

THE AUGMENTED TASK SPACE APPROACH FOR REDUNDANT MANIPULATOR CONTROL

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Abstract. Redundant manipulators are currently receiving a considerable interest in the robotics research community. The extra (redundant) degree-of-freedom's (DOF's) in a structure which is redundant with respect to a given task can be used to obtain a more versatile configuration in terms of the interaction of the structure with the environment. Two approaches are commonly followed to control robotic manipulators; one is to solve first the inverse kinematics and then to design a dynamic control in the joint space, the other one is to design a dynamic control directly in the task space. The former is adopted for end-effector motion control, whereas the latter is to be preferred for end-effector tasks that involve constrained motion and active force control. The task space augmentation approach is an alternative strategy to the usual methods based on the use of generalized inverses. The supporting idea is to augment the direct kinematics by the inclusion of a number of functional constraints which characterize the degree of dexterity required for the redundant structure. In this work it is shown how this method serves as an effective tool to control redundant manipulators, according to the above two approaches.

Keywords. Robots; manipulation; redundancy; inverse kinematics; closed-loop systems; nonlinear dynamic decoupling control.

INTRODUCTION

Kinematically redundant manipulators may offer a number of potential advantages over current nonredundant designs. The human arm constitutes a tangible model of the ability of a redundant arm to execute more dexterous motions. The task space formulation is usually adopted to develop motion and force control of the end effector (Khatib, 1987). When a manipulator is redundant with respect to a given task, the space of redundancy can be conveniently exploited to meet a number of constraints on the solution to the inverse kinematic problem (Hanafusa, Yoshikawa, and Nakamura, 1981). Typical goals are the avoidance of obstacles in the workspace (Maciejewski and Klein, 1985), mechanical joint limits (Liegeois, 1977), kinematic singularities (Nakamura and Hanafusa, 1986; Wampler, 1986), or the minimization of actuator joint forces (Vukobratovic and Kircanski, 1984), manipulability measures - kinematic (Yoshikawa, 1985b) and dynamic (Yoshikawa, 1985a), dexterity measures (Klein and Blaho, 1987), task compatibility indices (Chiu, 1987). Previous approaches in the literature have been aimed at solving redundancy at kinematic level in terms of optimizing quadratic type criteria along with the use of generalized inverses of the manipulator Jacobian (Whitney, 1972). Essentially the joint velocity solution vector is formed by two terms, a locally minimum norm term plus a term in the space of redundancy (the null space of the Jacobian matrix) which is used for local optimization purposes (Klein and Huang, 1983).

The strategy proposed here instead is to augment the task space vector in a systematic manner up to span the whole space of redundancy eventually, so as to include all above kinds of constraints. A requirement is to functionally characterize the constraints in terms of the joint variables with respect to the same base frame associated with the manipulator (Siciliano, 1986; Sciavicco and Siciliano, 1987a, 1987b, 1988b, 1989; Sciavicco, Siciliano, and Chiacchio, 1988). Similar approaches have been proposed by Baillieul (1985), Chang (1987), and Egeland (1987).

In general, however, it is not possible to arbitrarily choose the constraints so that the joint displacements satisfy the constraints and depend continuously on the trajectory assigned at the end-effector (Baillieul and co-workers, 1987). To avoid this drawback, a task-priority strategy has been independently suggested by Maciejewski and Klein (1985) and Nakamura, Hanafusa, and Yoshikawa (1987).

A possible solution to the control problem can be achieved in two stages. First the end effector desired motion is solved into joint motions by adopting a recently proposed algorithm which is based only on the computation of the direct (augmented) kinematics (Siciliano, 1986). Once a solution is obtained for the joint variables, the controller can be designed directly in the joint space as it would be in the case of a nonredundant manipulator. If the constraints are not properly formulated, however, the task-priority strategy introduced above is recommendable. Priority is given to the primary task - typically the end-effector - and the secondary task - typically the constraints - is satisfied only on condition that it does not disturb the primary task.

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This approach, indeed, is independent of the end-effector dynamic behavior since it is based purely on kinematic and static properties which are solved for in the first stage. So, if a task involves combined motion and contact forces, the dynamic control of end-effector motion and contact forces is not achieved with this two-stage technique (Khatib, 1987). Therefore, a different approach is derived accordingly which allows for the design of the controller directly in the task space, with the inherent advantage of dynamically controlling the end-effector motion and contact forces along with the additional task space variables. In particular a nonlinear decoupling control strategy can be pursued which decouples the manipulator dynamics by compensating for the available estimates of the nonlinear terms.

MANIPULATOR MODELING

The kinematic and dynamic models of a manipulator are recalled in the following in order to provide the basic definitions and the nomenclature throughout the paper.

Kinematics

The direct kinematic equation describes the mapping of the $(n \times 1)$ joint vector q into the $(m \times 1)$ end-effector task vector x_e as (Denavit and Hartenberg, 1955)

$$x_e = f_e(q) \quad (1)$$

where f_e is a continuous nonlinear vector function, whose structure and parameters are known.

The kinematic equation (1) can be differentiated with respect to time, yielding the mapping between the joint velocity vector $\dot{q}$ and the end-effector task velocity vector $\dot{x}_e$, through the $(m \times n)$ Jacobian matrix $J_e(q) = \partial f_e / \partial q$, i.e.

$$\dot{x}_e = J_e(q) \dot{q} \quad (2)$$

For a kinematically redundant manipulator it is $m < n$. Redundancy is usually resolved by optimizing quadratic type criteria along with the use of generalized inverses of the Jacobian matrix J_e (Whitney, 1972). It can be shown that a general solution to (2) is given by

$$\dot{q} = J_e^+(q) \dot{x}_e + [I - J_e^+(q) J_e(q)] \dot{q}_0 \quad (3)$$

where J_e^+ is the $(n \times m)$ Moore-Penrose pseudo-inverse matrix of J_e defined as $J_e^+ = J_e^T (J_e J_e^T)^{-1}$, I is the $(n \times n)$ identity matrix, and $\dot{q}_0$ is an $(n \times 1)$ arbitrary joint velocity vector. It can be noticed that the solution (3) composes of the least-square solution term of minimum norm plus a homogeneous solution term created by the projection operator $(I - J_e^+ J_e)$ which selects the components of $\dot{q}_0$ in the null space of the mapping J_e (Klein and Huang, 1983). Hence, the vector $\dot{q}_0$ can be used to optimize some additional criterion, such as obstacle avoidance (Maciejewski and Klein, 1985), limited joint range (Liegeois, 1977), actuator joint energy (Vukobratovic and Kircanski, 1984), kinematic singularity robustness (Nakamura and Hanafusa, 1986; Wampler, 1986), manipulability measures - kinematic (Yoshikawa, 1985b) and dynamic (Yoshikawa, 1985a) -, dexterity measures (Klein and Blaho, 1987), task compatibility indices (Chiu, 1987), to mention only the most popular uses.

Dynamics

The manipulator equations of motion in the joint space are given by (Luh, Walker, and Paul, 1980)

$$H(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q) = \tau \quad (4)$$

where $H(q)$ is the $(n \times n)$ kinetic energy matrix, $C(q, \dot{q}) \dot{q}$ is the $(n \times 1)$ vector of Coriolis and centrifugal forces, $g(q)$ is $(n \times 1)$ vector of gravitational forces, and τ is the $(n \times 1)$ vector of input forces, all in the joint space.

The static equation of the manipulator, derived from (2) along with the use of the principle of virtual work, relates the generalized forces between the joint space and the task space. It describes the mapping of the $(m \times 1)$ end-effector task force vector γ into the $(n \times 1)$ joint force vector τ as (Whitney, 1972)

$$\tau = J_e^T(q) \gamma \quad (5)$$

If the manipulator is nonredundant ($m = n$), by plugging (5) in (4), the end-effector equations of motion in the task space can be written in the form (Khatib, 1987)

$$J_e^T(q) [H(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q)] = \gamma \quad (6)$$

and then, in terms of the end-effector task vector x_e , as

$$\Lambda(x_e) \ddot{x}_e + \mu(x_e, \dot{x}_e) \dot{x}_e + p(x_e) = \gamma \quad (7)$$

where $\Lambda(x_e)$ is the $(n \times n)$ kinetic energy matrix, $\mu(x_e, \dot{x}_e)$ is the $(n \times 1)$ vector of Coriolis and centrifugal forces, and $p(x_e)$ is the $(n \times 1)$ vector of gravitational forces, all in the task space.

If the manipulator is redundant ($m < n$), the dynamic behavior of the system is not entirely described by a dynamic model formally identical to (7) and $\Lambda(x_e)$ represents only a pseudo-kinetic energy matrix. Khatib (1987) showed that (7) can be rewritten as

$$\tilde{J}_e^T(q) [H(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q)] = \gamma \quad (8)$$

where

$$\tilde{J}_e(q) = H^{-1}(q) J_e^T(q) [J_e(q) H^{-1}(q) J_e^T(q)]^{-1} \quad (9)$$

is actually a generalized inverse of the Jacobian matrix J_e corresponding to the solution that minimizes the manipulator's instantaneous kinetic energy. It is quite straightforward to recognize that the mapping (5), which is the dual of (2), can be modified into

$$\tau = \tilde{J}_e^T(q) \gamma + [I - \tilde{J}_e^T(q) \tilde{J}_e(q)] \tau_0 \quad (10)$$

where τ_0 is an arbitrary joint force vector.

TASK SPACE AUGMENTATION

As previously anticipated, a redundant manipulator offers the possibility of setting a number of constraints on its motion, which make the structure more versatile. The most widely adopted method to control a redundant manipulator is based on the use of generalized inverses, see eqs. (3) and (10); the constraints are incorporated into the homogeneous solution term.

The task augmentation approach is quite different from the generalized inverse methods. The main idea is to impose a number of functional differentiable constraints in terms of the joint vector, expressed in the same base frame associated with the manipulator kinematic equation (1), as (Siciliano, 1986)

$$x_c = f_c(q) \quad (11)$$

where f_c is an $(r \times 1)$ vector, with $r \leq (n - m)$. Therefore, the augmented kinematic equation results

$$\dot{x} = \begin{bmatrix} \dot{x}_e \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} \dot{f}_e(q) \\ \dot{f}_c(q) \end{bmatrix} = \dot{f}(q) \quad (12)$$

whose joint vector solution q places the end-effector at the desired location x_e as well as it meets the required constraints, specified by x_c . Differentiating both sides of (12) provides the differential kinematic mapping of the joint velocity vector into the augmented task velocity vector, i.e.

$$\dot{x} = J(q)\dot{q} \quad (13)$$

where the matrix

$$J(q) = \begin{bmatrix} J_e(q) \\ J_c(q) \end{bmatrix} \quad (14)$$

with $J_c(q) = \partial f_c / \partial q$, is called the augmented Jacobian matrix.

CONTROL IN THE JOINT SPACE

If it is desired to control the end-effector motion of a redundant manipulator in the joint space, the inverse kinematic problem has to be solved first. In other words, it is necessary to transform the end-effector desired task motion x_{ed} into the corresponding joint motion q , which constitutes the set of reference variables for the actual dynamic control to be designed in the joint space.

If a generalized inverse method is adopted, the inverse kinematic problem is solved in terms of the joint velocity vector $\dot{q}$ as in (3), which is then integrated over time to provide q .

A different approach to the inverse kinematic problem is given by the closed-loop scheme based on the feedback law

$$\dot{q} = J_e^T(q)K_e[x_{ed} - f_e(q)] \quad (15)$$

whose convergence properties and choice of the positive definite feedback gain matrix K_e are described in detail by Siciliano (1986). It is easy to recognize that eq. (15), which involves the sole computation of direct kinematic functions, yields a general computational scheme to solve the inverse kinematic problem for any manipulator structure. This solution is particularly suitable to the augmented task space approach described in the previous section, since it seems natural to replace the end-effector task vector and Jacobian defined in (1) and (2) with the augmented task vector and Jacobian defined in (12-14), respectively, i.e.

$$\dot{q} = J^T(q)K[x_d - f(q)] \quad (16)$$

where $K = \text{diag}(K_e, K_c)$, with K_c positive definite as well.

This approach has been applied to the case of dexterity measure constraints (Sciavicco and Siciliano, 1987b) and to the case of obstacle and/or limited joint range constraints (Sciavicco and Siciliano, 1988b). However, as evidenced in the particular case study discussed by Sciavicco, Siciliano and Chiacchio (1988), it is necessary to augment the task space in such a way that the augmented Jacobian matrix $J(q)$ in (14) presents at most the same singularities as the end-effector Jacobian matrix $J_e(q)$ in (2) (Baillieul, 1985). If

$J(q)$ is not guaranteed to have full rank $(m + r)$ for all possible configurations q 's (except the end-effector kinematic singularities), the task-priority strategy (Maciejewski and Klein, 1985; Nakamura, Hanafusa, and Yoshikawa, 1987) is advisable, as discusses above and illustrated in details by Sciavicco and Siciliano (1989). With reference to the inverse kinematic solution (16) for the augmented task case, the Jacobian matrix to use is modified into (Sciavicco and Siciliano, 1989)

$$\tilde{J}(q) = \begin{bmatrix} J_e(q) \\ \tilde{J}_c(q) \end{bmatrix} \quad (17a)$$

$$\tilde{J}_c = J_c(I - J_e^+ J_e) \quad (17b)$$

where only the components of the rows of J_c into the null space of the mapping J_e are selected. This guarantees that the end-effector task is correctly executed, while the constraint task is performed only along those directions in the null space of the end-effector Jacobian.

At this point, it can be recognized that the closed-loop scheme based on eq. (16) yields a solution for the joint displacements q as well as for the joint velocities $\dot{q}$. This is seen to be sufficient if simple linear positional PD controllers are to be designed for each joint of the robot based on the assumption that the joints are decoupled and linear; this is often the case for conventional industrial robot controllers. More effectively, a computed-torque control scheme can be designed (Luh, Walker, and Paul, 1980), which utilizes nonlinear feedback to decouple the manipulator by using the arm dynamic model (4) and compensating, on-line, for the dynamic and gravitational forces that vary as the arm configuration changes. The control torque τ in (4) is computed as

$$\tau = \hat{H}(q)u + \hat{C}(q, \dot{q})\dot{q} + \hat{g}(q) \quad (18a)$$

$$u = \ddot{q}_d + K_v(\dot{q}_d - \dot{q}) + K_p(q_d - q) \quad (18b)$$

where K_p and K_v are standard proportional and derivative feedback gain matrices and the caret indicates that the estimated arm dynamic model is used in the computation. The vectors q_d , $\dot{q}_d$ and $\ddot{q}_d$ which constitute the desired references for the decoupled linear joint controllers in (18b) are to be provided by an inverse kinematic scheme. Therefore, it is necessary to extend the "first"-order scheme based on eq. (16) to a "second"-order scheme which generates the joint acceleration vector $\ddot{q}$, too. If a pseudoinverse of the Jacobian J in (14), or the Jacobian $\tilde{J}$ in (17), is used in lieu of the transpose, a solution can be obtained quite in a straightforward manner from the second order scheme proposed by Sciavicco and Siciliano (1988a) for nonredundant manipulators. Preliminary results have shown that even the transpose can be utilized, but more research work needs to be done.

CONTROL IN THE TASK SPACE

If the assigned task involves combined motion and contact forces of the end-effector, it is necessary to control the manipulator directly in the task space. If the manipulator is nonredundant, the dynamic model of the end-effector (7) can be considered and different control strategies can be developed. According to the nonlinear dynamic decoupling approach that lead to the computed-torque control method in (18), the control γ in (7) can be selected as (Khatib, 1987)

$$\gamma = \hat{\Lambda}(x)v + \hat{\mu}(x, \dot{x}) + \hat{p}(x) \quad (19)$$

where $\hat{\Lambda}(x)$, $\hat{\mu}(x, \dot{x})$, and $\hat{p}(x)$ represent the estimates of $\Lambda(x)$, $\mu(x, \dot{x})$, and $p(x)$ respectively and v is the new command vector of the decoupled end-effector. Various control structures can be selected for v , depending on the nature of the task specified for the end-effector (pure motion, constrained motion involving force control and so forth). An adaptive control for nonredundant manipulators in the task space was proposed by Balestrino, De Maria, and Sciavicco (1983).

On the other hand, if the manipulator is redundant, one should consider the dynamic model of the unconstrained end-effector (8), which is based on the generalized inverse matrix in (9). A nonlinear dynamic decoupling control can be designed in the same formal manner as in (19). The resulting joint force vector is given by (10), where the vector τ_0 plays the same role as the vector q_0 in (3) (Khatib, 1987).

A different strategy can be pursued according to the augmented task space formulation proposed in a previous section. Differentiating eq. (13) with respect to time yields

$$\dot{\tilde{x}} = J(q)\dot{\tilde{q}} + \dot{J}(q)\dot{q} \quad (20)$$

with J defined in (14). If the matrix $J(q)$ has full rank ($m + r$), it is possible to solve (20) for the joint acceleration vector $\ddot{q}$ as

$$\ddot{q} = J^+(q)[\dot{\tilde{x}} - \dot{J}(q)\dot{q}] \quad (21)$$

and then plug (21) into the joint space dynamic model (4), i.e.

$$H(q)J^+(q)[\dot{\tilde{x}} - \dot{J}(q)\dot{q}] + C(q, \dot{q})\dot{q} + g(q) = \tau. \quad (22)$$

According to the nonlinear dynamic decoupling control strategy adopted above, the control τ can be thought of as

$$\tau = \hat{H}(q)J^+(q)[v - \dot{J}(q)\dot{q}] + \hat{C}(q, \dot{q})\dot{q} + \hat{g}(q) \quad (23)$$

where v is chosen as in (Khatib, 1987). Parallel research efforts have been produced by Egeland (1987) who designed an augmented task space controller for a macro-micro robot system, and by Hsu, Hauser, and Sastry (1988) who achieved tracking of a given end-effector trajectory and control of the joint velocities in the null space of the end-effector Jacobian.

On the other hand, if the matrix $J(q)$ in (14) has not full rank ($m + r$), a generalized inverse J^I of J can be adopted. It can be proved that a computationally simple expression of J^I is given by

$$J^I = [(I - \tilde{J}_c^I J_c)J_e^I \quad \tilde{J}_c^I] \quad (24)$$

with $\tilde{J}$ given in (17b), as found by Maciejewski and Klein (1985) and Nakamura, Hanafusa, and Yoshikawa (1987). In particular, if the end-effector Jacobian J is free of singularities, J^I can be used in lieu of J^+ . If $\tilde{J}$ is also free of singularities - that is to say that J has full rank, when the constraints are well-posed - $\tilde{J}_c^I$ can be used in lieu of $\tilde{J}_c$.

CONCLUSIONS

The problem of controlling a redundant manipulator has been considered in this work in the context of an augmented task space formulation. Namely, the end-effector task vector is suitably augmented by a set of functional task constraints which span at most the space of redundancy available. If the end-effector motion is in free space, a two-stage strategy can be adopted; first solve for the inverse augmented kinematics, then design a dynamic

control in the joint space. On the other hand, if the end-effector motion is constrained by the environment, the joint accelerations are solved from the augmented kinematic equation and plugged into the manipulator dynamic model, leading to the design of a dynamic control in the task space. In an augmented task space setting, it has been argued how the addition of functional constraints may be critical, in which case a task-priority strategy has been adopted as an effective means to guarantee that at least the end-effector task is accomplished. The nonlinear dynamic decoupling strategy has been used for the design of the dynamic controls, respectively in the joint space and in the task space. Issues like dynamic parameter adaptation and robustness to uncertainties and/or noncompensated dynamic terms (Slotine and Li, 1987) have not been addressed here, and constitute then a guarantee for further research work.

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