67).

The PD position feedback gains for all cases were set to $k_P = 10^5 \text{ N m}^{-1}$ and $k_D = 10^4 \text{ N s m}^{-1}$ so as to guarantee a well-damped behaviour for the unconstrained motion of the manipulator.

The PI force feedback gains for cases (b–d) were set to $k_F = 9$ and $k_I = 24 \text{ s}^{-1}$ so as to achieve a satisfactory behaviour during the constrained motion. With the above values it was possible to satisfy

- (a, b) design conditions (58) and (62) for control law (14) with some $\rho > 0$ for available estimates¹⁵ of λ_M , k_C and Φ
- (c) design conditions (58), (62) and (63) for control law (28) with some $\rho > 0$ for available estimates of λ_M , k_C , $\hat{\Phi}$, δ and $\hat{\delta}$
- (d) design conditions (78)–(82) for adaptive control law (64), (67) with some $\beta > 0$ for available estimates of λ_M , k_C and Ψ_1 .

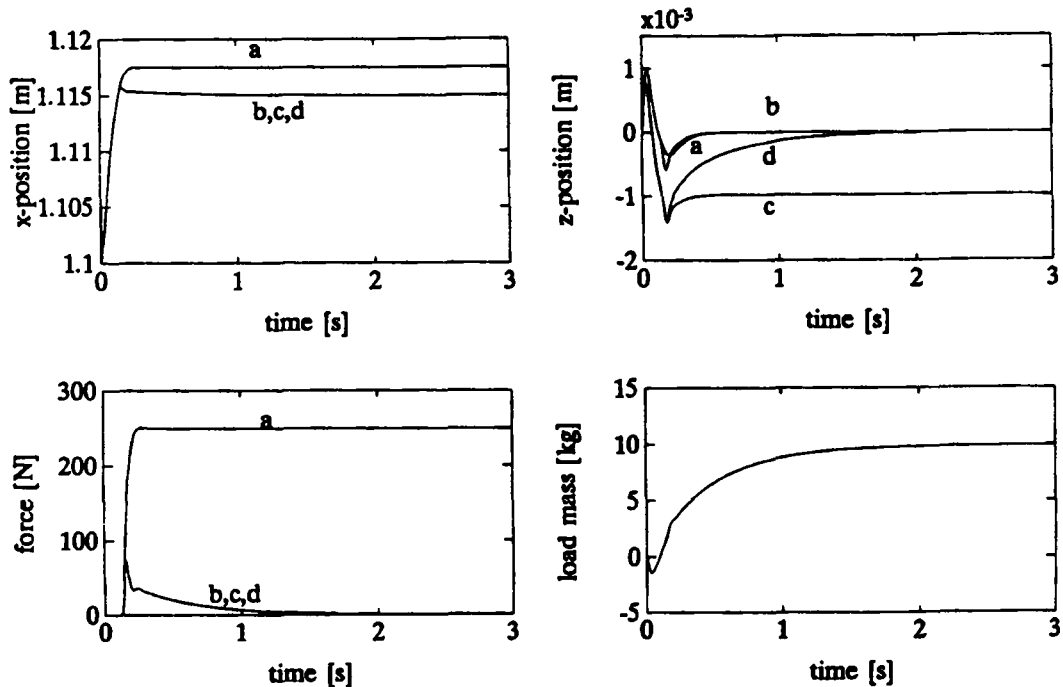


Figure 2. Time histories of position x -component and z -component, contact force and load mass estimate

The numerical results for the above four cases are illustrated in Figure 2 in terms of the time histories of the x - and z -components of position, the contact force (purely along x) and the estimated load mass, which is significant only for case (d).

The x -position and force reveal that without force feedback, case (a), finite steady state errors occur for both x -position and force; with force feedback, cases (b–d), the x -position and force behaviour is essentially the same for all controllers and null steady state errors are guaranteed. On the other hand, the z -position behaviour differs for each controller. It is driven to zero at steady state in all cases except (c); in fact, in the latter case the imperfect gravity compensation affects the vertical component of the tip position. Remarkably, the adaptation mechanism, case (d), works satisfactorily to recover the initially non-compensated load mass and the performance compares favourably with that of the controller with perfect compensation, case (b).

7. CONCLUSIONS

Stable force/position regulation of robot manipulators in contact with an elastically compliant surface was presented in this work. The controller consists of a PD action on the position loop, a PI action on the force loop, together with gravity compensation and desired contact force feed forward.

If compensation is perfect, asymptotic stability around the system equilibrium state is ensured. If compensation is imperfect, a different equilibrium position is asymptotically reached. To overcome this drawback, an adaptation mechanism is effectively added that acts on the gravity parameter vector so as to recover the original equilibrium state asymptotically.

The stability proofs are based on Lyapunov functions of the system state containing off-diagonal terms which can be considered as a generalization of the Lyapunov functions in Reference 23 to the case of stability of force/position regulators. The positive constant appearing in some of the off-diagonal terms serves as an additional degree of freedom to satisfy design conditions on the feedback gains, but remarkably is not used by the control law. In the case of gravity parameter adaptation, another positive constant is needed which turns out to be very useful to make the adaptation mechanism operate on feedback quantities already available to the controller.

A case study was developed for a three-joint industrial elbow manipulator whose tip experiences an unexpected impact with the environment on the way towards the target position. An unknown payload was added to test the performance of imperfect gravity compensation as well as of parameter adaptation. The numerical results of four sets of simulations confirmed the theoretical derivation.

Future work will be devoted to investigating the performance of the scheme for tasks that entail tracking of the tip position along unconstrained directions. In this respect it is desired to design PID force/position controllers that will guarantee good performance in surface edge-following tasks.²⁴

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