

ROBUST CONTROL OF INDUSTRIAL ROBOTS

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Abstract. The dynamic control of industrial robots is one of the most important issues in today's advanced robotics research. This paper first illustrates a new control algorithm and some relative schemes in order to guarantee perfect trajectory tracking both in the task space, usually the Cartesian one, and in the joint space. Such control allows joint servoing decoupling and proves so robust that practical implementation is realizable without requiring on-line accurate evaluation of robot model and load. The kinematic control, that is the coordinate transformation problem, is then faced up and a very general solution algorithm which only makes use of direct kinematics is derived.

Keywords. Robot kinematics and dynamics; nonlinear control systems; model reference adaptive control.

INTRODUCTION

The task of an industrial robot's controller consists in the control of the robot's end effector in such a way that it is able to follow a desired motion trajectory, which is described in a coordinate frame X which identifies the so-called Task Oriented Space. The reference frame is usually a Cartesian one and the state of the end effector is described through a wrist position vector p and a hand orientation unit vector triple s, a, n . The mechanical structure is a series of rigid bodies linked together by joints, on which generalized forces act; these forces are generated by actuators and can be considered as control inputs, whereas the relative position of the adjacent rigid bodies are the variables to be controlled so that the mechanical structure matches a desired configuration. Such displacements, i.e. joint variables, are usually taken as a generalized coordinate system θ which identifies the space in which control actions are developed (Drive Oriented Space). The direct relationship between the two spaces (DOS $\rightarrow$ TOS) is

$$x = f(\theta) \quad (1)$$

where f is a continuous nonlinear function, whose structure and parameters are known. It is unique, while the inverse transformation

$$\theta = f^{-1}(x) \quad (2)$$

is not unique and, because of complexity of (1), is hardly ever analytically known.

To control a robot means to be able to solve (2) since, given the goal trajectory in the task space, it is necessary to find the joint variable values which represent the set points for the position servos at each degree of freedom. The solution of (2) is a very complicated problem and a general solution algorithm does not exist indeed. In order to overcome the drawback that $f^{-1}(x)$ is not analytically definable, one can consider the relationship between velocities, that is

$$\dot{x} = J(\theta)\dot{\theta} \quad (3)$$

where $J(\theta)$ is the Jacobian matrix, derived from $f(\theta)$, which is almost everywhere invertible in case of nonredundant kinematical structures. (3) can be used to solve for the inverse velocity equation, that is a relationship between position increments

$$\Delta\theta = J^{-1}(\theta)\Delta x. \quad (4)$$

Both the above approaches, once the end effector desired trajectory $\hat{x}(t)$ is known, allows the determination of the corresponding values of generalized variables $\hat{\theta}(t)$ to be used as set points for the joint servos. In practice, however, the trajectory is divided into elementary paths and the coordinate transformation is performed for a finite number of via points (Paul, 1982); because of the computational burden required, the maximum sampling frequency achievable is below 100 Hz. Since the joint position servos require set points which are updated at a higher frequency, the controller is asked to execute some kind of interpolation in the DOS.

Such a control, known as kinematic control, is very often used for industrial robots; in practice it is a decoupled control of the degrees of freedom, whose coordination acts at inverse kinematic level. It does not account, indeed, for masses, inertias, backlashes and elasticities which modify the robot's behavior; the above technique, however, is invalidated whenever higher performance, in terms of operational speed and tracking accuracy, is required.

quired. In such cases one must consider the robot dynamical model which, on the assumption of rigid bodies, for an n -degree of freedom mechanical structure is in the form of

$$M(t) = B(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + E(\theta) \quad (5)$$

where θ is the n -vector of joint variables, $B(\theta)$ is the generalized positive definite inertia matrix, $C(\theta, \dot{\theta})$ is the vector taking into account centrifugal and Coriolis forces, $E(\theta)$ is the vector of gravitational forces, and $M(t)$ is the vector of generalized forces which act on joints.

On the other hand, with reference to the TOS, the robot can also be described through the model (Balestrino and co-workers, 1983a)

$$F(t) = J^{T-1}(x)B(x)J^{-1}(x)\ddot{x} + C_x(x, \dot{x})\dot{x} + J^T(x)E(x) \quad (6)$$

where F are the generalized forces applied to the robot's end effector. The derivation of models (5) and (6) is based on the assumption that rigid bodies are concerned; considering lumped elasticity phenomena leads to double the number of state variables needed.

By the term robot dynamical control one is referred to those techniques which are used for the control system synthesis on the basis of robot dynamical equations. The main guidelines are: control via inversion of dynamic equations, decoupled dynamic control, and control via a reference model. All those techniques have not been applied so far in robot industry. The control via dynamic inversion is based on the fact that, if the desired trajectory in terms of $\hat{\theta}, \ddot{\theta}$ is available as the result of a kinematic inversion procedure, on the assumption of negligible disturbance forces, one can evaluate $M(t)$ in (5), that is the generalized forces to be applied to the joints in order to obtain the trajectory itself. Since such control technique is open loop, it generally requires a closed loop compensation which acts in the neighborhood of superimposed nominal motions (Whitney, 1969), (Paul, 1972), (Markiewicz, 1973), (Luh and co-workers, 1980) and (Vukobratovich and Stokic, 1982). On the other hand, with a decoupled dynamic control, the typical drawbacks due to the high nonlinear interaction degree are overcome by recurring to suitable nonlinear position and velocity feedback control laws (Freund, 1977), (Lentini and co-workers, 1981). Both the above techniques, however, require a continuous accurate knowledge of the coefficients of the robot mathematical model; thus they involve a high computational burden. The control technique which makes use of a reference model is strictly part of the adaptive control theory; it allows a decoupled control for the degrees of freedom, without requiring a precise knowledge of robot parameters (Dubowsky and DesForges, 1979), (Horowitz and Tomizuka, 1983).

On the basis of the above remarks which are concerned with the control of industrial robots, robust control laws will be formulated in the sequel, and a general solution algorithm for the inverse kinematic problem will be lastly proposed (Balestrino and co-workers, 1983b), (Balestrino

and co-workers, 1984), (Balestrino and co-workers, 1983c) and (Balestrino and co-workers, 1985).

MODEL REFERENCE ADAPTIVE CONTROL

As it clearly emerges from what previously said, an efficient control system for industrial robots must have the following requirements:

- i) easy implementation, without need of using on-line an accurate robot mathematical model,
- ii) a satisfactory tracking accuracy, independently of load variations,
- iii) chance of performing a decoupled control on the degrees of freedom.

Among the several dynamic control techniques which have been proposed for industrial robots, the model reference adaptive control seems the best fitting the above requirements. In order to avoid the on-line solution of the linear reference model, an inverse model technique can be adopted which, from the knowledge of desired position, velocity and acceleration, yields a term corresponding to a linear feedforward action. The resulting control scheme (Balestrino and co-workers, 1983a) is reported in Fig. 1, where $r, \dot{r}, \ddot{r}$ are vectors of opportune dimensions which represent the reference position, velocity and acceleration, y and $\dot{y}$ are the robot input position and velocity vectors, and the vector v is a linear combination of e and $\dot{e}$ via opportune coefficients; F represents the robot input generalized forces, u_M is a linear combination of $r, \dot{r}, \ddot{r}$, and finally K_P and K_U are specific compensating actions, feedforward and feedback respectively.

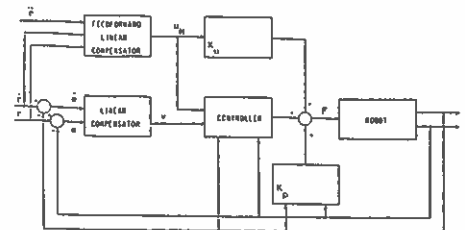


Fig. 1. General Adaptive Control Scheme.

Since for each kind of industrial robot the structural properties concerning its mathematical model (perfect model following conditions and positive definiteness of generalized inertia matrix) always hold, it is always possible to determine an adaptive controller which ensures the trajectory tracking with $e = \dot{e} = 0$, even in case of high performance requirements, as far as operational speed is concerned (Dubowsky and DesForges, 1979), (Balestrino and co-workers, 1983a). The adaptation mechanism must be chosen in order to guarantee the stability of the control system; the resulting synthesis procedure makes use of hyperstability theory. A very easy control law which satisfies such property is

$$\phi(v, y, \dot{y}, u_M) = \frac{v}{\|v\|} (\alpha \Sigma (|y_1| + |\dot{y}_1|) + \beta \Sigma |u_{M1}|) \quad (7)$$

where $v = D_1 e + D_2 \dot{e}$ with D_1 and D_2 diagonal matrices

determined via Lyapunov equation solution (Balestrino and co-workers, 1983). α and β are characteristic parameters of the adaptation mechanism which can be determined through an estimate of the maximum range of variation of robot parameters and actions K_p and K_v . As v tends to zero, ϕ represents a discontinuous forcing signal whose amplitude is modulated by the term $\alpha \Sigma + \beta \Sigma$, and whose frequency is infinite from a theoretical viewpoint, but high enough from a practical viewpoint. An adequate choice of feedforward and feedback actions can act on the amplitude of such signal. As it can be seen the adaptation mechanism is extremely simple to implement. The adaptation, indeed, is obtained not via parameter variations of a controller, determined by a self-tuning procedure, but through a high gain in the neighborhood of $v = 0$ in a fixed parameter structure control system. On the other hand, the nonlinear part of the adaptation mechanism tends to make the tracking error dynamics for each degree of freedom evolve on $v = 0$; such dynamics is determined by the equation

$$\dot{e} = D^{-1} D_1 e. \quad (8)$$

It must be outlined, moreover, that the input and the output variables of the above scheme can represent both the joint and the task space variables; in other words the proposed technique can be used to synthesize the control both in the DOS (Dubowsky and Des Forges, 1979) and in the TOS (Balestrino and co-workers, 1983a). In the following those two approaches are further illustrated.

DRIVE ORIENTED SPACE CONTROL

By means of the nonlinear feedforward actions previously indicated, the scheme of Fig. 2 proves adequate in order to minimize the factors α and β , which concur to determine the amplitude of the discontinuous input. The task of the nonlinear actions consists in counteracting gravity and inertia terms and decoupling, at the same time, the degrees of freedom. The goal of the adaptive controller is to compensate for Coriolis terms and counteract parameter variations and inaccurate modeling effects. This solution is equivalent to defining a nominal control for the robot of the type

$$M_n = B(\theta)\ddot{\theta} + E(\theta) \quad (9)$$

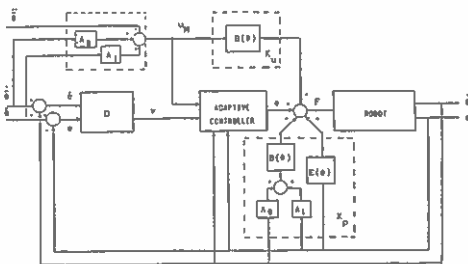


Fig. 2. Adaptive Control Scheme in the DOS with Nonlinear Compensation.

and a compensation term, given by the adaptation signal, which ensures the global asymptotic stability. The resulting control is decoupled for the singular degrees of freedom and one controller supervises all the joints; this is due both to the presence of blocks $B(\theta)$, $E(\theta)$ and to the structure itself of the signal ϕ (the term $v/\|v\|$ must be evaluated).

On the other hand one can obtain a completely different realization if a structure with independent controllers for each degree of freedom is desired. In this case it is necessary to set up $K_p = K_v = 0$ since, due to the diagonally dominant character of matrix $B(\theta)$, the control signal becomes

$$\phi = \gamma \operatorname{sgn} v \quad (10)$$

where

$$\gamma = \alpha \Sigma (|\dot{\theta}_{imax}| + |\dot{\phi}_{imax}|) + \beta \Sigma u_{Mimax}. \quad (11)$$

Since each component of the error vector v does only depend on position and velocity of the corresponding joint, the decentralized, and decoupling at the same time, structure of Fig. 3 is achieved. A comparison with the previous solution bears evidence of the simplicity and modularity of the implementation of the solution just presented. It must be noted, however, that in this case the amplitude of the discontinuous signal is constant since it is obtained through maximization. Consequently the choice between the two solutions must be made on the basis of a compromise between implementation simplicity and actuators cost.

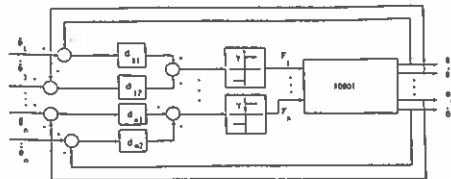


Fig. 3. Decoupled Control Scheme in the DOS.

TASK ORIENTED SPACE CONTROL

On the basis of the robot model in the TOS (6), the control scheme of Fig. 4 can be obtained. Also in this case nonlinear actions have been introduced in order to reduce the amplitude of the discontinuous input signal generated by the adaptation mechanism. From a theoretical viewpoint this scheme corresponds to the one of Fig. 2. The matrices $J^{-1}(\theta)$ and $J^T(\theta)$ are needed to transform linear accelerations into angular accelerations and end effector forces into joint moments respectively. The matrix $J(\theta)$ and the vector $f(\theta)$ have been introduced in order to reconstruct Cartesian position and velocity measurements from the corresponding joint ones.

Setting $K_p = K_v = 0$ leads to the scheme of Fig. 5 which, even if in principle seems to correspond to the one of Fig. 3, in reality deeply differs from

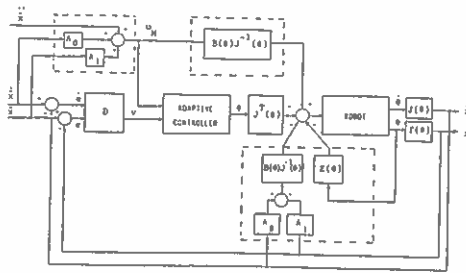


Fig. 4. Adaptive Control Scheme in the TOS with Nonlinear Compensation.

that one. In this case, indeed, the decoupled control in terms of task variables cannot be realized by means of a set of independent controllers, since the decoupling in X corresponds to interaction in θ . The most interesting features of such a technique, however, are the following ones:

- i) the computational burden, even if not negligible, refers to a control scheme which does not require any coordinate transformation procedure,
- ii) the only nonlinear functions to be evaluated on-line with adequate accuracy are the direct kinematics ones,
- iii) the adaptive controller counteracts the effects due to nonconsidered terms and parameter variations; as a consequence the matrix B and the vector E to be determined on-line can be evaluated even with low accuracy.

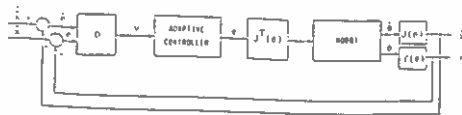


Fig. 5. Adaptive Control Scheme in the TOS without Compensation.

COORDINATE TRANSFORMATION

In order to provide the DOS robot controller with the reference joint variables, a coordinate transformation from the TOS to the DOS is needed (inverse kinematic problem). Any coordinate transformation technique must be evaluated in terms of a compromise between computation time and precision requirements. So as it results from literature there are no general coordinate transformation algorithms; in other words the transformation software is specifically designed for each kinematical structure.

A very general coordinate transformation algorithm is the one suggested in Fig. 6. If the system is asymptotically stable it is evident that, resulting a null steady-state error, the vector θ corre-

sponding to $\hat{x}$ can be determined on the basis of direct relationship $f(\theta)$. If the controller is capable to ensure a null error for all trajectories $\hat{x}(t)$ satisfying certain constraints, one can obtain

$$\theta(t) = f^{-1}(x) = f^{-1}(\hat{x}) = \hat{\theta}(t). \quad (12)$$

In other words the dynamic system of Fig. 6 represents a coordinate transformation algorithm if and only if it is possible to determine a controller which ensures that $x(t)$ tends to $\hat{x}(t)$ (Balestrino and co-workers, 1983).

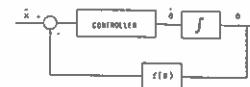


Fig. 6. Coordinate Transformation General Scheme.

For most industrial robots the position of the wrist is independent of the orientation of the hand; in this case the coordinate transformation problem can be conveniently divided into two subproblems, one relative to the wrist position and the other one to the hand orientation. To this end let

$$x_p = f_p(\theta_p) \quad \dot{x}_p = J_p(\theta_p) \dot{\theta}_p \quad (13)$$

$$x_h = f_h(\theta_p, \theta_h) \quad \dot{x}_h = J_{hp}(\theta_p, \theta_h) \dot{\theta}_p + J_{hh}(\theta_p, \theta_h) \dot{\theta}_h \quad (14)$$

be the direct kinematic relations between TOS and DOS positions and velocities, where subscripts p and h refer to wrist and hand respectively. x_p represents the vector of Cartesian wrist coordinates which depends on the angular displacements of the preceding joints, whereas x_h is a six component vector which is specified by any two of the three unit vectors s, a, n which usually define the hand orientation. By making use of the structural properties of the above Jacobian matrices and recurring to a control system approach based on the direct method of Lyapunov (Balestrino and co-workers, 1985), the scheme of Fig. 7 is achieved; it ensures that

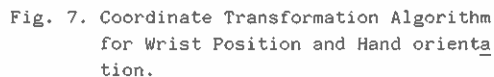
$$\lim_{t \rightarrow \infty} (\hat{x}_p - x_p) = 0 \quad \forall \hat{x}_p \quad (15)$$

$$\lim_{t \rightarrow \infty} (\hat{x}_h - x_h) = 0 \quad \forall \hat{x}_h,$$

that is $e_p = 0$, $e_h = 0$ are stable equilibrium points. This means that, if the initial conditions on $\hat{x}_p$ and θ_p are so that $e_p(0) = e_h(0) = 0$ (so as it is reasonable for a coordinate transformation algorithm), the tracking errors maintain identically zero along the whole trajectory. It must be emphasized that only the direct kinematic relations must be evaluated on-line, and there is no problem of nonuniqueness of the solution. The controllers γ_p and γ_h which ensure the convergence to zero of errors e_p and e_h are respectively

$$\gamma_p = \alpha_p + \frac{e_p^T J_p^T}{e_p^T J_p J_p^T e_p}, \quad \alpha_p > 0 \quad (16)$$

where $\Lambda(A)$ and $\lambda(A)$ denote the maximum and the minimum eigenvalue of matrix A respectively.



In the previous sections robust control laws, both in the Task Oriented Space and in the Drive Oriented Space, and coordinate transformation procedures have been derived for continuous time systems. Since such algorithms are to be implemented on reduced complexity computer systems, time discretization and quantization problems naturally arise. The effects of the sampling period have been evaluated by means of simulating the proposed control laws for a three degree of freedom robot; the tracking error along a trajectory line of 1.5 m with a maximum velocity of 1.5 m/s has been shown to be limited to 5 mm in the worst case corresponding to a sampling frequency of 100 Hz, and to .5 mm with a sampling frequency of 1KHz. The steady-state error has always resulted negligible. Analogously, as far as the inverse kinematic problem is concerned, simulation results for a six degree of freedom robot have shown the effectiveness of the proposed scheme, giving a wrist position error less than 1 mm and a hand orientation error less than 1° .

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