

Kineto-Static Analysis of Cooperative Robot Manipulators Achieving Dexterous Configurations

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INTRODUCTION

Adopting a system of cooperative manipulators is believed to offer enhanced performance in robotic applications. The use of multiple arms becomes necessary for the execution of all those tasks that go beyond the capabilities of a single arm, e.g. manipulation of large, heavy or non-rigid objects.

Cooperation can be successfully performed only when an effective coordination of the arms is realized. This demands for accurate synchronization of relative motions, solution of collision avoidance problems, ease of robot programming, and coordinated control.

In this paper, the basic case of a two-arm planar system is considered. An effective description of the cooperative task space is derived in terms of suitable absolute and relative task variables. The former are naturally expressed in a common base frame for the system while the latter are conveniently referred to a reference frame attached to the object.

If kinematic redundant degrees of freedom are available in the system, multiple joint configuration solutions exist. Here the closed-loop inverse kinematics scheme for constrained redundant manipulators with task space augmentation and task priority strategy [1] is suitably employed to solve the inverse kinematics problem for the cooperative arm system. The goal is to reconfigure the system in more dexterous postures [2] for the execution of a given task [3]. This is accomplished by considering the global task space external force manipulability ellipsoid [4] that gives a measure of the ratio between resulting object forces and applied joint torques; this ratio is maximized along the given task space direction.

Case studies are developed to show the effectiveness of the approach. Different types of redundancy are introduced by releasing one or more task space variables, and the system is reconfigured to more dexterous configurations to execute the given task.

COOPERATIVE TASK VARIABLES

Consider the planar system composed of two arms manipulating a common object sketched in Fig. 1. A suitable choice of absolute and relative task variables can be obtained as follows. Such variables allow an effective description of the cooperation

between the two arms and the object, although they are not directly related to the variables of the usual object space symmetric formulation [5].

The absolute position of the object is computed as

$$\mathbf{p}_a = 0.5(\mathbf{p}_1 + \mathbf{p}_2) \quad (1)$$

where $\mathbf{p}_1$ and $\mathbf{p}_2$ are the 2×1 position vectors of the end-effectors of the two arms expressed in the common base frame (x_b, y_b) . Let consider the reference frame (x_o, y_o) fixed to the object with origin in $\mathbf{p}_a$ and x_o -axis aligned with the vector $\mathbf{w} = \mathbf{p}_2 - \mathbf{p}_1$. Then, the absolute orientation of the object can be described as the angle formed by the x_o -axis with the x_b -axis

$$\varphi_a = \text{atan2}(w_{xb}, w_{yb}). \quad (2)$$

Next, the relative variables must be specified. First, the relative position of the two end-effectors can be described by the 2×1 vector $\mathbf{w}$. However, if $\mathbf{w}$ is conveniently expressed in the object frame, only the x_o component of $\mathbf{w}$ is significant; this leads to considering one scalar variable (the signed distance between the two end-effectors) to describe this portion of the task. Then

$$p_r = w_{xb} \cos \varphi_a + w_{yb} \sin \varphi_a. \quad (3)$$

As for the remaining variables, the grasp angle of each arm relative to the x_o -axis can be selected, i.e.

$$\varphi_{r1} = \varphi_a - \varphi_1 \quad (4)$$

$$\varphi_{r2} = \pi + \varphi_a - \varphi_2 \quad (5)$$

where φ_1 and φ_2 represent the orientation angles of the two end-effectors with respect to the base frame; by the way, each angle φ_i is given by the sum of the respective joint coordinates of each arm.

INVERSE KINEMATICS SCHEME

Suppose that each arm has n_i , $i = 1, 2$, degrees of freedom. If $\bar{\mathbf{x}}$ denotes the reference value for the $M \times 1$ vector of cooperative task variables, the inverse kinematics solution of the system can be obtained in terms of the joint velocities by adopting the closed-loop scheme proposed in [1]

$$\dot{\mathbf{q}} = \mathbf{J}^\dagger(\mathbf{q})(\dot{\bar{\mathbf{x}}} + \mathbf{K}(\bar{\mathbf{x}} - \mathbf{k}(\mathbf{q}))) + (\mathbf{I} - \mathbf{J}^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q}))\dot{\mathbf{q}}_0 \quad (6)$$

where $\mathbf{q}$ is the $N \times 1$ vector ($N = n_1 + n_2$) of joint variables for the whole system, $\mathbf{k}(\cdot)$ is the known direct kinematics function that allows the computation of task variables from joint variables, and $\mathbf{J}^\dagger$ denotes the pseudoinverse of the Jacobian $\mathbf{J}$ associated to the task. Notice that, in case of kinematic redundancy, i.e. $N > M$, solution (6) features an extra term where $\dot{\mathbf{q}}_0$ is an arbitrary vector of joint velocities; this is projected onto the null space of the Jacobian, and then can be used to satisfy additional constraints on the solution. Further, it is easy to recognize that solution (6), thanks to the feedback correction term $-\mathbf{K}$ is a suitable positive diagonal feedback matrix—

guarantees asymptotic convergence of $\mathbf{k}(\mathbf{q})$ to $\bar{\mathbf{x}}$ which in turn implies that of $\mathbf{q}$ to the sought solution $\bar{\mathbf{q}}$.

In view of utilizing the above inverse kinematics scheme, the Jacobians for the cooperative task variables are derived below. For the absolute variables, one obtains:

$$\mathbf{J}_{pa} = 0.5 \begin{pmatrix} \mathbf{J}_1 & \mathbf{J}_2 \end{pmatrix} \quad (7)$$

$$\mathbf{J}_{\varphi a} = (\boldsymbol{\nu}^T \mathbf{J}_1 \quad -\boldsymbol{\nu}^T \mathbf{J}_2) \quad (8)$$

where $\mathbf{J}_1$ ($\mathbf{J}_2$) is the $2 \times n_1$ ($2 \times n_2$) end-effector position Jacobian matrix relative to $\mathbf{p}_1$ ($\mathbf{p}_2$), and $\boldsymbol{\nu}^T = (w_{xb}^2 + w_{yb}^2)^{-1} (w_{yb} \quad -w_{xb})$.

On the other hand, for the relative variables one obtains:

$$\mathbf{J}_{pr} = (-w_{xb} \sin \varphi_a + w_{yb} \cos \varphi_a) \mathbf{J}_{\varphi a} + (\cos \varphi_a \quad \sin \varphi_a) \begin{pmatrix} -\mathbf{J}_1 & \mathbf{J}_2 \end{pmatrix} \quad (9)$$

$$\mathbf{J}_{\varphi r1} = \mathbf{J}_{\varphi a} + \begin{pmatrix} -1 & -1 & -1 & 0 & 0 & 0 \end{pmatrix} \quad (10)$$

$$\mathbf{J}_{\varphi r2} = \mathbf{J}_{\varphi a} + \begin{pmatrix} 0 & 0 & 0 & -1 & -1 & -1 \end{pmatrix}. \quad (11)$$

Part or all of these Jacobians can be conveniently compacted to form the task Jacobian matrix used in (6). This depends on the choice of cooperative variables made to functionally characterize the given task.

If the system possesses redundant degrees of freedom with respect to the given task, solution (6) allows to choose the null space velocity vector in a proper way. Let then $\gamma(\mathbf{q})$ denote a scalar function of the joint variables which describes an additional constraint task. Local minimization of γ can be obtained by choosing for $\dot{\mathbf{q}}_0$ the gradient-type law [6]

$$\dot{\mathbf{q}}_0 = -k_\gamma \nabla_{\mathbf{q}} \gamma(\mathbf{q}) \quad (12)$$

where k_γ is a positive scalar.

For the purpose of the present work, it is desired to locally maximize the intersection of the global task space external force manipulability ellipsoid [4] along a given task space direction [3]; this corresponds to minimizing the scalar quantity

$$\gamma(\mathbf{q}) = \mathbf{u}^T \mathbf{J}_a(\mathbf{q}) \mathbf{J}_a^T(\mathbf{q}) \mathbf{u}, \quad (13)$$

where $\mathbf{J}_a^T$ is the $N \times 2$ Jacobian transpose representing the mapping between external forces on the object and applied joint torques [5], and $\mathbf{u}$ is the 2×1 unit vector identifying the chosen direction in the base frame. For the given system, the core of the ellipsoid can be explicitly derived by composing the end-effector Jacobians as [4]

$$\mathbf{J}_a \mathbf{J}_a^T = 0.25(\mathbf{J}_1 \mathbf{J}_1^T + \mathbf{J}_2 \mathbf{J}_2^T). \quad (14)$$

CASE STUDIES

A cooperative system of two equal three-degree-of-freedom arm is chosen to develop a number of case studies. The links have all unit length. The manipulated object is a disk of radius 0.5.

A task space of dimension less than *six* must be considered in order to have kinematic redundancy in the system. The initial configuration in all cases is chosen so that an isotropic force ellipsoid is obtained; the values of the task variables introduced in (1-5) are

$$\begin{aligned} \mathbf{p}_a^T &= (1.5 \quad 1) & \varphi_a &= 0 \\ p_r &= 1 & \varphi_{r1} &= 0 & \varphi_{r2} &= \pi/2 \end{aligned}$$

It is required that some task space variables remain constant while the degrees of freedom gained with the exclusion of the remaining variables are exploited to reconfigure the cooperative system minimizing the constraint (13) with

$$\mathbf{u}^T = (1 \quad 0),$$

i.e. achieving a dexterous configuration to exert forces on the object along the horizontal direction. The time history of the constraint function plus the initial and final configurations with force ellipsoids are illustrated for each case.

First, the object orientation φ_a is set free: The constraint is minimized within the limits of the available cooperative system workspace for the assigned task; notice that the left arm cannot further move without changing the grasp angle or loosing the grasp (Fig. 2).

Next, the object position $\mathbf{p}_a$ is set free: The local minimum of the constraint is lower than above, because the object can move now; this is confirmed by the stretched shape of the force ellipsoid (Fig. 3).

Finally, besides $\mathbf{p}_a$, both the grasp angles φ_{r1} and φ_{r2} are released and the end-effectors realize a rolling contact with the object: In this particular case, the system is allowed to reach the global minimum for the constraint which corresponds to a degenerate force ellipsoid (Fig. 4).

CONCLUSIONS

Dexterous reconfiguration of a redundant two-arm planar robot system has been discussed in this work. An inverse kinematics solution scheme has been developed with local minimization of a scalar constraint based on the external force ellipsoid. Different types of redundancy have been tested in simulated case studies for the cooperative system with respect to a suitable description of absolute and relative task variables. It is understood that other kinds of constraints can be considered on condition that they are expressed as a function of the joint variables of the overall system. More complex cases of spatial cooperation are under investigation.

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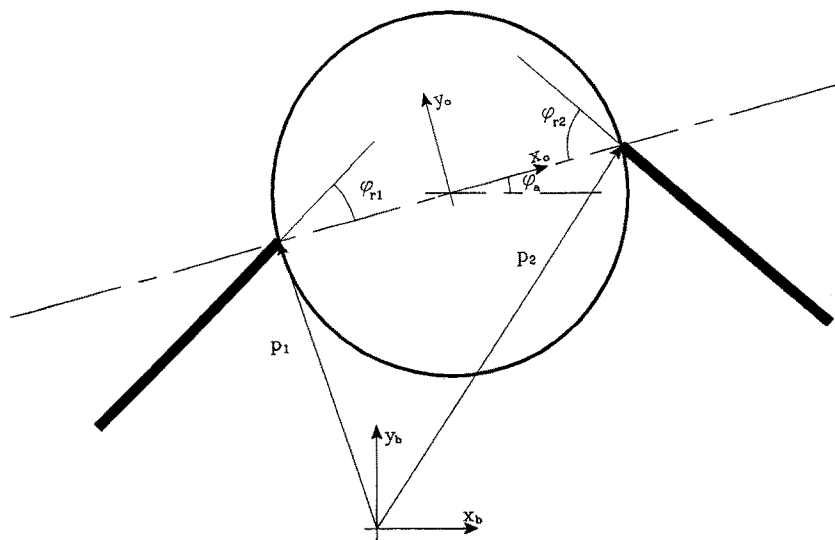


Fig. 1 — A two-arm robot cooperative system.

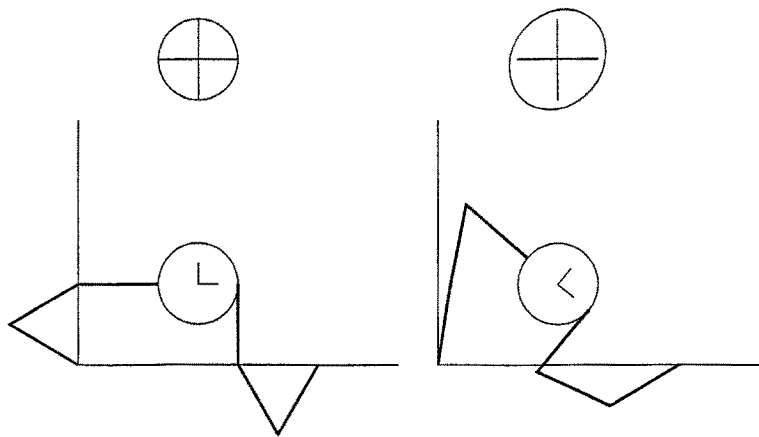
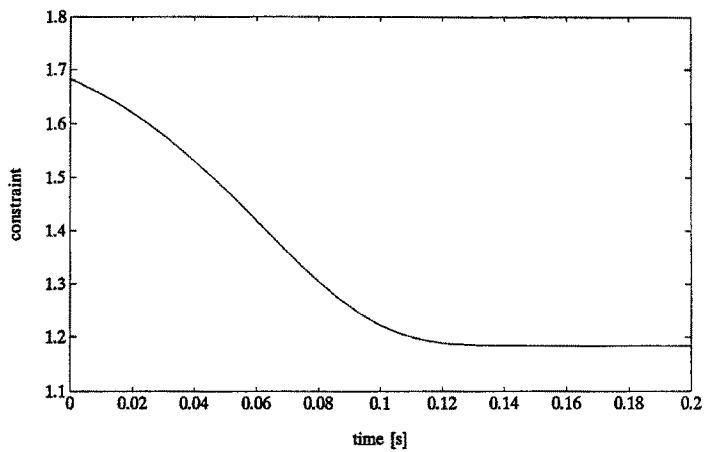


Fig. 2 — Time history of constraint value + initial and final configurations with force ellipsoids (φ_a is free).

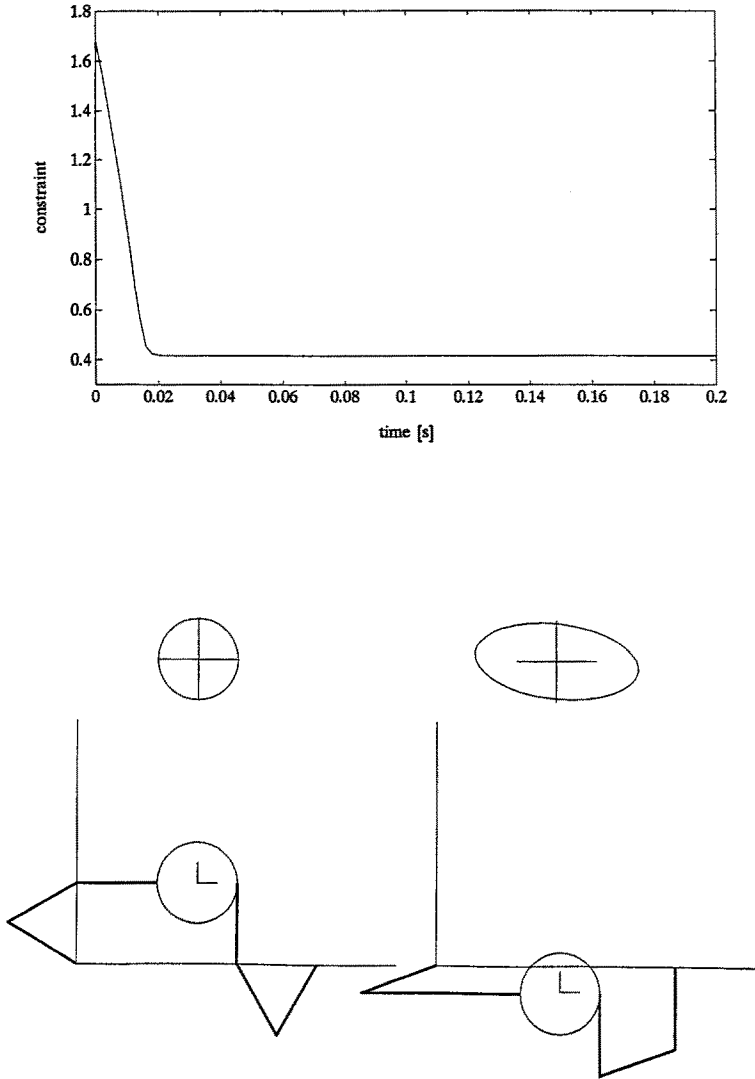


Fig. 3 — Time history of constraint value + initial and final configurations with force ellipsoids (p_a is free).

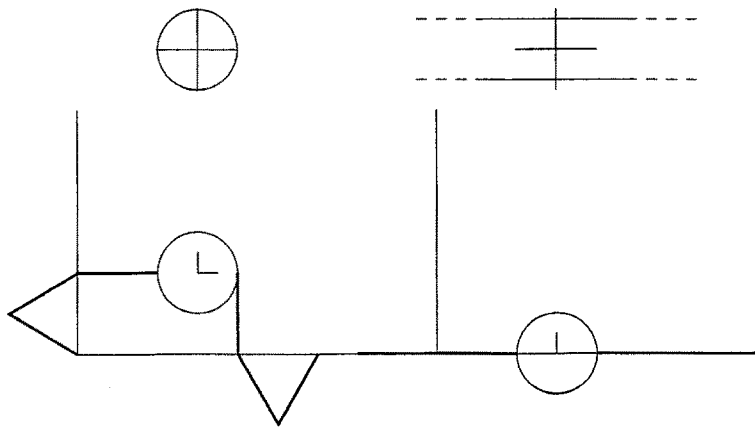
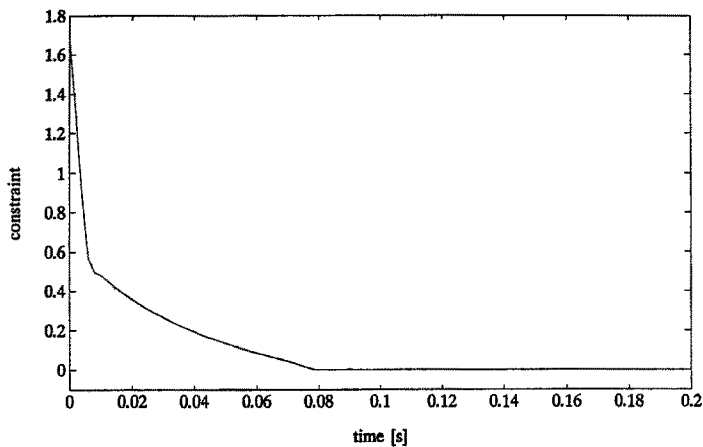


Fig. 4 — Time history of constraint value + initial and final configurations with force ellipsoids ($\mathbf{p}_a$, φ_{r1} , and φ_{r2} are free).