

Kinematics, Dynamics and Control for a Class of Parallel Robots

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Abstract

This paper presents a survey on kinematics, dynamics and control for a class of parallel robots having a hybrid kinematic design, i.e. a three-degree-of-freedom (DOF) parallel structure with a radial link of variable length, and a 3-DOF spherical wrist. The inverse kinematics equation is obtained by using spherical coordinates. The inverse differential kinematics is derived, and a Jacobian-based Closed-Loop Direct Kinematics (CLDK) algorithm is adopted to solve the direct kinematics problem along a given trajectory. Then, a dynamic model is computed to be used for both simulation and control purposes. Finally, a model-based control is derived aimed at enforcing a 6-DOF impedance behaviour at the end effector to manage interaction with the environment.

1 Introduction

Parallel robots are constituted by a fixed base and a mobile base, connected by a number of independent kinematic chains [1]. They have lately been receiving quite a deal of attention not only in the academic community but also in the industrial community; their main advantage regards the possibility to obtain high structural stiffness and perform high-speed motions. Two recent examples of parallel structures for industrial robots are the Tricept robot designed by Neumann [2] and the Flexpicker robot based on the Delta design by Clavel [3].

In this work the class of parallel robots having the kinematic structure of the so-called Tricept robot is considered (figure 1). This class is characterized by a hybrid kinematic design, comprising a 3-DOF parallel structure with a radial link of variable length connected to the mobile base, and a 3-DOF spherical wrist.

The kinematics of this class of robots has to be analyzed with specific concern to the parallel structure. Spherical coordinates are used to obtain the inverse kinematics equation of the structure. Then, the inverse differential kinematics

mapping is derived in terms of the inverse Jacobian [4]. This is at the basis of a Closed-Loop Direct Kinematics (CLDK) algorithm which employs either the transpose of the inverse Jacobian or the direct Jacobian [5].

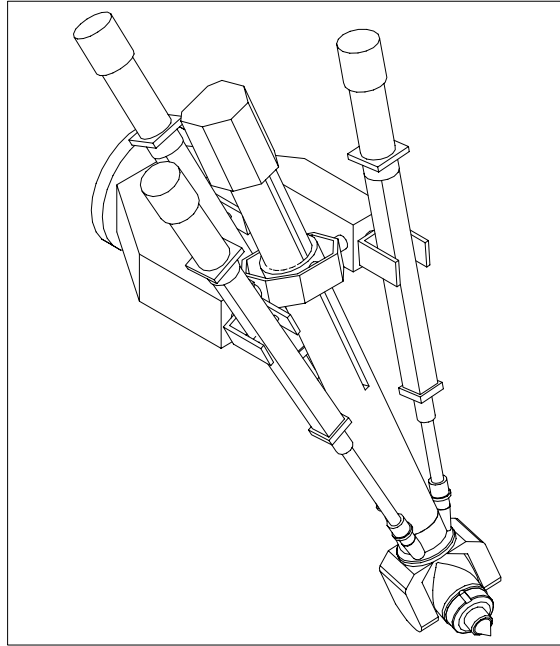


Figure 1: 3D drawing of the Tricept robot

On the other hand, the availability of a dynamic model is crucial for computation of direct dynamics and ultimately for parameter identification and model-based dynamic control purposes [6]. A symbolic dynamic model is derived via the Lagrangian formulation under simplifying assumptions leading to an approximate solution [7]. Such a model is adopted to develop a model-based impedance control strategy to manage the interaction of the robot with the environment [8]. In particular, a spatial impedance control [9] is designed by resorting to an inner position/orientation loop, ensuring good robustness to disturbances and modelling approximations, and an outer loop, which confers a compliant behaviour of the end effector with suitable dynamic features, characterized by a desired inertia, damping and stiffness [10].

2 Inverse kinematics

The Tricept robot has a 3-DOF structure of parallel type to execute translational motions and a 3-DOF spherical wrist to execute rotational motions.

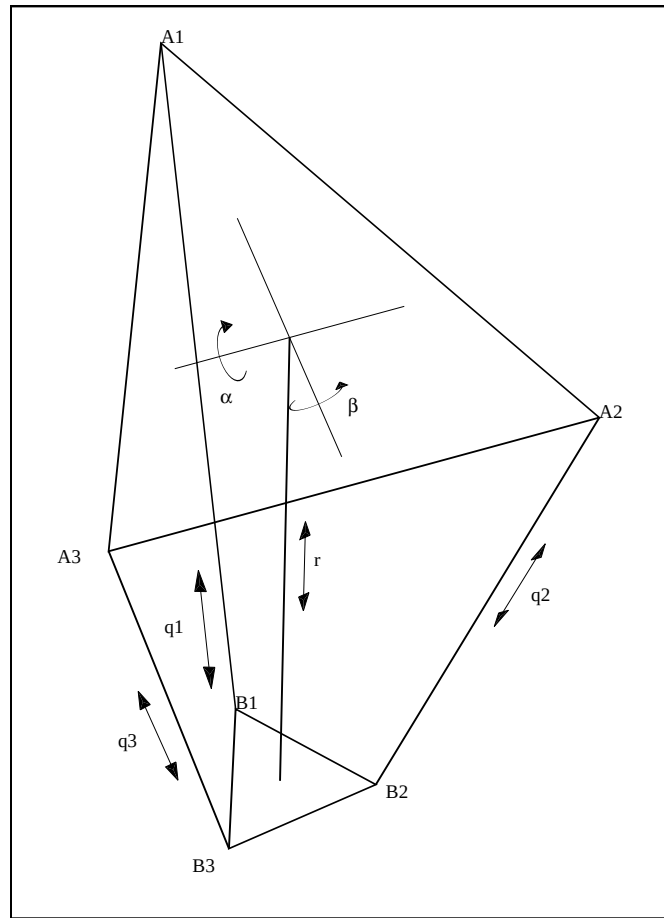


Figure 2: Sketch of the parallel kinematic structure

A sketch of the parallel kinematic structure is illustrated in figure 2. This consists of three identical outer links with actuated prismatic joints which can be grouped into

$$\mathbf{q}_p = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}.$$

The fixed base is part of the supporting structure of the manipulator. Each link is connected to the fixed base by means of a Cardan joint allowing the link to rotate about two axes that are both orthogonal to the link. On the other end, the links are connected to the mobile base by three spherical joints. Besides the above three outer links, a radial link is present. This is connected to the fixed base by a 3-DOF joint allowing the link to rotate by the angles α , β about two mutually orthogonal axes (one of which is orthogonal itself to the link), and to translate by r along a through-hole axis; the connection of the radial link to the mobile base is fixed and orthogonal so as to avoid axial rotations and determine the reference point for the attachment of the spherical wrist.

It is easy to recognize that the parallel structure has 3 DOF's which can be effectively described by the vector of spherical coordinates

$$\mathbf{x}_p = \begin{bmatrix} r \\ \alpha \\ \beta \end{bmatrix}.$$

These, together with the vector of the three joint angles of the wrist

$$\mathbf{x}_o = \begin{bmatrix} q_4 \\ q_5 \\ q_6 \end{bmatrix},$$

constitute the 6 DOF's of the robot that can be collected into the vector of coordinates

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_p \\ \mathbf{x}_o \end{bmatrix}.$$

Hence, the kinematic model of the manipulator can be seen as that of an equivalent open-chain spherical arm, characterized by the three joint coordinates r , α and β , with a spherical wrist, characterized by the three joint coordinates q_4 , q_5 and q_6 .

A suitable Jacobian matrix $\mathbf{J}$ relates the velocity vector $\dot{\mathbf{x}}$ to the joint velocities $\dot{q}_i$ ($i=1, \dots, 6$) collected into the vector $\dot{\mathbf{q}}$:

$$\dot{\mathbf{q}} = \begin{bmatrix} \mathbf{J}_p^{-1}(\mathbf{x}_p) & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{I}_3 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}}_p \\ \dot{\mathbf{x}}_o \end{bmatrix} = \mathbf{J}^{-1}(\mathbf{x})\dot{\mathbf{x}}, \quad (1)$$

where the Jacobian $\mathbf{J}_p$ can be computed through the differential kinematics of the structure [5].

Let $\mathbf{p}_e$ be the position of a frame attached to the end effector, and $\mathbf{R}_e$ the rotation matrix expressing the orientation of the frame. Then, the kinematic model is completed by expressing the relationship between $\dot{\mathbf{x}}$ and the linear and angular velocity of the end-effector frame $\mathbf{v}_e = [\dot{\mathbf{p}}_e^T \ \dot{\boldsymbol{\omega}}_e^T]^T$, i.e.

$$\mathbf{v}_e = \mathbf{J}_e(\mathbf{x})\dot{\mathbf{x}} \quad (2)$$

where $\mathbf{J}_e$ coincides with the geometric Jacobian of the equivalent open-chain manipulator defined above, which can be computed via standard techniques [6].

3 Closed-Loop Direct Kinematics Algorithm

The direct kinematics problem for a parallel robot consists of finding the vector of position coordinates $\mathbf{x}$ as a function of the vector of joint variables $\mathbf{q}$. Typically, such a problem does not admit closed-form solutions and numerical algorithms should be used. A crucial point with using a numerical algorithm [11] regards the possibility of computing all the feasible solutions to the problem, and preventing solutions to jump from one branch to another. For the parallel structure at issue in this work, up to eight solutions can be found; this result can be derived by referring to the direct kinematics of the parallel manipulator analyzed in [12] when three of the six links have fixed length and intersect at a common point.

A conceptually different approach to the problem can be pursued as follows. Instead of seeking one (or more) position solution corresponding to the given set of joint variables, the direct kinematics problem can be formulated as that to determine the motion of the mobile base as a function of the joint motion. In other words, assume that the initial joint configuration is assigned and a corresponding feasible position is known, e.g. via an off-line analytical or numerical procedure; assume, also, that a joint trajectory is assigned as a function of time (joint positions and velocities). Then, the goal is to compute on-line the resulting trajectory of the mobile base (position and velocity) starting from the initial posture of the robot.

Such approach involves the use of the robot differential kinematics as in the well-known Closed-Loop Inverse Kinematics (CLIK) scheme [13] developed for open-chain robot manipulators, which provides an inverse kinematics algorithm whose convergence is ensured through the stability of a closed-loop dynamic system of the tracking error.

Therefore, an effective solution to the direct kinematics problem for a parallel robot can be devised by transposing the above CLIK algorithm as explained below. Since the spherical wrist is standard and its direct and inverse kinematics is well known, the algorithm will be derived only for the parallel structure, i.e. to compute the position of the mobile base $\mathbf{x}_p$ from the joint positions $\mathbf{q}_p$.

Let $\mathbf{q}_{pd}$ denote a set of desired joint variables and

$$\mathbf{e} = \mathbf{q}_{pd} - \mathbf{q}_p \quad (3)$$

denote the error between $\mathbf{q}_{pd}$ and the computed joint variables $\mathbf{q}_p$. Differentiating (3) with respect to time and accounting for (2) yields

$$\dot{\mathbf{e}} = \dot{\mathbf{q}}_{pd} - \mathbf{J}_p^{-1}(\mathbf{x}_p)\dot{\mathbf{x}}_p. \quad (4)$$

This error dynamics equation is at the basis of a Closed-Loop Direct Kinematics (CLDK) algorithm based on the geometric Jacobian. In particular, taking

$$\dot{\mathbf{x}}_p = \mathbf{J}_p(\mathbf{x}_p)(\dot{\mathbf{q}}_{pd} + \mathbf{K}\mathbf{e}) \quad (5)$$

leads to the linear system

$$\dot{\mathbf{e}} + \mathbf{K}\mathbf{e} = \mathbf{0}, \quad (6)$$

and the choice of a positive definite (symmetric) matrix gain $\mathbf{K}$ guarantees that the error uniformly converges to zero, i.e. $\mathbf{x}_p$ that can be computed by integration of (5) is a solution to the direct kinematics problem. Notice that both position and velocity are obtained for given joint position and velocities; the closed loop on the tracking error of the algorithm guarantees convergence and eliminates the steady-state errors, as in typical open-loop resolved rate schemes instead. The resulting CLDK algorithm can be represented in block scheme form as in figure 3 which shows that the inverse of the Jacobian inverse has to be computed besides the inverse kinematics function in the return path.

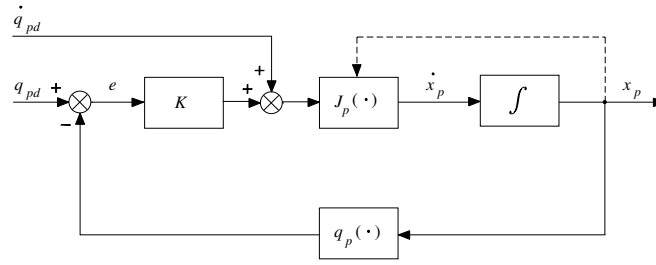


Figure 3: Block scheme of CLDK algorithm using the inverse of the Jacobian inverse

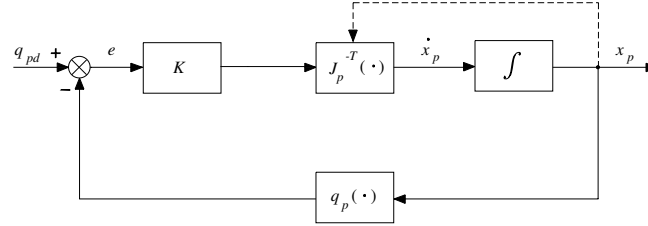


Figure 4: Block scheme of CLDK algorithm using the transpose of the Jacobian inverse

An alternative and computationally more efficient solution can be devised which avoids the inversion of the inverse geometric Jacobian to compute $\mathbf{J}_p$ in (5). In fact, the choice

$$\dot{\mathbf{x}}_p = \mathbf{J}_p^{-T}(\mathbf{x}_p) \mathbf{K} \mathbf{e} \quad (7)$$

is based on the transpose of the inverse geometric Jacobian. In this case, convergence of the algorithm can be ascertained by resorting to a Lyapunov argument [5]. The resulting CLDK algorithm can be represented in block scheme form as in figure 4 which shows that the transpose of the Jacobian inverse has to be computed besides the inverse kinematics function in the return path. Notice that, differently from the solution (5), the solution (7) does not require the inversion of the inverse geometric Jacobian and thus it may work well also in the neighbourhood of kinematic singularities.

4 Dynamics

The dynamic model in symbolic form can be determined based on the Lagrangian formulation under the following simplifying assumptions [7]:

- the center of mass of the cylinder of each outer link coincides with the center of the Cardan joint;
- the center of mass of the piston of each outer link coincides with the center of the spherical joint;
- the rotational contribution of the outer links to the kinetic energy is neglected.

Taking $\mathbf{x}$ as the vector of generalized coordinates, the equations of motion can be written in the form

$$\mathbf{B}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{g}(\mathbf{x}) = \boldsymbol{\gamma} - \mathbf{J}_e^T(\mathbf{x})\mathbf{h} \quad (8)$$

where $\mathbf{B}$ is the inertia matrix, $\mathbf{C}\dot{\mathbf{x}}$ is the vector of Coriolis and centrifugal forces, $\mathbf{g}$ is the vector of gravitational forces, and $\mathbf{h}$ is the vector of end-effector contact forces. The expressions of the various terms can be found in [7].

The vector $\boldsymbol{\gamma}$ contains the generalized forces related to the joint actuating forces via the kinetostatic mapping

$$\boldsymbol{\tau} = \mathbf{J}^T \boldsymbol{\gamma}, \quad (9)$$

where $\mathbf{J}$ is the Jacobian matrix in (1).

An exact dynamic model can be derived by using the virtual work principle [14]. The resulting equations can be computed only numerically in view of the difficulties concerned with the derivation of a symbolic solution. Because of the heavy computational burden, this model is not suitable for control purposes. Hence, in the following the approximate symbolic model will be adopted in the design of the impedance control.

5 Impedance Control

According to the well-known inverse dynamics strategy, the vector of generalized forces can be chosen as [6]

$$\boldsymbol{\gamma} = \mathbf{B}(\mathbf{x})\mathbf{y} + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{g}(\mathbf{x}) + \mathbf{J}_e^T(\mathbf{x})\mathbf{h} \quad (10)$$

where $\mathbf{y}$ is a resolved acceleration given by

$$\mathbf{y} = \mathbf{J}_e^{-1}(\mathbf{x}) (\mathbf{a} - \dot{\mathbf{J}}_e(\mathbf{x}, \dot{\mathbf{x}}) \dot{\mathbf{x}}) \quad (11)$$

with

$$\mathbf{a} = \begin{bmatrix} \mathbf{a}_p \\ \mathbf{a}_o \end{bmatrix}. \quad (12)$$

Folding (10), (11) and (12) in (8) and accounting for the time derivative of (2) yields

$$\ddot{\mathbf{p}}_e = \mathbf{a}_p \quad (13)$$

$$\dot{\boldsymbol{\omega}}_e = \mathbf{a}_o. \quad (14)$$

In order to track a position trajectory for a reference frame c , $\mathbf{a}_p$ can be selected as

$$\mathbf{a}_p = \ddot{\mathbf{p}}_c + \mathbf{K}_{Dp} \Delta \dot{\mathbf{p}}_{ce} + \mathbf{K}_{Pp} \Delta \mathbf{p}_{ce} \quad (15)$$

where $\mathbf{K}_{Dp}$, $\mathbf{K}_{Pp}$ are suitable positive definite matrix gains and $\Delta \mathbf{p}_{ce} = \mathbf{p}_c - \mathbf{p}_e$ is the position error between the reference and the end-effector frame. On the other hand, tracking of an orientation trajectory for the reference frame c is achieved by choosing $\mathbf{a}_o$ as [15]

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_c + \mathbf{K}_{Do} \Delta \boldsymbol{\omega}_{ce} + \mathbf{K}_{Po} \mathbf{R}_e {}^e \boldsymbol{\epsilon}_{ce} \quad (16)$$

where $\mathbf{K}_{Do}$, $\mathbf{K}_{Po}$ are suitable positive definite matrix gains, $\mathbf{R}_e$ is the rotation matrix of the end-effector frame and ${}^e \boldsymbol{\epsilon}_{ce}$ is the vector part of the unit quaternion expressing the mutual orientation between the reference and the end-effector frame when referred to the base frame. The interested reader is referred to [5] for details on the use of the unit quaternion for robot control.

Typically, a position and orientation trajectory for a desired frame is specified in terms of $\mathbf{p}_d$ and $\mathbf{R}_d$. In the case of end-effector contact with the environment, it is worth introducing a mechanical impedance between the desired and the reference frame so as to contain the values of the interaction force and moment $\mathbf{h} = [\mathbf{f}^T \quad \boldsymbol{\mu}^T]^T$. Hence, the commanded motion for the reference frame c can be computed via the impedance equations so as to achieve a suitable compliant

behaviour for the end-effector [5]. Figure 5 illustrates a typical interaction task where the motion of the end effector is obstructed by a surface along the vertical direction.

In particular, the position $\mathbf{p}_c$ can be computed via the translational impedance equation

$$\mathbf{M}_p \Delta \ddot{\mathbf{p}}_{dc} + \mathbf{D}_p \Delta \dot{\mathbf{p}}_{dc} + \mathbf{K}_p \Delta \mathbf{p}_{dc} = \mathbf{f}, \quad (17)$$

where

$$\Delta \mathbf{p}_{dc} = \mathbf{p}_d - \mathbf{p}_c, \quad (18)$$

and $\mathbf{M}_p$, $\mathbf{D}_p$ and $\mathbf{K}_p$ are positive definite matrices representing the mass, damping, and stiffness characterizing the impedance.

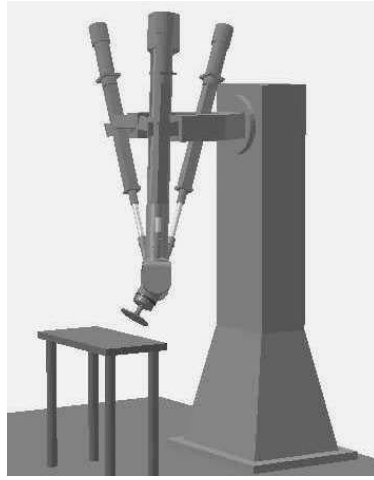


Figure 5: The Tricept robot executing an interaction task

In the classical 6-DOF approaches, the orientation of the reference frame $\mathbf{R}_c$ is computed via an impedance equation formally equal to (17), where a minimal representation of the end-effector orientation (i.e., in terms of three Euler angles) is used. However, as shown in [9], the rotational stiffness may become not well defined even for small orientation displacements and cannot be specified in a consistent way with the task geometry. This problem can be solved by resorting to a class of geometrically meaningful representations of the mutual orientation

between the desired frame and the reference frame. In detail, a geometrically consistent impedance equation for the rotational part is [9]

$$\mathbf{M}_o \Delta {}^c \dot{\boldsymbol{\omega}}_{dc} + \mathbf{D}_o \Delta {}^c \boldsymbol{\omega}_{dc} + \mathbf{K}'_o {}^c \boldsymbol{\epsilon}_{dc} = {}^c \boldsymbol{\mu}, \quad (19)$$

where ${}^c \boldsymbol{\epsilon}_{dc}$ is the vector part of the unit quaternion that can be extracted from ${}^c \mathbf{R}_d = \mathbf{R}_\ell^T \mathbf{R}_d$. The rotational stiffness matrix in (19) is

$$\mathbf{K}'_o = 2\mathbf{E}^T(\eta_{dc}, {}^c \boldsymbol{\epsilon}_{dc})\mathbf{K}_o, \quad (20)$$

with

$$\mathbf{E}(\eta_{dc}, {}^c \boldsymbol{\epsilon}_{dc}) = \eta_{dc} \mathbf{I} - \mathbf{S}({}^c \boldsymbol{\epsilon}_{dc}), \quad (21)$$

being $\mathbf{S}(\cdot)$ the matrix operator performing the cross product between two (3×1) vectors. The resulting impedance control can be represented in block scheme form as in figure 6.

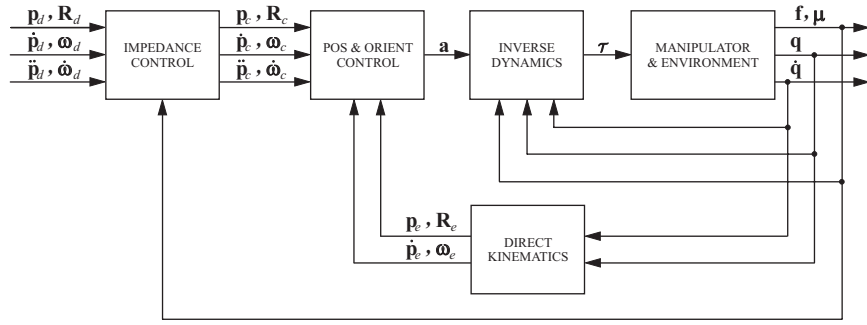


Figure 6: Block scheme of impedance control

The relationship between $\mathbf{K}'_o$ and $\mathbf{K}_o$ can be investigated by considering the decomposition of $\mathbf{K}_o$ as

$$\mathbf{K}_o = \mathbf{U}_o \boldsymbol{\Gamma}_o \mathbf{U}_o^T, \quad (22)$$

where $\boldsymbol{\Gamma}_o = \text{diag}\{\gamma_{e1}, \gamma_{e2}, \gamma_{e3}\}$ is the eigenvalue matrix and $\mathbf{U}_o = [\mathbf{u}_{e1} \ \mathbf{u}_{e2} \ \mathbf{u}_{e3}]$ is the (orthogonal) eigenvector matrix. An orientation displacement $\{\cos(\vartheta_{dr}/2), \sin(\vartheta_{dr}/2)\mathbf{u}_{ei}\}$ about the i -th eigenvector leads to

$$\mathbf{K}'_o {}^r \boldsymbol{\epsilon}_{dr} = \gamma_{oi} \sin \vartheta_{dr} \ \mathbf{u}_{oi} \quad (23)$$

which represents an elastic moment about the same $\mathbf{u}_{ei}$ axis. This implies that the rotational stiffness matrix $\mathbf{K}_o$ can be expressed in terms of three parameters γ_{ei} representing the stiffness about three principal axes $\mathbf{u}_{ei}$, and in turn it allows the rotational stiffness to be specified in a consistent way with the task geometry. Needless to say, task geometric consistency holds for the translational stiffness too.

6 Conclusion

In this paper kinematics, dynamics and control for a class of parallel robots have been surveyed. The inverse Jacobian has been employed to characterize the inverse differential kinematics mapping. The direct kinematics problem has been solved by resorting to a closed-loop algorithm based either on the inverse or on the transpose of the inverse Jacobian. The Lagrangian formulation has been adopted to derive the dynamic model. Finally, a geometrically consistent 6-DOF impedance has been designed to manage the interaction with the environment.

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