

S. Nicosia, B. Siciliano, A. Bicchi and P. Valigi (Eds)

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**Articulated and Mobile Robots
for SERVICES and TECHNOLOGY**



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Professor Salvatore Nicosia
Dipartimento di Informatica Sistemi e Produzione
Università degli Studi di Roma "Tor Vergata"
Via Tor Vergata 110 00133 Roma, Italy

Professor Bruno Siciliano
Dipartimento di Informatica e Sistemistica
Università degli Studi di Napoli Federico II
Via Claudio 21, 80125 Napoli, Italy

Professor Antonio Bicchi
Centro "E. Piaggio"
Università degli Studi di Pisa
Via Diotisalvi 2, 56126 Pisa, Italy

Professor Paolo Valigi
Dipartimento di Ingegneria Elettronica e dell'Informazione
Università degli Studi di Perugia
Via G. Duranti 93, 06125 Perugia, Italy

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Interaction Control

Fabrizio Caccavale, Ciro Natale, Bruno Siciliano, and Luigi Villani

PRISMA Lab, Dipartimento di Informatica e Sistemistica
Università degli Studi di Napoli Federico II
<http://www.prisma.unina.it>

In the framework of interaction control of robotic systems, impedance control represents one of the most effective strategies when the model of the environment is not available. In this chapter, a new impedance control strategy is presented for six-degree-of freedom (six-DOF) tasks. The main features are geometric consistency and absence of singularities. The case of a single manipulator interacting with the environment is considered first. Then, the case of redundant manipulators is analysed, and an algorithm ensuring stabilisation of null-space velocities is presented. The redundant degrees of mobility are exploited to optimise an additional task function. Finally, the case of cooperative robots manipulating a common object is addressed: both the problems of loose and tight cooperation are considered. The theoretical findings are validated in experiments on a dual-robot industrial setup.

1 Introduction

Control of interaction between a robot manipulator and the environment is crucial for successful execution of a number of practical tasks where the robot end effector has to manipulate an object or perform some operation on a surface. Typical examples include polishing, deburring, machining or assembly. During interaction, the environment sets constraints on the geometric paths that can be followed by the end effector. In such a case, the use of pure motion control for controlling interaction is a candidate to fail, while it is expected that enhanced performance can be achieved with an interaction control provided that force measurements are available [25].

Interaction control strategies can be grouped in two categories; those performing indirect force control and those performing direct force control. The main difference between the two categories is that in the former force control is achieved via motion control, without explicit closure of a force feedback loop; in the latter, instead, the contact force is controlled to to a desired value, thanks to the closure of a force feedback loop.

To the first category belong compliance control [21], stiffness control [23] and also impedance control [14], where the position error is related to the contact force through a mechanical stiffness or impedance of adjustable parameters.

If a detailed model of the environment is available, a widely adopted strategy belonging to the second category is the hybrid position/force control [22],

[27], [17], [18]. In most practical situations, the model of the environment is not available; in such a case, effective strategies still in the second category are the inner/outer motion/force control [12] and the parallel control [10].

The focus of the present chapter is on impedance control for six-degree-of-freedom (six-DOF) tasks. Impedance control is aimed at imposing a desired dynamic behaviour to the end effector of a robot manipulator in the presence of external forces and moments, described by a mechanical impedance. For three-degree-of-freedom tasks (i.e. involving the sole end-effector position), the impedance consists in three linear second order differential equations corresponding to a mechanical system with desired mass, damping and stiffness. If a minimal representation of the end-effector orientation is adopted (e.g. Euler angles), the generalisation of the impedance to six-DOF tasks may result to be geometrically inconsistent; moreover, representation singularities may occur. In order to guarantee geometric consistency, the rotational part of the impedance equation is expressed in terms of a class of geometrically meaningful orientation representations based on the equivalent axis and angle of rotation. Singularities are avoided by using the unit quaternion, which is a four parameter angle/axis representation. An energy-based formulation is adopted to derive the impedance equation whose rotational part results to be nonlinear [7].

The geometrically consistent approach is applied also to the impedance control of redundant manipulators, characterised by a number of degrees of mobility greater than the number of degrees of freedom required for the execution of a given task. The method adopted for redundancy resolution allows complete decoupling of internal motion control from forces and moments acting on the end effector. Thus internal motion can be exploited to meet additional task requirements, e.g. to improve robot dexterity, without affecting or being affected by the execution of the interaction task [20].

In the framework of interaction control of cooperative robots manipulating a common object, both the problems of loose and tight cooperation are addressed.

A cooperative control strategy can be termed loose when the manipulation task is executed by controlling each robot in an independent fashion. Cooperation is realised only at the task planning level, e.g. mating rigid parts such as a dual-robot assembly in a workcell. In this case limited mating forces are achieved by adopting a geometrically consistent impedance control strategy for one of the two manipulators [5].

A cooperative control strategy can be termed tight when the manipulation task is executed by controlling the robots in a coordinated fashion. Cooperation is realised not only at the task planning level, but also at the control level. This is the typical task of two robots whose end effectors tightly grasp a commonly held rigid object, thus creating a closed-kinematic chain. In this case, interaction of the grasped object with the environment is successfully

managed by enforcing a geometrically consistent impedance behaviour at the object level [4].

The control strategies presented in this chapter have been experimentally tested on a set-up composed by two industrial robots Comau SMART-3S each of them equipped with a force/torque sensor ATI FT-30/100 mounted at the wrist. A number of experiments for representative robotic tasks is illustrated.

2 Motion Control

In this section the kinematic and dynamic modelling of a robot manipulator are briefly presented. Moreover, the motion control problem in the task space is analysed, which is the base for developing interaction control.

2.1 Modelling

The kinematic model of an open-chain robot manipulator gives the relationship between the $(n \times 1)$ vector of joint variables and the (3×1) position vector $\mathbf{p}_e$ and the (3×3) rotation matrix $\mathbf{R}_e$, i.e. $\mathbf{p}_e = \mathbf{p}_e(\mathbf{q})$ and $\mathbf{R}_e = \mathbf{R}_e(\mathbf{q})$.

The quantities $\mathbf{p}_e$, $\mathbf{R}_e$ characterise the end-effector frame $\Sigma_e(O_e-X_eY_eZ_e)$ with respect to a fixed base frame $\Sigma_b(O_b-X_bY_bZ_b)$ and no superscript is used; instead, if a matrix or vector quantity is to be referred to a frame other than the base frame, then a proper frame superscript shall precede the quantity.

Let $\dot{\mathbf{q}}$ denote the vector of joint velocities, $\dot{\mathbf{p}}_e$ the vector of end-effector linear velocity, and $\boldsymbol{\omega}_e$ the (3×1) vector of end-effector angular velocity. The differential kinematics model gives the relationship between $\dot{\mathbf{q}}$ and $\mathbf{v}_e = [\dot{\mathbf{p}}_e^T \quad \boldsymbol{\omega}_e^T]^T$ in the form

$$\mathbf{v}_e = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \quad (2.1)$$

where $\mathbf{J}$ is the $(6 \times n)$ end-effector geometric Jacobian matrix. A general motion task for the end-effector position and orientation requires m degrees of freedom with $m \leq 6$. Whenever the number of joints exceeds the number of degrees of freedom, i.e. $n > m$, the manipulator is said kinematically redundant.

The dynamic model can be written in the form

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} - \mathbf{J}^T(\mathbf{q})\mathbf{h}, \quad (2.2)$$

where $\mathbf{B}$ is the (6×6) symmetric positive definite inertia matrix, $\mathbf{C}\dot{\mathbf{q}}$ is the (6×1) vector of Coriolis and centrifugal torques, $\mathbf{g}$ is the (6×1) vector of gravitational torques, $\boldsymbol{\tau}$ is the (6×1) vector of driving torques, $\mathbf{h} = [\mathbf{f}^T \quad \boldsymbol{\mu}^T]^T$ is the (6×1) vector of contact force exerted by the end effector on the environment.

2.2 Motion Control in Task Space

The motion control problem for a robot manipulator can be formulated as finding the joint torques which ensure that the end effector attains a desired position and orientation. Since the present chapter is focused on the problem of controlling the interaction between the manipulator end effector and the environment, direct feedback of task-space variables (i.e. end-effector position and orientation) is utilised.

A classical control strategy is inverse dynamics control, which is aimed at linearising and decoupling the manipulator dynamics via feedback. The joint torques are chosen as

$$\boldsymbol{\tau} = \mathbf{B}(\mathbf{q})\boldsymbol{\alpha} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{J}^T(\mathbf{q})\mathbf{h} \quad (2.3)$$

where $\boldsymbol{\alpha}$ is a new control input to be properly designed.

Folding the control law (2.3) into the system model (2.2), and taking into account that $\mathbf{B}(\mathbf{q})$ is always nonsingular, yields

$$\ddot{\mathbf{q}} = \boldsymbol{\alpha} \quad (2.4)$$

which constitutes a linear and decoupled system, where the quantity $\boldsymbol{\alpha}$ represents a resolved acceleration in terms of joint variables.

The new control input $\boldsymbol{\alpha}$ in (2.4) can be chosen as

$$\boldsymbol{\alpha} = \mathbf{J}^{-1}(\mathbf{q}) \left(\mathbf{a} - \dot{\mathbf{J}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \right) \quad (2.5)$$

which leads to

$$\dot{\mathbf{v}}_e = \mathbf{a} \quad (2.6)$$

where the time derivative of (2.1) has been used. The vector $\mathbf{a}$ attains the meaning of a resolved acceleration in terms of task space variables.

In deriving (2.5), a nonredundant manipulator ($n = 6$) moving in a singularity-free region of the workspace has been considered to compute the inverse of the Jacobian.

It is appropriate to partition the vector $\mathbf{a}$ into its linear and angular components, i.e. $\mathbf{a} = [\mathbf{a}_p^T \ \mathbf{a}_o^T]^T$, where $\mathbf{a}_p$ and $\mathbf{a}_o$ are (3×1) vectors. Therefore, Equation (2.6) can be rewritten as

$$\ddot{\mathbf{p}}_e = \mathbf{a}_p \quad (2.7)$$

$$\dot{\boldsymbol{\omega}}_e = \mathbf{a}_o, \quad (2.8)$$

where $\mathbf{a}_p$ and $\mathbf{a}_o$ shall be designed so as to ensure tracking of the desired end-effector position and orientation trajectory, respectively.

The desired position trajectory is specified in terms of the position vector $\mathbf{p}_d(t)$, linear velocity vector $\dot{\mathbf{p}}_d(t)$ and linear acceleration vector $\ddot{\mathbf{p}}_d(t)$;

the desired orientation trajectory is specified in terms of the rotation matrix $\mathbf{R}_d(t)$, angular velocity vector $\boldsymbol{\omega}_d(t)$ and angular acceleration vector $\dot{\boldsymbol{\omega}}_d(t)$. The quantities $\mathbf{p}_d$ and $\mathbf{R}_d$ characterise the origin and the orientation of a desired frame Σ_d .

A position error between the desired and the actual end-effector position can be defined as $\Delta \mathbf{p}_{de} = \mathbf{p}_d - \mathbf{p}_e$ where the operator Δ denotes that a vector difference has been taken, and the double subscript denotes the corresponding frames. Then, the resolved linear acceleration can be chosen as

$$\mathbf{a}_p = \ddot{\mathbf{p}}_d + \mathbf{K}_{Dp} \Delta \dot{\mathbf{p}}_{de} + \mathbf{K}_{Pp} \Delta \mathbf{p}_{de} \quad (2.9)$$

ensuring exponential tracking of the desired position trajectory for any choice of the positive definite matrix gains $\mathbf{K}_{Dp}$ and $\mathbf{K}_{Pp}$.

The most natural way of defining an orientation error is to consider an expression analogous to the position error, i.e. $\Delta \boldsymbol{\varphi}_{de} = \boldsymbol{\varphi}_d - \boldsymbol{\varphi}_e$, where $\boldsymbol{\varphi}_d$ and $\boldsymbol{\varphi}_e$ are the set of Euler angles that can be extracted respectively from the rotation matrices $\mathbf{R}_d$ and $\mathbf{R}_e$ describing the orientation of Σ_d and Σ_e .

The resolved angular acceleration based on the Euler angles error can be chosen as

$$\mathbf{a}_o = \mathbf{T}(\boldsymbol{\varphi}_e)(\ddot{\boldsymbol{\varphi}}_d + \mathbf{K}_{Do} \Delta \dot{\boldsymbol{\varphi}}_{de} + \mathbf{K}_{Po} \Delta \boldsymbol{\varphi}_{de}) + \dot{\mathbf{T}}(\boldsymbol{\varphi}_e, \dot{\boldsymbol{\varphi}}_e) \dot{\boldsymbol{\varphi}}_e \quad (2.10)$$

where $\mathbf{T}$ is the transformation matrix relating the time derivative of the Euler angles $\dot{\boldsymbol{\varphi}}_e$ and the end-effector angular velocity $\boldsymbol{\omega}_e$, i.e.

$$\boldsymbol{\omega}_e = \mathbf{T}(\boldsymbol{\varphi}_e) \dot{\boldsymbol{\varphi}}_e. \quad (2.11)$$

The control law (2.10) ensures exponential tracking of the desired orientation trajectory for any choice of the positive definite matrix gains $\mathbf{K}_{Do}$ and $\mathbf{K}_{Po}$, provided that $\mathbf{T}(\boldsymbol{\varphi}_e)$ is nonsingular.

The above Euler angles error becomes ill-conditioned when the actual end-effector orientation $\boldsymbol{\varphi}_e$ becomes close to a representation singularity, i.e. a configuration for which $\mathbf{T}(\boldsymbol{\varphi}_e)$ becomes singular. In order to overcome this drawback, an alternative Euler angles error can be considered which is based on the rotation matrix describing the mutual orientation between Σ_d and Σ_e , i.e. ${}^e\mathbf{R}_d = \mathbf{R}_e^T \mathbf{R}_d$. Let $\boldsymbol{\varphi}_{de}$ denote the set of Euler angles that can be extracted from ${}^e\mathbf{R}_d$. In this case, the resolved angular acceleration can be chosen as

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_d + \mathbf{T}_e(\boldsymbol{\varphi}_{de})(\mathbf{K}_{Do} \dot{\boldsymbol{\varphi}}_{de} + \mathbf{K}_{Po} \boldsymbol{\varphi}_{de}) - \dot{\mathbf{T}}_e(\boldsymbol{\varphi}_{de}, \dot{\boldsymbol{\varphi}}_{de}) \dot{\boldsymbol{\varphi}}_{de} \quad (2.12)$$

where $\mathbf{T}_e = \mathbf{R}_e \mathbf{T}$. The control law (2.12) ensures exponential tracking of the desired orientation trajectory for any choice of the positive definite matrix gains $\mathbf{K}_{Do}$ and $\mathbf{K}_{Po}$, provided that $\mathbf{T}(\boldsymbol{\varphi}_{de})$ is nonsingular. The advantage of the alternative over the classical Euler angles error is that, by adopting a representation $\boldsymbol{\varphi}_{de}$ for which $\mathbf{T}(\mathbf{0})$ is nonsingular (e.g. the XYZ Euler angles), representation singularities occur only for large orientation errors.

A different definition of orientation error can be obtained using an angle/axis representation. The mutual orientation between Σ_d and Σ_e is described by ${}^e\mathbf{R}_d$, and thus the orientation error can be defined in terms of the general expression

$${}^e\mathbf{o}_{de} = f(\vartheta_{de}) {}^e\mathbf{r}_{de}, \quad (2.13)$$

where ϑ_{de} and ${}^e\mathbf{r}_{de}$ are respectively the rotation and the unit vector corresponding to ${}^e\mathbf{R}_d$, and $f(\cdot)$ is a suitable scalar function with $f(0) = 0$. Common choices for $f(\vartheta)$ are reported in Tab. 2.1.

Representation	$f(\vartheta)$
Classical angle/axis	$\sin(\vartheta)$
Quaternion	$\sin(\vartheta/2)$
Rodrigues parameters	$\tan(\vartheta/2)$
Simple rotation	ϑ

Table 2.1. Common choices for $f(\vartheta)$

A special type of angle/axis representation of the orientation error is obtained with the quaternion, i.e.

$${}^e\mathbf{o}_{de} = \sin \frac{\vartheta_{de}}{2} {}^e\mathbf{r}_{de} = {}^e\boldsymbol{\epsilon}_{de} \quad (2.14)$$

corresponding to the vector part of the quaternion $\mathcal{Q}_{de} = \{\eta_{de}, {}^e\boldsymbol{\epsilon}_{de}\}$ that can be extracted from the rotation matrix ${}^e\mathbf{R}_d$. A brief review of the basic properties of the unit quaternion can be found, e.g. in [25].

The resolved angular acceleration based on the quaternion error can be chosen as

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_d + \mathbf{K}_{Do} \Delta \boldsymbol{\omega}_{de} + \mathbf{K}_{Po} \mathbf{R}_e {}^e\boldsymbol{\epsilon}_{de} \quad (2.15)$$

where $\Delta \boldsymbol{\omega}_{de} = \boldsymbol{\omega}_d - \boldsymbol{\omega}_e$ is the relative angular velocity between Σ_d and Σ_e . Control law (2.15) ensures asymptotic tracking of the desired trajectory when the feedback gains are taken as scalar matrices, i.e. $\mathbf{K}_{Do} = k_{Do} \mathbf{I}$ and $\mathbf{K}_{Po} = k_{Po} \mathbf{I}$ [6].

3 Six-DOF Impedance Control

When the manipulator moves in free space, the end effector is required to match a desired frame Σ_d . Instead, when the end effector interacts with the environment, it is worth considering another (compliant) frame Σ_c specified by $\mathbf{p}_c$ and $\mathbf{R}_c$; then, a mechanical impedance can be introduced which is aimed at imposing a dynamic behaviour for the position and orientation displacements between the above two frames.

In the following, six-DOF impedance control schemes are derived using different types of orientation displacements.

3.1 Classical Impedance Control

The mutual position between the compliant frame and the desired frame can be described by the position displacement $\Delta \mathbf{p}_{dc} = \mathbf{p}_d - \mathbf{p}_c$. The impedance equation is typically chosen so as to enforce an equivalent mass-damper-spring behaviour for the end-effector position displacement under an external force $\mathbf{f}$ acting on the end effector

$$\mathbf{M}_p \Delta \ddot{\mathbf{p}}_{dc} + \mathbf{D}_p \Delta \dot{\mathbf{p}}_{dc} + \mathbf{K}_p \Delta \mathbf{p}_{dc} = \mathbf{f}, \quad (3.1)$$

where $\mathbf{M}_p$, $\mathbf{D}_p$ and $\mathbf{K}_p$ are positive definite matrices.

In order to ensure a proper end-effector behaviour for the successful execution of an interaction task, the selection of the stiffness matrix plays a key role. Therefore, it is worth analysing the elastic term from a geometric point of view. The stiffness matrix $\mathbf{K}_p$ can be decomposed as $\mathbf{K}_p = \mathbf{U}_p \mathbf{\Gamma}_p \mathbf{U}_p^T$ where $\mathbf{\Gamma}_p = \text{diag}\{\gamma_{p1}, \gamma_{p2}, \gamma_{p3}\}$ and $\mathbf{U}_p = [\mathbf{u}_{p1} \ \mathbf{u}_{p2} \ \mathbf{u}_{p3}]$ are respectively the eigenvalue matrix and the (orthogonal) eigenvector matrix. Then, considering a position displacement of length λ along the i -th eigenvector $\mathbf{u}_{pi}$ leads to an elastic force

$$\mathbf{f}_E = \mathbf{K}_p \Delta \mathbf{p}_{dc} = \gamma_{pi} \lambda \mathbf{u}_{pi} \quad (3.2)$$

which is aligned with the same axis. This implies that the translational stiffness matrix can be expressed in terms of three parameters γ_{pi} representing the stiffness along three principal axes $\mathbf{u}_{pi}$, and in turn it establishes the property of task geometric consistency for the elastic force in (3.2).

The end-effector orientation displacement can be computed as a difference of Euler angles, i.e. $\Delta \phi_{dc} = \phi_d - \phi_c$, where ϕ_c and ϕ_d denote the set of Euler angles corresponding to $\mathbf{R}_c$ and $\mathbf{R}_d$, respectively. In this case the rotational part of the impedance at the end effector can be formally defined in the same way as for the positional part (3.1), i.e.

$$\mathbf{M}_o \Delta \ddot{\boldsymbol{\varphi}}_{dc} + \mathbf{D}_o \Delta \dot{\boldsymbol{\varphi}}_{dc} + \mathbf{K}_o \Delta \boldsymbol{\varphi}_{dc} = \mathbf{T}^T(\boldsymbol{\varphi}_c) \boldsymbol{\mu}, \quad (3.3)$$

where $\mathbf{M}_o$, $\mathbf{D}_o$, $\mathbf{K}_o$ are positive definite matrices describing the generalised inertia, rotational damping, rotational stiffness, respectively, and $\boldsymbol{\mu}$ is the contact moment at the end effector; all the above quantities have been referred to the base frame. Notice that, differently from (3.1), the dynamic behaviour for the rotational part is not absolutely determined by the choice of the impedance parameters but it does also depend on the orientation of the compliant frame with respect to the base frame through the matrix $\mathbf{T}^T(\boldsymbol{\varphi}_c)$. Moreover, Equation (3.3) becomes ill-defined in the neighborhood of a representation singularity; in particular, at such a singularity, moment components in the null space of $\mathbf{T}^T$ do not generate any contribution to the dynamics of the orientation displacement, leading to a possible build-up of large values of contact moment. The effect of the rotational stiffness can be better understood by considering an infinitesimal orientation displacement between Σ_d

and Σ_c . From (3.3), in the absence of representation singularities, the elastic moment is $\boldsymbol{\mu}_E = \mathbf{T}^{-T}(\boldsymbol{\varphi}_c) \mathbf{K}_o \Delta \boldsymbol{\varphi}_{dc}$. In the case of an infinitesimal orientation displacement about $\boldsymbol{\varphi}_c$, it is

$$d(\Delta \boldsymbol{\varphi}_{dc}) = (\dot{\boldsymbol{\varphi}}_d - \dot{\boldsymbol{\varphi}}_c) \Big|_{\boldsymbol{\varphi}_d = \boldsymbol{\varphi}_c} dt = \mathbf{T}^{-1}(\boldsymbol{\varphi}_c) \Delta \boldsymbol{\omega}_{dc} dt, \quad (3.4)$$

where $\Delta \boldsymbol{\omega}_{dc} = \boldsymbol{\omega}_d - \boldsymbol{\omega}_c$ is the relative angular velocity between Σ_d and Σ_c . Hence, the elastic moment for an infinitesimal displacement $d(\Delta \boldsymbol{\varphi}_{dc})$ is

$$\boldsymbol{\mu}_E = \mathbf{T}^{-T}(\boldsymbol{\varphi}_c) \mathbf{K}_o \mathbf{T}^{-1}(\boldsymbol{\varphi}_c) \Delta \boldsymbol{\omega}_{dc} dt. \quad (3.5)$$

Equation (3.5) reveals that the relationship between the orientation displacement and the elastic moment depends on the orientation of Σ_c . It follows that the property of task geometric consistency of the elastic force (3.2) is lost, that is, the eigenvectors of the matrix $\mathbf{K}_o$ do not represent the three principal axes for the rotational stiffness.

The drawbacks discussed above can be mitigated by adopting the alternative Euler angles displacement $\boldsymbol{\varphi}_{dc}$ that can be extracted from the rotation matrix ${}^c \mathbf{R}_d = \mathbf{R}_c^T \mathbf{R}_d$. Then, a rotational impedance at the end effector can be defined as

$$\mathbf{M}_o \ddot{\boldsymbol{\varphi}}_{dc} + \mathbf{D}_o \dot{\boldsymbol{\varphi}}_{dc} + \mathbf{K}_o \boldsymbol{\varphi}_{dc} = \mathbf{T}^T(\boldsymbol{\varphi}_{dc}) {}^c \boldsymbol{\mu} \quad (3.6)$$

where $\mathbf{M}_o$, $\mathbf{D}_o$ and $\mathbf{K}_o$ are defined in a similar way to (3.3) and ${}^c \boldsymbol{\mu}$ is referred to Σ_c . An advantage with respect to (3.3) is that now the impedance behaviour for the rotational part depends only on the relative orientation between Σ_d and Σ_c through the matrix $\mathbf{T}^T(\boldsymbol{\varphi}_{dc})$. Hence, if XYZ Euler angles are adopted, representation singularities have a mitigated effect since they occur for large end-effector orientation displacements. From (3.6) the elastic moment is ${}^c \boldsymbol{\mu}_E = \mathbf{T}^{-T}(\boldsymbol{\varphi}_{dc}) \mathbf{K}_o \boldsymbol{\varphi}_{dc}$. The infinitesimal orientation displacement about $\boldsymbol{\varphi}_{dc} = \mathbf{0}$ is

$$d\boldsymbol{\varphi}_{dc} = \dot{\boldsymbol{\varphi}}_{dc} \Big|_{\boldsymbol{\varphi}_{dc} = \mathbf{0}} dt = \mathbf{T}^{-1}(\mathbf{0}) \Delta {}^c \boldsymbol{\omega}_{dc} dt. \quad (3.7)$$

Hence, the elastic moment for an infinitesimal displacement $d\boldsymbol{\varphi}_{dc}$ is

$${}^c \boldsymbol{\mu}_E = \mathbf{T}^{-T}(d\boldsymbol{\varphi}_{dc}) \mathbf{K}_o \mathbf{T}^{-1}(\mathbf{0}) \Delta {}^c \boldsymbol{\omega}_{dc} dt = \mathbf{T}^{-T}(\mathbf{0}) \mathbf{K}_o \mathbf{T}^{-1}(\mathbf{0}) \Delta {}^c \boldsymbol{\omega}_{dc} dt \quad (3.8)$$

where the first-order approximation $\mathbf{T}^{-T}(d\boldsymbol{\varphi}_{dc}) dt \simeq \mathbf{T}^{-T}(\mathbf{0}) dt$ has been made. Equation (3.8) reveals that the relationship between the orientation displacement and the elastic moment is independent of the orientation of Σ_c . Notice, however, that the choice of Euler angles affects the resulting stiffness through the matrix $\mathbf{T}(\mathbf{0})$ which must be invertible. It is convenient to adopt the XYZ representation, which gives $\mathbf{T}(\mathbf{0}) = \mathbf{I}$ and thus, for an infinitesimal displacement,

$${}^c \boldsymbol{\mu}_E \simeq \mathbf{K}_o \Delta {}^c \boldsymbol{\omega}_{dc} dt. \quad (3.9)$$

As regards the property of task geometric consistency for the elastic moment, when $\mathbf{K}_o$ is a diagonal matrix and the XYZ representation of Euler angles is adopted, the i -th eigenvector $\mathbf{u}_{oi}$ of $\mathbf{K}_o = \text{diag}\{\gamma_{o1}, \gamma_{o2}, \gamma_{o3}\}$ is the i -th column of the identity matrix. Hence, the orientation displacement of an angle ϑ_{dc} about $\mathbf{u}_{oi}$ is described by $\boldsymbol{\varphi}_{dc} = \vartheta_{dc}\mathbf{u}_{oi}$, which, in view of the expression of $\mathbf{T}(\boldsymbol{\varphi}_{dc})$ for XYZ Euler angles [25], leads to

$${}^c\boldsymbol{\mu}_E = \gamma_{oi}\vartheta_{dc}\mathbf{u}_{oi}, \quad (3.10)$$

representing an elastic moment about the same $\mathbf{u}_{oi}$ axis; thus the vectors $\mathbf{u}_{oi}$ have the meaning of rotational stiffness principal axes. It can be recognised that the same property does not hold in general for a nondiagonal $\mathbf{K}_o$ [7].

3.2 Geometrically Consistent Impedance Control

The above analysis has revealed that the adoption of minimal representations of orientation for the rotational part of the impedance does not preserve properties analogous to those of the translational part. The main objective of this section is to define a six-DOF impedance with a rotational part matching the following desirable properties:

- the velocity used in the impedance equation should be dual to the moment $\boldsymbol{\mu}$ acting on the end effector, i.e. with no need of a transformation matrix depending on the actual end-effector orientation;
- the equivalent rotational stiffness in case of small orientation displacements should be always well defined;
- the elastic contribution should allow the specification of a rotational stiffness matrix in a consistent way with the task geometry.

A class of geometrically meaningful representations of the mutual orientation between Σ_d and Σ_c can be given in terms of the angle/axis displacement

$${}^c\mathbf{o}_{dc} = f(\vartheta_{dc}){}^c\mathbf{r}_{dc}, \quad (3.11)$$

where ϑ_{dc} and ${}^c\mathbf{r}_{dc}$ correspond to ${}^c\mathbf{R}_d$, and $f(\vartheta_{dc})$ is any of the functions listed in Tab. 2.1. Those are strictly increasing smooth functions in an interval $(-\vartheta_M, \vartheta_M)$ with $\vartheta_M > 0$. Hence, the derivative $f'(\vartheta_{dc})$ of f with respect to ϑ_{dc} is strictly positive in that interval.

Differentiating (3.11), gives [7]

$${}^c\dot{\mathbf{o}}_{dc} = \boldsymbol{\Omega}({}^c\mathbf{r}_{dc}, \vartheta_{dc})\Delta{}^c\boldsymbol{\omega}_{dc} \quad (3.12)$$

with

$$\boldsymbol{\Omega} = f'(\vartheta_{dc}){}^c\mathbf{r}_{dc}{}^c\mathbf{r}_{dc}^T + \frac{1}{2}f(\vartheta_{dc})\left(\cot(\vartheta_{dc}/2)(\mathbf{I} - {}^c\mathbf{r}_{dc}{}^c\mathbf{r}_{dc}^T) - \mathbf{S}({}^c\mathbf{r}_{dc})\right), \quad (3.13)$$

where $\mathbf{S}(\cdot)$ is the (3×3) skew-symmetric matrix operator performing the cross product. Notice that the following property of $\boldsymbol{\Omega}$ holds

$$\boldsymbol{\Omega}({}^c\mathbf{r}_{dc}, 0) = f'(0)\mathbf{I} \quad (3.14)$$

which will be useful in the following.

In order to derive the impedance equation for the rotational part, it is convenient to refer to the following energy-based argument. Let

$$\mathcal{T}_o = \frac{1}{2} \Delta^c \boldsymbol{\omega}_{dc}^T \mathbf{M}_o \Delta^c \boldsymbol{\omega}_{dc} \quad (3.15)$$

express a rotational pseudo-kinetic energy of a rigid body with inertia tensor $\mathbf{M}_o$ and angular velocity $\Delta^c \boldsymbol{\omega}_{dc}$. Then consider the potential energy

$$\mathcal{U}_o = \psi {}^c \mathbf{o}_{dc}^T \mathbf{K}_o {}^c \mathbf{o}_{dc}, \quad (3.16)$$

where ψ is a positive constant depending on the particular choice of $f(\cdot)$, and $\mathbf{K}_o$ is a symmetric positive definite matrix. Having defined the various energy contributions, the terms in the rotational impedance equation can be derived by considering the associated powers. Taking the time derivative of (3.15) yields

$$\dot{\mathcal{T}}_o = {}^c \boldsymbol{\mu}_I^T \Delta^c \boldsymbol{\omega}_{dc} \quad (3.17)$$

where

$${}^c \boldsymbol{\mu}_I = \mathbf{M}_o \Delta^c \dot{\boldsymbol{\omega}}_{dc} \quad (3.18)$$

is the inertial moment. Further, taking the time derivative of (3.16) and accounting for (3.12) yields

$$\dot{\mathcal{U}}_o = {}^c \boldsymbol{\mu}_E^T \Delta^c \boldsymbol{\omega}_{dc} \quad (3.19)$$

where

$${}^c \boldsymbol{\mu}_E = 2\psi \boldsymbol{\Omega}^T({}^c \mathbf{r}_{dc}, \vartheta_{dc}) \mathbf{K}_o {}^c \mathbf{o}_{dc} \quad (3.20)$$

is the elastic moment. Finally, a dissipative contribution can be added as

$${}^c \boldsymbol{\mu}_D = \mathbf{D}_o \Delta^c \boldsymbol{\omega}_{dc}, \quad (3.21)$$

where $\mathbf{D}_o$ is a positive definite matrix characterising a rotational damping at the end effector.

Therefore, a rotational impedance at the end effector can be defined by adding the contributions (3.18), (3.21) and (3.20), i.e.

$$\mathbf{M}_o \Delta^c \dot{\boldsymbol{\omega}}_{dc} + \mathbf{D}_o \Delta^c \boldsymbol{\omega}_{dc} + \mathbf{K}'_o {}^c \mathbf{o}_{dc} = {}^c \boldsymbol{\mu}, \quad (3.22)$$

where the equality ${}^c \boldsymbol{\mu} = {}^c \boldsymbol{\mu}_I + {}^c \boldsymbol{\mu}_D + {}^c \boldsymbol{\mu}_E$ has been imposed, and

$$\mathbf{K}'_o = 2\psi \boldsymbol{\Omega}^T({}^c \mathbf{r}_{dc}, \vartheta_{dc}) \mathbf{K}_o. \quad (3.23)$$

Notice that the rotational part of the impedance equation has been derived in terms of quantities all referred to Σ_c ; this allows the impedance

behaviour to be effectively expressed in terms of the relative orientation between Σ_d and Σ_c , no matter what the absolute orientation of the compliant frame with respect to the base frame is. It is worth remarking that the velocity used in the impedance equation is dual to the moment ${}^c\boldsymbol{\mu}$ exerted by the end effector, i.e. there is no need of a transformation matrix depending on the actual end-effector orientation.

In the following, the analysis for small orientation displacements is carried out and consistency with the task geometry is investigated. Consider an infinitesimal orientation displacement expressed as

$$d^c\mathbf{o}_{dc} = {}^c\dot{\mathbf{o}}_{dc} \Big|_{\vartheta_{dc}=0} dt = \boldsymbol{\Omega}({}^c\mathbf{r}_{dc}, 0) \Delta^c\boldsymbol{\omega}_{dc} dt = f'(0) \Delta^c\boldsymbol{\omega}_{dc} dt \quad (3.24)$$

where the property (3.14) has been exploited. Folding (3.24) into (3.20), written for an infinitesimal displacement about $\vartheta_{dc} = 0$, gives

$${}^c\boldsymbol{\mu}_E = 2\psi \boldsymbol{\Omega}^T({}^c\mathbf{r}_{dc}, d\vartheta_{dc}) \mathbf{K}_o d^c\mathbf{o}_{dc} \quad (3.25)$$

$$\simeq 2\psi (f'(0))^2 \mathbf{K}_o \Delta^c\boldsymbol{\omega}_{dc} dt = \mathbf{K}_o \Delta^c\boldsymbol{\omega}_{dc} dt, \quad (3.26)$$

where the first-order approximation $\boldsymbol{\Omega}({}^c\mathbf{r}_{dc}, d\vartheta_{dc}) \simeq f'(0)\mathbf{I}$ has been considered and the choice $\psi = 1/2(f'(0))^2$ has been made. Equation (3.25) clearly shows how the relationship between the orientation displacement and the elastic moment is independent of the orientation of Σ_c , and the problem of representation singularities is not of concern since $f'(0)$ is finite.

As regards the property of task geometric consistency, the stiffness matrix in (3.20) can be decomposed as $\mathbf{K}_o = \mathbf{U}_o \boldsymbol{\Gamma}_o \mathbf{U}_o^T$, where $\boldsymbol{\Gamma}_o = \text{diag}\{\gamma_{o1}, \gamma_{o2}, \gamma_{o3}\}$ and $\mathbf{U}_o = [\mathbf{u}_{o1} \ \mathbf{u}_{o2} \ \mathbf{u}_{o3}]$ are respectively the eigenvalue matrix and the (orthogonal) eigenvector matrix. Then, considering an orientation displacement by an angle ϑ_{dc} about the i -th eigenvector

$${}^c\mathbf{o}_{dc} = f(\vartheta_{dc}) \mathbf{u}_{oi}, \quad (3.27)$$

and taking into account the expression of $\boldsymbol{\Omega}$ in (3.13), yields

$${}^c\boldsymbol{\mu}_E = 2\psi f'(\vartheta_{dc}) f(\vartheta_{dc}) \gamma_{oi} \mathbf{u}_{oi}. \quad (3.28)$$

This represents an elastic moment about the same $\mathbf{u}_{oi}$ axis which is in the same direction of the orientation displacement since $f'(\vartheta_{dc}) > 0$. Therefore, the rotational stiffness matrix can be expressed in terms of three parameters γ_{oi} representing the stiffness about three principal axes $\mathbf{u}_{oi}$, i.e. in a consistent way with the task geometry.

A special case among angle/axis representations of orientation displacement is constituted by the quaternion displacement. Such a representation has the advantage over other angle/axis representations to avoid representation singularities. The mutual orientation between Σ_d and Σ_c can be described by the quaternion $\mathcal{Q}_{dc} = \{\eta_{dc}, {}^c\boldsymbol{\epsilon}_{dc}\}$ extracted from ${}^c\mathbf{R}_d$. Indeed, the orientation displacement to be considered in (3.22) is given by the vector part ${}^c\boldsymbol{\epsilon}_{dc}$ and, in view of (3.16), the expression of the potential energy becomes

$$\mathcal{U}_o = 2 {}^c \boldsymbol{\epsilon}_{dc}^T \mathbf{K}_o {}^c \boldsymbol{\epsilon}_{dc}, \quad (3.29)$$

where it has been set $\psi = 2$. Even though the potential energy is expressed in terms of the vector part of the quaternion, it can be shown that $\mathcal{U}_o$ coincides with the rotational elastic energy associated with a torsional spring of stiffness $\mathbf{K}_o$ acting so as to align $\boldsymbol{\Sigma}_c$ with $\boldsymbol{\Sigma}_d$ [7].

In view of (3.22), the resulting impedance equation for the rotational part becomes

$$\mathbf{M}_o \Delta^c \dot{\boldsymbol{\omega}}_{dc} + \mathbf{D}_o \Delta^c \boldsymbol{\omega}_{dc} + \mathbf{K}_o' {}^c \boldsymbol{\epsilon}_{dc} = {}^c \boldsymbol{\mu}, \quad (3.30)$$

where the rotational stiffness matrix is

$$\mathbf{K}_o' = 2 \mathbf{E}^T (\eta_{dc}, {}^c \boldsymbol{\epsilon}_{dc}) \mathbf{K}_o \quad (3.31)$$

where $\mathbf{E} = \eta_{dc} \mathbf{I} - \mathbf{S}({}^c \boldsymbol{\epsilon}_{dc})$.

A generalisation of the proposed approach leads to an impedance equation characterised by coupling elastic terms between the translational and the rotational part; they may be useful for the execution of assembly tasks. The details can be found in [8].

3.3 Inner Motion Control Loop

The impedance behaviour at the end-effector can be enforced by resorting to a model-based control law designed so as to give closed-loop dynamic equations, for both position and orientation, coinciding with those describing the desired impedance [24]. However, the selection of good impedance parameters that guarantee a satisfactory compliant behaviour during the interaction may turn out to be inadequate to ensure accurate tracking of the desired position and orientation trajectory when the end effector moves in free space. A solution to this drawback can be devised by separating the motion control action from the impedance control action as follows. The motion control action is purposefully made stiff so as to enhance disturbance rejection but, rather than ensuring tracking of the desired end-effector position and orientation, it shall ensure tracking of a reference position and orientation resulting from the impedance control action [1]. In other words, the desired position and orientation together with the measured contact force and moment are input to the impedance equation which, via a suitable integration, generates the position and orientation to be used as a reference for the motion control action.

In detail, the control input $\mathbf{a}_p$ in (2.6) has to be designed to match the desired impedance for the translational part and the rotational part, respectively. In view of (3.1), $\mathbf{a}_p$ is taken as in (2.9), by simply replacing the variables referring to the desired frame (subscript d) with the variables referring to the compliant frame (subscript c). The latter can be computed by forward integration of the translational impedance equation (3.1) with input $\mathbf{f}$ available from the force sensor.

As regards the orientation loop, $\mathbf{a}_o$ can be chosen according to the different representations of orientation displacement illustrated above. Similarly to what has been conceived for $\mathbf{a}_p$, the control input $\mathbf{a}_o$ is taken as in (2.10), (2.12) or (2.15) by simply replacing the variables referring to the desired frame (subscript d) with the variables referring to the compliant frame (subscript c). The latter can be computed by forward integration of the corresponding rotational impedance equations (3.3), (3.6) or (3.30) with input $\boldsymbol{\mu}$ available from the force sensor.

3.4 Redundancy Resolution

In case the manipulator is kinematically redundant, there exist infinite joint motions that produce the same end-effector motion. In particular, even when the end effector is at rest, it is possible to generate an internal motion at the joints. As a minimal requirement, such motion should be made stable [16]. In addition, it could be exploited to meet additional task requirements besides the execution of the end-effector trajectory, thus providing redundancy resolution [19].

Redundancy can be solved either at kinematic level, that is in the first stage of a kinematic control strategy, or at dynamic level by suitably modifying the inverse dynamics control law [13]. The latter approach is pursued hereafter.

Since the Jacobian matrix for a redundant manipulator has more columns than rows ($n > 6$), a suitable right inverse of $\mathbf{J}$ is to be used in lieu of $\mathbf{J}^{-1}$. Hence, in lieu of (2.5), the new control input in (2.3) can be chosen as

$$\boldsymbol{\alpha} = \mathbf{J}^\dagger(\mathbf{q}) \left(\mathbf{a} - \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}} \right) + \boldsymbol{\alpha}_n \quad (3.32)$$

where

$$\mathbf{J}^\dagger = \mathbf{W}^{-1} \mathbf{J}^T \left(\mathbf{J} \mathbf{W}^{-1} \mathbf{J}^T \right)^{-1} \quad (3.33)$$

denotes the right pseudo-inverse of $\mathbf{J}$ weighted by the positive definite ($n \times n$) matrix $\mathbf{W}$. Also, in (3.32), $\boldsymbol{\alpha}_n$ denotes a joint acceleration vector lying in the null space of $\mathbf{J}$ which is available for redundancy resolution. It can be shown that plugging (3.32) in (2.4) yields the same end-effector resolved acceleration as in (2.6) [20].

The matrix projecting arbitrary joint accelerations into the null space of $\mathbf{J}$ is given by $(\mathbf{I} - \mathbf{J}^\dagger \mathbf{J})$, no matter what choice is made for the weighting matrix $\mathbf{W}$ in (3.33). Therefore, it is significant to choose $\mathbf{W}$ so that the redundancy resolution scheme for motion control should not be altered when interaction with the environment occurs. To this purpose, from (2.2) the joint accelerations induced by the external end-effector force and moment are given by

$$\ddot{\mathbf{q}}_e = -\mathbf{B}^{-1}(\mathbf{q}) \mathbf{J}^T(\mathbf{q}) \mathbf{h}. \quad (3.34)$$

Projecting these accelerations in the null space of the Jacobian gives

$$\ddot{\mathbf{q}}_{en} = - \left(\mathbf{I} - \mathbf{J}^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q}) \right) \mathbf{B}^{-1}(\mathbf{q})\mathbf{J}^\mathrm{T}(\mathbf{q})\mathbf{h}. \quad (3.35)$$

Choosing $\mathbf{W} = \mathbf{B}$ in (3.33) and plugging the resulting $\mathbf{J}^\dagger$ in (3.35) yields $\ddot{\mathbf{q}}_{en} = \mathbf{0}$, meaning that the external force and moment produce no null space joint accelerations. Therefore, in view of this choice, in the design of the joint resolved acceleration in (3.32) the vector $\boldsymbol{\alpha}_n$ can be used to solve redundancy independently of the occurrence of interaction with the environment. The matrix

$$\mathbf{J}^\dagger = \mathbf{B}^{-1}\mathbf{J}^\mathrm{T} \left(\mathbf{J}\mathbf{B}^{-1}\mathbf{J}^\mathrm{T} \right)^{-1} \quad (3.36)$$

weighted by the inertia matrix is termed dynamically consistent pseudo-inverse of the Jacobian.

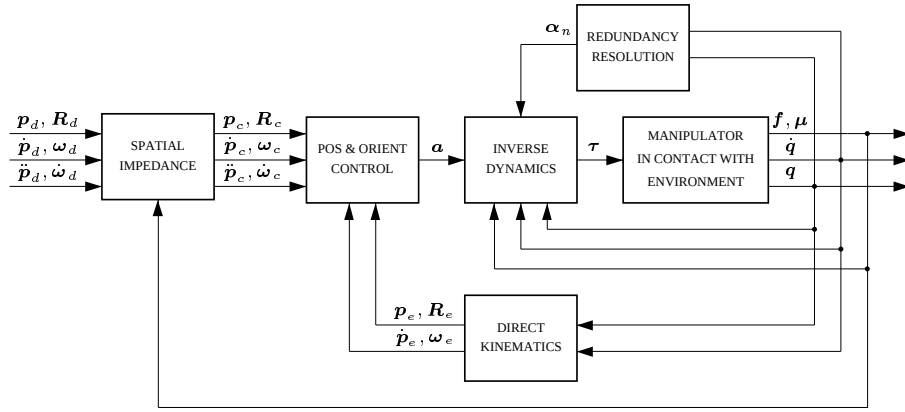


Figure 3.1. Spatial impedance control with redundancy resolution

The next step consists of designing a redundancy resolution control in terms of the null space joint accelerations $\boldsymbol{\alpha}_n$ in (3.32). To this purpose, $\boldsymbol{\alpha}_n$ shall be chosen so as to ensure stabilisation of the null space motion and possibly optimisation of an additional task function. Let

$$\mathbf{e}_n = \left(\mathbf{I} - \mathbf{J}^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q}) \right) (\boldsymbol{\beta} - \dot{\mathbf{q}}) \quad (3.37)$$

denote the null space velocity error where $\boldsymbol{\beta}$ is a joint velocity vector which is available for redundancy resolution. The goal is to make $\mathbf{e}_n$ asymptotically converge to zero. Choosing

$$\boldsymbol{\alpha}_n = (\mathbf{I} - \mathbf{J}^\dagger\mathbf{J}) \left(\dot{\boldsymbol{\beta}} - \dot{\mathbf{J}}^\dagger\mathbf{J}(\boldsymbol{\beta} - \dot{\mathbf{q}}) + \mathbf{B}^{-1}(\mathbf{K}_n\mathbf{e}_n + \mathbf{C}\mathbf{e}_n) \right) \quad (3.38)$$

where $\mathbf{K}_n$ is a positive definite matrix, it can be shown that $\mathbf{e}_n \rightarrow \mathbf{0}$ asymptotically [20].

Regarding the utilisation of redundancy, a typical choice for β is

$$\beta = k_\beta \mathbf{B}^{-1} \left(\frac{\partial w(\mathbf{q})}{\partial \mathbf{q}} \right) \quad (3.39)$$

where k_β is a signed scalar and $w(\mathbf{q})$ is an additional task function that can be locally optimised.

A block diagram summarising the overall spatial impedance control with redundancy resolution is sketched in Figure 3.1.

4 Cooperative Robots

4.1 Loose Cooperation

Consider a system of two robots manipulating an object. A cooperative control strategy can be termed loose when the manipulation task is executed by controlling the two robots in an independent fashion. Cooperation is realised only at the task planning level.

A typical task requiring loose cooperative control is constituted by mating rigid parts such as dual-robot assembly in a workcell. The archetype is the classical peg-in-hole, where one robot carries the peg and the other holds the hollow part. It should be clear that the task is successfully executed provided that mating forces are suitably reduced during the insertion so as to avoid undesirable jamming and wedging. This concept can be brought to fruition by resorting to special mechanical devices such as the Remote Center of Compliance in [28] or the compliant end effectors in [11], [15].

An alternative strategy [5] is to assign complementary roles to the two robots, i.e. to operate one robot using pure positional control while controlling the other so as to achieve a programmable impedance at the end effector. In detail, the motion of the position controlled robot is planned to match the nominal requirements of the assigned task, while the active compliant behaviour imposed to the impedance controlled robot is devoted to mitigating the effects of imperfect knowledge of the task geometry and unavoidable tracking errors.

The position controlled robot can be operated using the standard industrial control unit, i.e. by exploiting the set of motion planning instructions of the native programming language.

On the other hand, assuming that an open control architecture is available for the other robot, an impedance control is realised at the end-effector level where the object is either the peg or the hollow part; the end-effector frame of such a robot coincides with the object frame.

4.2 Tight Cooperation

A cooperative control strategy can be termed tight when the manipulation task is executed by controlling the robots in a coordinated fashion. Cooperation is realised not only at the task planning level, but also at the control level. This is the typical task of two robots whose end effectors tightly grasp a commonly held rigid object, thus creating a closed-kinematic chain.

Consider a two-robot system tightly grasping a rigid object in contact with the environment. Let Σ_o be the frame attached to the object; its origin and orientation with respect to the base frame are characterised by the (3×1) position vector $\mathbf{p}_o$ and the (3×3) rotation matrix $\mathbf{R}_o$, respectively.

The object motion must be related to the motions of the end effectors of the robots. This can be done by resorting to the task-oriented formulation for coordinated motion of dual-robot systems developed in [9], [3].

For each manipulator ($k = 1, 2$) let Σ_k denote a frame attached to the end effector; its origin and orientation with respect to the base frame are characterised by the (3×1) position vector $\mathbf{p}_k$ and the (3×3) rotation matrix $\mathbf{R}_k$, respectively. Then, $\mathcal{Q}_k = \{\eta_k, \boldsymbol{\epsilon}_k\}$ represents the unit quaternion corresponding to $\mathbf{R}_k$. Let also $\mathbf{v}_k = [\dot{\mathbf{p}}_k^T \ \boldsymbol{\omega}_k^T]^T$ be the (6×1) end-effector (linear and angular) velocity vector.

The object position is chosen as

$$\mathbf{p}_o = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2), \quad (4.1)$$

while the rotation matrix defining the orientation of Σ_o is chosen as

$$\mathbf{R}_o = \mathbf{R}_1^{-1} \mathbf{R}({}^1\mathbf{r}_{21}, \vartheta_{21}/2), \quad (4.2)$$

where ${}^1\mathbf{r}_{21}$ and ϑ_{21} are respectively the unit vector and the angle that realise the rotation described by

$${}^1\mathbf{R}_2 = \mathbf{R}_1^T \mathbf{R}_2 \quad (4.3)$$

and ${}^1\mathbf{R}({}^1\mathbf{r}_{21}, \vartheta_{21}/2)$ is the rotation matrix corresponding to a rotation of $\vartheta_{21}/2$ about the axis ${}^1\mathbf{r}_{21}$. Then the absolute orientation can be expressed as

$$\mathcal{Q}_o = \mathcal{Q}_1 * \left\{ \cos \frac{\vartheta_{21}}{4}, \sin \frac{\vartheta_{21}}{4} {}^1\mathbf{r}_{21} \right\} \quad (4.4)$$

where $\mathcal{Q}_o$ is the unit quaternion corresponding to $\mathbf{R}_o$ and “*” denotes the quaternion product (see the appendix in [25]); the second factor on the right-hand side of (4.4) is the unit quaternion extracted from ${}^1\mathbf{R}({}^1\mathbf{r}_{21}, \vartheta_{21}/2)$.

From (4.1) and (4.2), the object linear velocity $\dot{\mathbf{p}}_o$ and angular velocity $\boldsymbol{\omega}_o$ can be expressed as

$$\mathbf{v}_o = \frac{1}{2}(\mathbf{v}_1 + \mathbf{v}_2) \quad (4.5)$$

where $\mathbf{v}_o = [\dot{\mathbf{p}}_o^T \ \boldsymbol{\omega}_o^T]^T$.

Let $\mathbf{f}_k$ and $\boldsymbol{\mu}_k$ ($k = 1, 2$) respectively denote the (3×1) end-effector force and moment vectors for either manipulator. Then, according to the kineto-statics duality concept [26] applied to (4.5), the object force and moment can be expressed as

$$\mathbf{h}_o = \mathbf{h}_1 + \mathbf{h}_2 \quad (4.6)$$

where $\mathbf{h}_k = [\mathbf{f}_k^T \ \boldsymbol{\mu}_k^T]^T$ and $\mathbf{h}_o = [\mathbf{f}_o^T \ \boldsymbol{\mu}_o^T]^T$.

In order to fully describe a coordinated motion, the position and orientation of one manipulator relative to the other is also of concern. The mutual position between the two end effectors is the vector

$$\Delta \mathbf{p}_{21} = \mathbf{p}_2 - \mathbf{p}_1. \quad (4.7)$$

The mutual orientation between the two end effectors is defined with reference to Σ_1 in terms of the rotation matrix ${}^1\mathbf{R}_2$, and then in terms of the quaternion product

$$\mathcal{Q}_{21} = \mathcal{Q}_1^{-1} * \mathcal{Q}_2, \quad (4.8)$$

where $\mathcal{Q}_1^{-1} = \{\eta_1, -\boldsymbol{\epsilon}_1\}$ is the unit quaternion corresponding to $\mathbf{R}_1^T$ (i.e. the conjugate of $\mathcal{Q}_1$).

From (4.3) and (4.7), the mutual velocity can be expressed as

$$\Delta \mathbf{v}_{21} = \mathbf{v}_2 - \mathbf{v}_1. \quad (4.9)$$

Once the position and orientation variables describing the cooperative system have been defined, reference end-effector position $\mathbf{p}_{r,k}$ and orientation $\mathcal{Q}_{r,k}$ variables, as well as reference end-effector velocities $\mathbf{v}_{r,k}$ ($k = 1, 2$), must be generated with a twofold objective; namely, realising an impedance behaviour at the object level, while assigning a mutual position and orientation between the two end effectors that is compatible with the object geometry.

The first objective can be fulfilled as follows. Let the desired object position $\mathbf{p}_d$ and orientation $\mathcal{Q}_d$ (extracted from the desired rotation matrix $\mathbf{R}_d$) be assigned with the associated linear and angular velocities and accelerations. Also, the object force and moment can be computed from (4.6) with the end-effector forces and moments available from the wrist force/torque sensors. Then, the translational and rotational impedance equations (3.1) and (3.30) are integrated, with input $\mathbf{f}_o$ and ${}^c\boldsymbol{\mu}_o$, to compute $\ddot{\mathbf{p}}_c$ and ${}^c\dot{\boldsymbol{\omega}}_c$, $\dot{\mathbf{p}}_c$ and ${}^c\boldsymbol{\omega}_c$, and then $\mathbf{p}_c$ and $\mathcal{Q}_c$ via the quaternion propagation.

The second objective can be fulfilled by assigning a reference mutual position $\Delta \mathbf{p}_{r,21}$ and orientation $\mathcal{Q}_{r,21}$. In particular, $\Delta \mathbf{p}_{r,21}$ and $\mathcal{Q}_{r,21}$ are taken as constant and equal to the initial values of $\Delta \mathbf{p}_{21}$ in (4.7) and $\mathcal{Q}_{21}$ extracted from (4.3), respectively, that can be computed via the direct kinematics of the two manipulators.

The two objectives are combined by choosing the above reference position and orientation for the two end effectors so as to satisfy (4.1), (4.7), and (4.4), (4.8), i.e.

$$\mathbf{p}_{r,1} = \mathbf{p}_c - \frac{1}{2}\Delta\mathbf{p}_{r,21} \quad (4.10)$$

$$\mathbf{p}_{r,2} = \mathbf{p}_c + \frac{1}{2}\Delta\mathbf{p}_{r,21} \quad (4.11)$$

$$\mathcal{Q}_{r,1} = \mathcal{Q}_c * \left\{ \cos \frac{\vartheta_{r,21}}{4}, -\sin \frac{\vartheta_{r,21}}{4} \mathbf{r}_{r,21} \right\} \quad (4.12)$$

$$\mathcal{Q}_{r,2} = \mathcal{Q}_{r,1} * \mathcal{Q}_{r,21}. \quad (4.13)$$

Further, the reference velocities for the two end effectors are chosen as

$$\mathbf{v}_{r,1} = \mathbf{v}_c - \frac{1}{2}\Delta\mathbf{v}_{r,21} \quad (4.14)$$

$$\mathbf{v}_{r,2} = \mathbf{v}_c + \frac{1}{2}\Delta\mathbf{v}_{r,21} \quad (4.15)$$

where $\mathbf{v}_c = [\dot{\mathbf{p}}_c^T \ \boldsymbol{\omega}_c^T]^T$. Then, the reference accelerations can be computed via a formal time derivative of the terms in (4.14) and (4.15).

The above reference trajectories can be tracked by resorting to an inner motion control loop based on an inverse dynamics strategy as for the case of the single manipulator.

5 Experimental Validation

The setup in the PRISMA Lab consists of two industrial robots Comau SMART-3 S (Figure 5.1). Each robot arm has a six-revolute-joint anthropomorphic geometry with nonnull shoulder and elbow offsets and non-spherical wrist. One arm is mounted on a sliding track which provides an additional degree of mobility. The joints are actuated by brushless motors via gear trains; shaft absolute resolvers provide motor position measurements. Each robot is controlled by the C3G 9000 control unit which has a VME-based architecture. Independent joint control is adopted where the individual servos are implemented as standard PID controllers. The native robot programming language is PDL 2, a high-level Pascal-like language with typical motion planning instructions.

An open version of the control unit is available [2] which allows testing of advanced control algorithms on a conventional industrial robot. Connection of the VME bus of the C3G 9000 unit to the ISA bus of a standard PC is made possible by a BIT 3 Computer bus adapter board; for the experiments, a PC Pentium MMX/233 was used.

Various operating modes are available in the control unit, allowing the PC to interact with the original controller both at trajectory generation level and at joint control level. To implement model-based control schemes, the operating mode number 4 is used in which the PC is in charge of computing the control algorithm and passing the references to the current servos

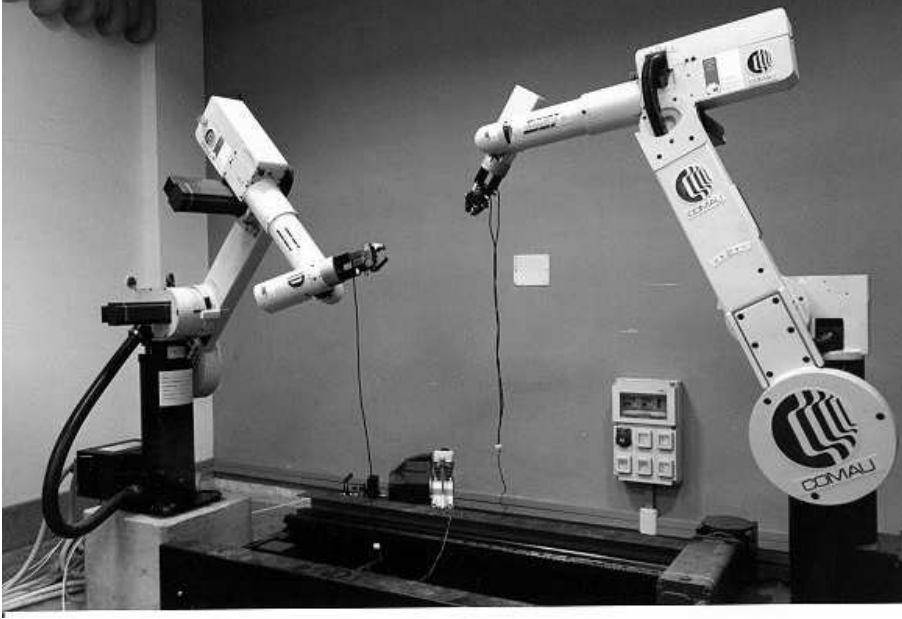


Figure 5.1. Experimental setup available in the PRISMA Lab

through the communication link at 1 ms sampling time. Joint velocities are reconstructed through numerical differentiation of joint position readings.

A six-axis force/torque sensor ATI FT30-100 with force range of ± 130 N and torque range of ± 10 N·m can be mounted at either arm's wrist. The sensor is connected to the PC by a parallel interface board which provides readings of six components of generalised force at 1 ms.

5.1 Single Manipulator

The above impedance control schemes have been tested in a number of experiments. An end effector has been built as a steel stick with a wooden disk of 5.5 cm radius at the tip. The end-effector frame has its origin at the center of the disk and its approach axis normal to the disk surface and pointing outwards.

First case study: Interaction with environment. The first case study has been developed to analyse interaction with environment. This is constituted by a flat plexiglas surface. The translational stiffness at the contact between the end effector and the surface is of the order of 10^4 N/m, while the rotational stiffness for small angles is of the order of 20 Nm/rad.

The task consists of taking the disk in contact with the surface at an angle of unknown magnitude (Figure 5.2). The end-effector desired position

is required to make a straight-line motion with a vertical displacement of -0.24 m along the Z_b -axis of the base frame. The trajectory along the path is generated according to a fifth-order interpolating polynomial with null initial and final velocities and accelerations, and a duration of 7 s. The end-effector desired orientation is required to remain constant during the task. The surface is placed (horizontally) in the X_bY_b -plane in such a way as to obstruct the desired end-effector motion, both for the translational part and for the rotational part.

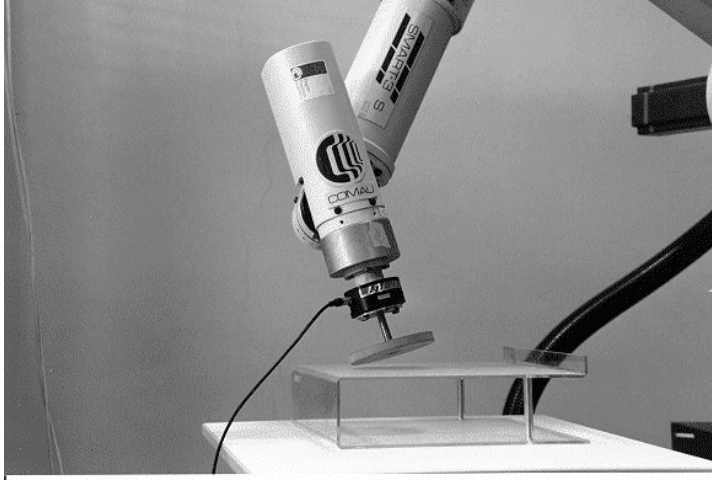


Figure 5.2. End effector in contact with plexiglas surface

The parameters of the translational part of the six-DOF impedance equation (3.1) have been set to $\mathbf{M}_p = 9\mathbf{I}$, $\mathbf{D}_p = 2000\mathbf{I}$, $\mathbf{K}_p = 700\mathbf{I}$, while the parameters of the rotational part of the six-DOF impedance equation (3.30) have been set to $\mathbf{M}_o = 0.4\mathbf{I}$, $\mathbf{D}_o = 5\mathbf{I}$, $\mathbf{K}_o = 2\mathbf{I}$. Notice that the stiffness matrices have been chosen so as to ensure a compliant behaviour at the end effector (limited values of contact force and moment) during the constrained motion, while the damping matrices have been chosen so as to guarantee a well-damped behaviour.

The gains of the inner motion control loop actions in (2.9),(2.15) have been set to $\mathbf{K}_{Pp} = 2025\mathbf{I}$, $\mathbf{K}_{Po} = 4500\mathbf{I}$, $\mathbf{K}_{Dp} = \mathbf{K}_{Do} = 65\mathbf{I}$.

The results in Figure 5.3 show the effectiveness of the quaternion-based six-DOF impedance control. After the contact, the component of the position error between Σ_d and Σ_e $\Delta\mathbf{p}_{de} = \mathbf{p}_d - \mathbf{p}_e$ along the Z_b -axis significantly deviates from zero, as expected, while small errors can be seen also for the components along the X_b - and the Y_b -axis due to contact friction. As for the orientation error, all the components of the orientation displacement be-

tween Σ_d and Σ_e (${}^e\epsilon_{de}$) significantly deviate from zero since the end-effector frame has to rotate with respect to the base frame after the contact in order to comply with the surface. Also, in view of the imposed task, a prevailing component of the contact force can be observed along the Z_b -axis after the contact, while the small components along the X_b - and the Y_b -axis arise as a consequence of the above end-effector deviation. As for the contact moment referred to Σ_d , the component about the Z_b -axis is small, as expected. It can be recognised that all the above quantities reach constant steady-state values after the desired motion is stopped. The oscillations on the force and moment during the transient can be mainly ascribed to slipping of the disk on the surface after the contact.

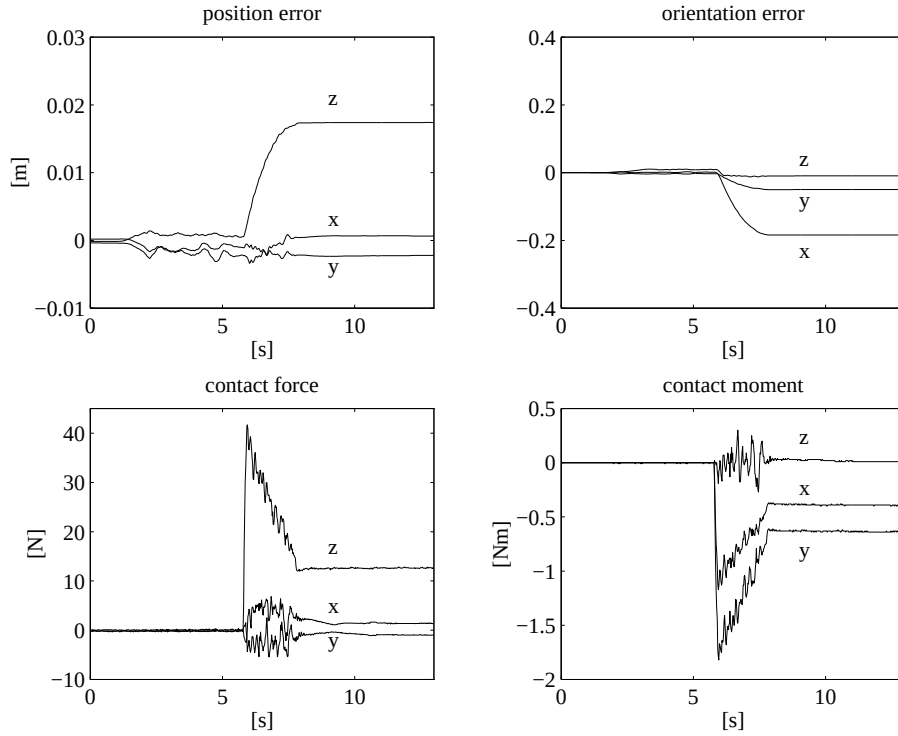


Figure 5.3. Experimental results under six-DOF impedance control based on quaternion in the first case study

In sum, it can be asserted that a compliant behaviour is successfully achieved. A similar performance has been obtained also with the six-DOF impedance control schemes based on the Euler angles error, i.e. by using either (3.3) or (3.6) in lieu of (3.30). This fact can be explained because both the absolute end-effector orientation in (3.3) and the relative orientation

in (3.6) keep far from representation singularities. The results are not reported here for brevity.

Second case study: Representation singularity. The second case study is aimed at testing the performance of the quaternion-based compared to the Euler angles-based six-DOF impedance control, when the end-effector orientation is close to a representation singularity of $\mathbf{T}$. The end effector and the surface are the same as in the previous case study.

The end-effector desired position is required to make a straight-line motion with a horizontal displacement of 0.085 m along the X_b -axis of the base frame. The trajectory along the path is generated according to a fifth-order interpolating polynomial with null initial and final velocities and accelerations, and a duration of 5 s. The end-effector desired orientation is required to remain constant during the task. The surface is now placed vertically in such a way as to obstruct the desired end-effector motion, only for the rotational part though. Therefore, no impedance control has been accomplished for the translational part, i.e. $\mathbf{p}_c$ in (2.9) coincides with $\mathbf{p}_d$.

The parameters of the quaternion-based impedance equations (3.1),(3.30) are set to $\mathbf{M}_p = 10\mathbf{I}$, $\mathbf{D}_p = 600\mathbf{I}$, $\mathbf{K}_p = 1000\mathbf{I}$, $\mathbf{M}_o = 0.25\mathbf{I}$, $\mathbf{D}_o = 3.5\mathbf{I}$, $\mathbf{K}_o = 2.5\mathbf{I}$. In order to carry out a comparison, the impedance control based on the Euler angles has also been tested. The parameters of the rotational impedance equation (3.3) have been set to the same values as for the quaternion. As regards the gains of the inner motion control loop, these have been chosen equal to those in the previous experiment for both types of impedance control schemes.

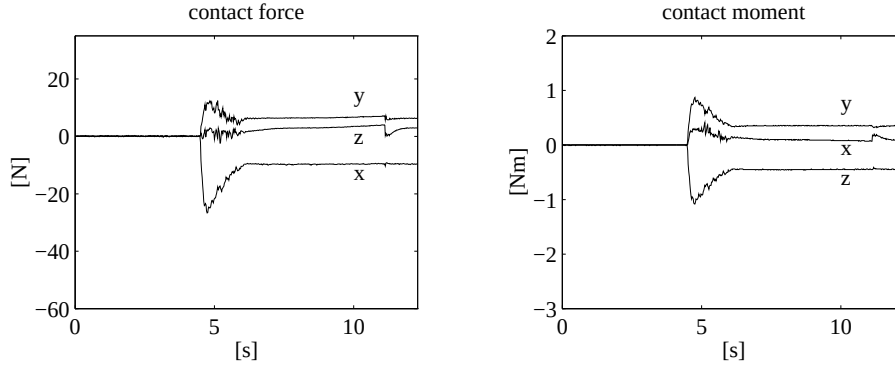


Figure 5.4. Experimental results under six-DOF impedance control based on quaternion in the second case study

The results in Figures 5.4 and 5.5 show the significant differences occurring in the performance of the two schemes. For the impedance control based

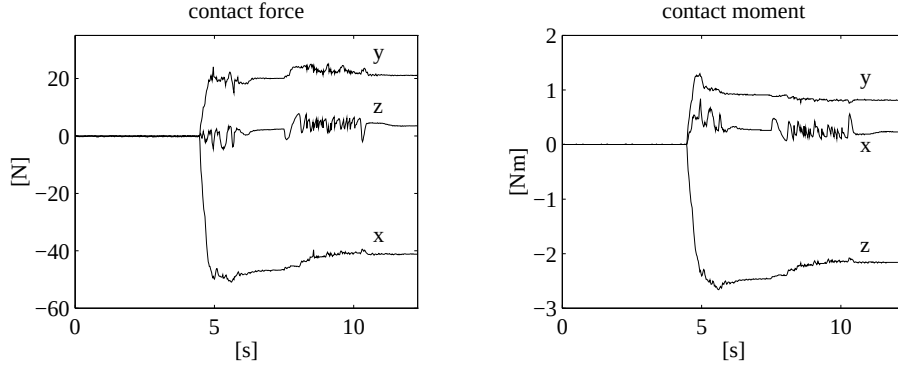


Figure 5.5. Experimental results under six-DOF impedance control based on the classical Euler angles in the second case study

on (3.3), large values of contact force and moment are generated since the rotational impedance equation suffers from ill-conditioning of the matrix $\mathbf{T}(\varphi_c)$; this phenomenon is not present for the quaternion-based impedance control based on (3.30) since representation singularities are not involved in the rotational impedance equation. On the other hand, testing of the impedance control based on the alternative Euler angles in (3.6) has revealed a performance as good as the quaternion-based impedance control, since the orientation displacement φ_{dc} is kept far from a representation singularity. Hence, the results are not reported here for brevity.

In sum it can be concluded that both the impedance control based on the alternative Euler angles and the quaternion-based impedance control perform better than the impedance control based on the classical Euler angles, as far as interaction with the environment is concerned.

Third case study: Task geometric consistency. Another case study has been developed to analyse task geometric consistency when an external moment is applied at the end effector. The quaternion-based impedance control and the impedance control based on Euler angles have been tested.

The stiffness matrices of the rotational impedance equations (3.30), (3.3) have been taken as diagonal matrices; $\mathbf{K}_o = \mathbf{U}_o^T \mathbf{\Gamma}_o \mathbf{U}_o$ has been chosen with $\mathbf{U}_o = \mathbf{I}$ and $\mathbf{\Gamma}_o = 2.5\mathbf{I}$ for both schemes. The remaining parameters of the rotational impedance have been set to $\mathbf{M}_o = 0.25\mathbf{I}$ and $\mathbf{D}_o = 1.5\mathbf{I}$ for both schemes. No impedance control has been accomplished for the translational part. The gains of the inner motion control loop have been chosen equal to those in the previous case study.

The position and orientation of the desired frame are required to remain constant, and a torque is applied about the approach axis of Σ_d ; the torque is taken from zero to 2.5 Nm according to a linear interpolating polynomial with 4th-order blends and a total duration of 1 s.

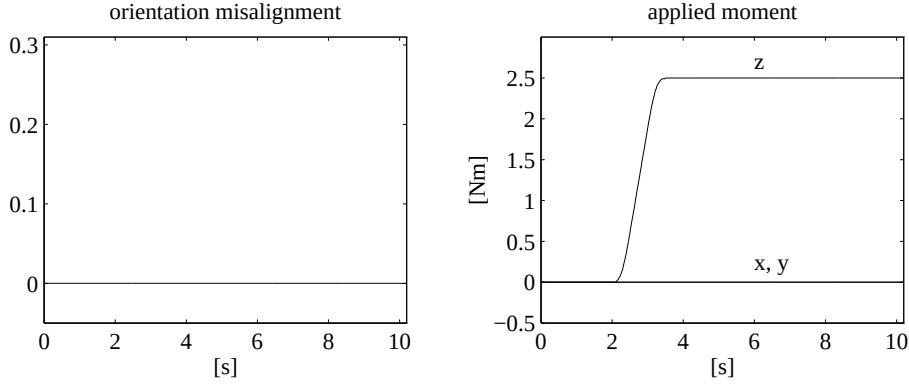


Figure 5.6. Experimental results under six-DOF impedance control based on quaternion in the third case study

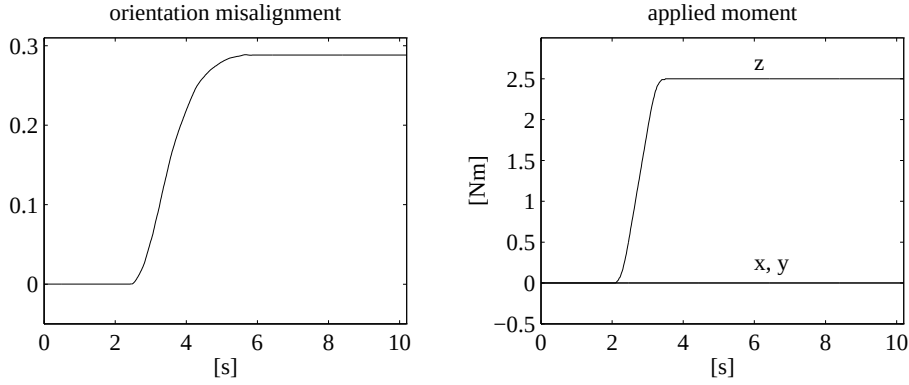


Figure 5.7. Experimental results under six-DOF impedance control based on classical Euler angles in the third case study

The results in Figures 5.6 and 5.7 show the different performance in terms of the orientation misalignment δ which has been defined as the norm of the vector product between the orientation error and the unit vector $\mathbf{u}_{o3}$ of the approach axis of Σ_d , i.e.

$$\delta = \|\mathbf{S}({}^e\boldsymbol{\epsilon}_{de})\mathbf{u}_{o3}\|.$$

For the impedance control based on (3.3) the instantaneous axis of rotation of Σ_e changes, while remarkably no misalignment occurs for the impedance control based on (3.30). The impedance control based on (3.6) has also been tested and its performance is as good as that of the quaternion-based control; hence, the results are not reported for brevity.

Fourth case study: Nondiagonal rotational stiffness. In the fourth case study, the quaternion-based impedance control and the impedance control based on

the alternative Euler angles have been tested when a nondiagonal rotational stiffness is chosen. The impedance control based on the classical Euler angles has been ruled out in view of the poor results of the previous experiment.

The principal axes of the stiffness matrices of the rotational impedance equations (3.30),(3.6) are rotated with respect to the coordinate axes of Σ_d ; $\mathbf{K}_o = \mathbf{U}_o^T \mathbf{F}_o \mathbf{U}_o$ has been chosen with

$$\mathbf{U}_o = \begin{bmatrix} 0.8047 & -0.3106 & 0.5059 \\ 0.5059 & 0.8047 & -0.3106 \\ -0.3106 & 0.5059 & 0.8047 \end{bmatrix} \quad \mathbf{F}_o = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}$$

for both schemes. The remaining parameters of the rotational impedance have been set to $\mathbf{M}_o = 0.25\mathbf{I}$ and $\mathbf{D}_o = 1.5\mathbf{I}$ for both schemes. As above, no impedance control has been accomplished for the translational part, and the gains of the inner motion control loop have been chosen equal to those in the previous case study. A torque has been applied about the axis whose unit vector is $\mathbf{u}_{o3}$; the torque is taken from zero to -1.5 Nm according to a linear interpolating polynomial with 4th-order blends and a total duration of 1 s.

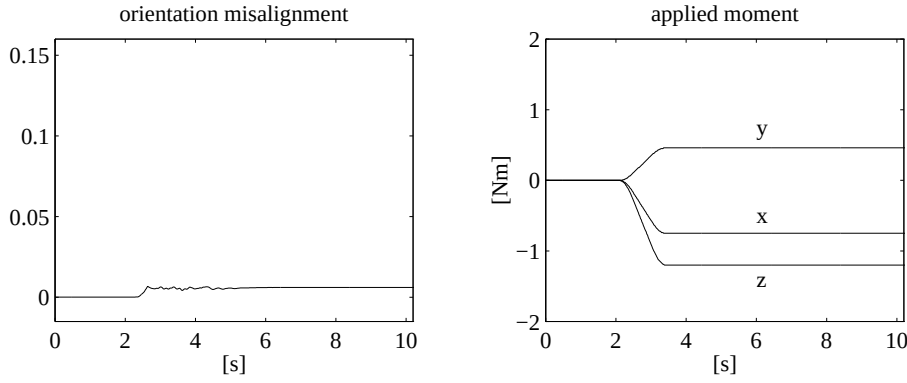


Figure 5.8. Experimental results under six-DOF impedance control based on quaternion in the fourth case study

The results in Figures 5.8 and 5.9 show the significant differences occurring in terms of the orientation misalignment δ . It can be seen that the instantaneous axis of rotation of Σ_e does not appreciably rotate with the impedance control based on (3.30), given the performance of the inner loop acting on the end-effector orientation error. Instead, a significant misalignment occurs with the impedance control based on (3.6).

In sum, it can be concluded that the quaternion-based impedance control performs better than both impedance control schemes based on the Euler angles as far as task geometric consistency is concerned.

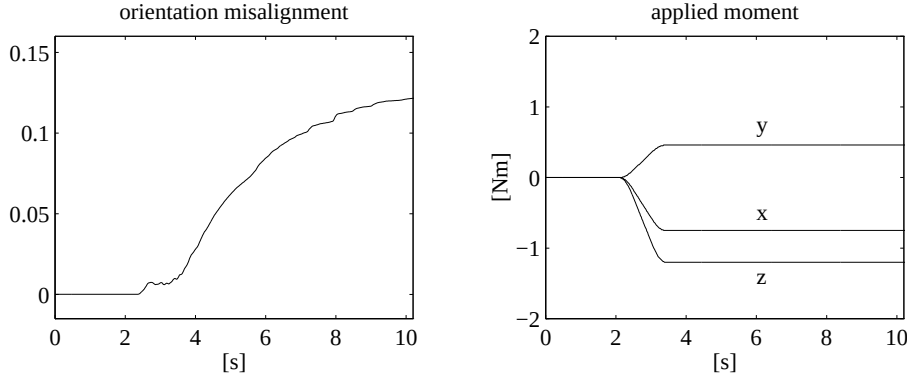


Figure 5.9. Experimental results under six-DOF impedance control based on alternative Euler angles in the fourth case study

Fifth case study: Redundancy resolution. The six-DOF quaternion-based impedance control has been tested in a case study when a redundancy resolution scheme is incorporated into the motion control.

The environment is constituted by a cardboard box. The translational stiffness at the contact between the end effector and the surface is of the order of 5000 N/m, while the rotational stiffness for small angles is of the order of 15 Nm/rad.

The task in the experiment consists of four phases; namely, reconfiguring the manipulator, approaching the surface, staying in contact, and leaving the surface. To begin, the additional task function in (3.39) has been chosen as

$$w(\mathbf{q}) = \frac{1}{2}(q_3 - q_{3d})^2$$

where q_3 is the elbow joint and q_{3d} is a desired trajectory from the initial value of q_3 to the final value of 1.1 rad in a time of 4 s with a fifth-order interpolating polynomial with null initial and final velocity and acceleration. This function is aimed at reconfiguring the manipulator in a more dexterous posture before contacting the surface. After a lapse of 4 s, the disk is taken in contact with the surface at an angle of $7\pi/36$ rad. The end-effector desired position is required to make a straight-line motion with a horizontal displacement of 0.08 m along the Z_b axis of the base frame. The trajectory along the path is generated according to a fifth-order interpolating polynomial with null initial and final velocities and accelerations, and a duration of 2 s. The end-effector desired orientation is required to remain constant during the task. The surface is placed (vertically) in the X_bY_b -plane of the base frame in such a way to obstruct the desired end-effector motion, both for the translational part and the rotational part. After a lapse of 13 s in contact, the end-effector motion is commanded back to the initial position with a duration of 4 s.

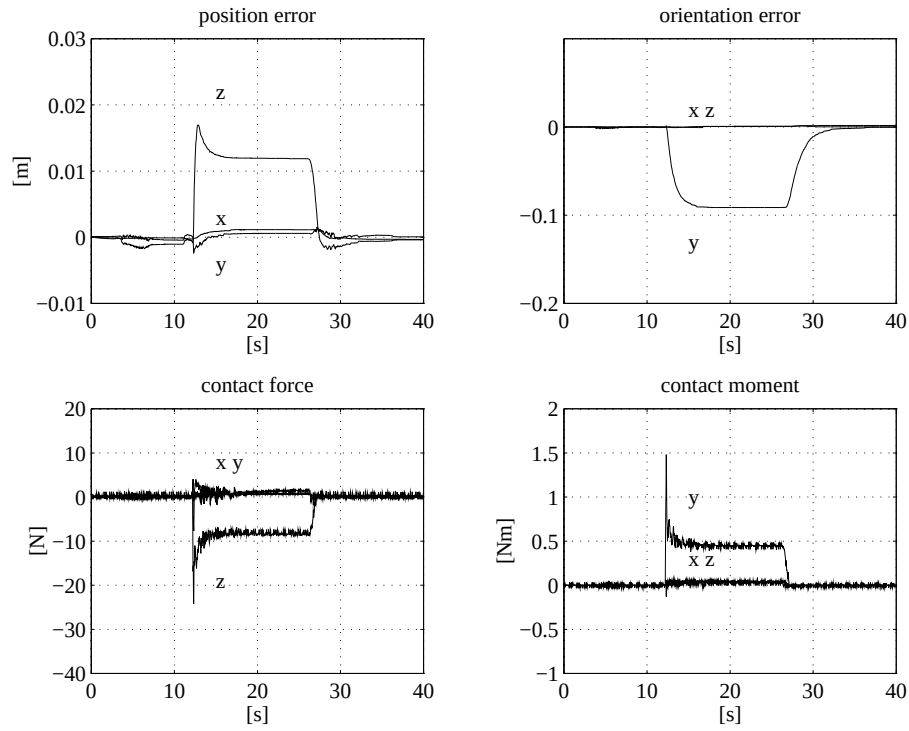


Figure 5.10. Experimental results under six-DOF impedance control based on quaternion with redundancy resolution in the fifth case study

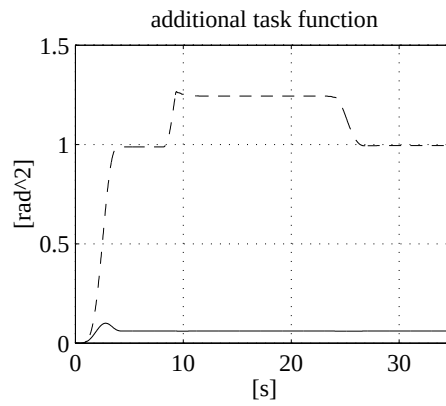


Figure 5.11. Additional task function in the fifth case study

The parameters of the translational impedance (3.1) have been set to $\mathbf{M}_p = 16\mathbf{I}$, $\mathbf{D}_p = \text{diag}\{800, 800, 250\}$ and $\mathbf{K}_p = \text{diag}\{1300, 1300, 800\}$, while the parameters of the rotational impedance (3.30) have been set to $\mathbf{M}_o = 0.7\mathbf{I}$ and $\mathbf{D}_o = 4\mathbf{I}$, $\mathbf{K}_o = 2.5\mathbf{I}$.

The gains of the inner motion control loop in (2.9),(2.15) have been set to $\mathbf{K}_{Pp} = 2250\mathbf{I}$ and $\mathbf{K}_{Po} = 4000\mathbf{I}$, $\mathbf{K}_{Dp} = 70\mathbf{I}$ and $\mathbf{K}_{Do} = 75\mathbf{I}$. The gains of the redundancy resolution control in (3.38),(3.39) have been set to $\mathbf{K}_n = 20\mathbf{I}$ and $k_\beta = 250$.

The results in Figure 5.10 show the effectiveness of the six-DOF impedance control with redundancy resolution. During the reconfiguration (8 s), the components of the position error $\Delta\mathbf{p}_{de} = \mathbf{p}_d - \mathbf{p}_e$ between Σ_d and Σ_e and of the orientation error ${}^e\epsilon_{de}$ between Σ_d and Σ_e are practically zero, meaning that the dynamics of the null space motion does not disturb the end-effector motion. Such error remains small during the approach (2 s). During the contact (13 s), the component of the position error along the Z_b -axis significantly deviates from zero, as expected; as for the orientation error, the component of the orientation error along the Y_e -axis significantly deviates from zero since Σ_e has to rotate about Y_e in order to comply with the surface. Also, in view of the imposed task, a prevailing component of the contact force can be observed along the Z_b -axis after the contact, whereas the sole component of the contact moment about the Y_e -axis is significant, as expected. During the takeoff (4 s), both the errors and the contact force and moment return to zero.

The same task has been executed again for the impedance control without redundancy resolution ($k_\beta = 0$). The performance in terms of the contact between the end effector and the surface is the same as above since the additional task does not interfere with the primary interaction task; hence, the time history of the relevant quantities is omitted for brevity. Nevertheless, a comparison between the two cases in Figure 5.11 shows that the task function is successfully optimised when redundancy is exploited (solid) other than when redundancy is not exploited (dashed).

5.2 Loose Cooperation

A typical assembly task has been performed to test the effectiveness of the quaternion-based impedance control strategy for execution of interaction tasks where the geometry of the contact plays a crucial role. The task is the classical peg-in-hole, which can be executed by a dual-arm system if one robot carries the peg and the other holds the hollow part.

The task for the seven-joint manipulator is planned as follows. From a given posture a joint space motion is commanded to reach a suitable intermediate posture which facilitates the subsequent phases of the task; then, a Cartesian space motion along a straight-line path is commanded to drive the tip of the peg in the proximity of the mouth of the hole and align the

approach axis of the peg with the axis of the hole, as accurately as possible. Finally, a straight-line motion along the approach axis —typically at a reduced speed with respect to the previous phase— is commanded to realise the insertion. The described task is specified in the following PDL 2 program:

```

PROGRAM insertion CONST spd=3
    dpt=50
VAR    int: JOINTPOS
    pro: POSITION
BEGIN
    --move joints to intermediate posture
    MOVE TO int
    --move tip to proximate pose
    MOVE LINEAR TO pro
    --set arm speed override at spd %
    $ARM_OVR := spd
    --move tip along approach axis by dpt mm
    MOVE RELATIVE VEC(0,0,dpt) IN TOOL
END insertion

```

where the joint position `int` and the end-effector pose `pro` are taught beforehand, while the constants `spd` and `dpt` are set according to task requirements.

The six-joint manipulator is controlled using the open operating mode so that the end effector behaves as the programmed six-DOF impedance. To this purpose, the force/torque sensor is mounted at the wrist.

For the task at issue, the position and orientation of the desired frame is taken as a constant, i.e. the six-joint manipulator is controlled to stay still. Whenever a contact force and/or moment is experienced at the end effector, this reacts according to the programmed impedance where the origin of the desired frame determines the location of the Remote Center of Compliance.

The peg is a wooden cylinder of 17 mm diameter and 80 mm height, while the hollow part is a wooden block with a hole of 18 mm diameter and 70 mm depth; that is, a 0.5 mm radial tolerance is present during the insertion.

The insertion task is programmed in terms of a planned motion for the seven-joint manipulator described by `spd = 3` and `dpt = 50`.

The six-joint manipulator is impedance-controlled so that the Remote Center of Compliance is located at $\mathbf{p}_d = \mathbf{p}_0 - [0 \ 165 \ 0]^T$ mm since the insertion direction is along the Y_b -axis of the base frame. Also, it is $\mathbf{R}_d = \mathbf{R}_0$. The parameters of the impedance equation in (3.1),(3.30) are respectively set to $\mathbf{M}_p = \text{diag}\{15, 40, 15\}$, $\mathbf{D}_p = \text{diag}\{300, 950, 300\}$, $\mathbf{K}_p = \text{diag}\{400, 1300, 400\}$, for the translational part, and $\mathbf{M}_o \mathbf{9I}$, $\mathbf{D}_o = 13.5\mathbf{I}$, $\mathbf{K}_o = \mathbf{I}$, for the rotational part.

Since the experiment is aimed at testing the robustness of the proposed strategy in the case of incorrect task planning, an additional misalignment has been intentionally introduced by rotating the end-effector frame of the seven-joint manipulator by 2 deg about the X_b - and the Z_b -axis of its base

frame. Further, the length of the path along the approach axis of the peg has been set to $d_{pt} = 90$, corresponding to a 20 mm overshoot beyond the bottom of the hole. The same impedance parameters as above are chosen.

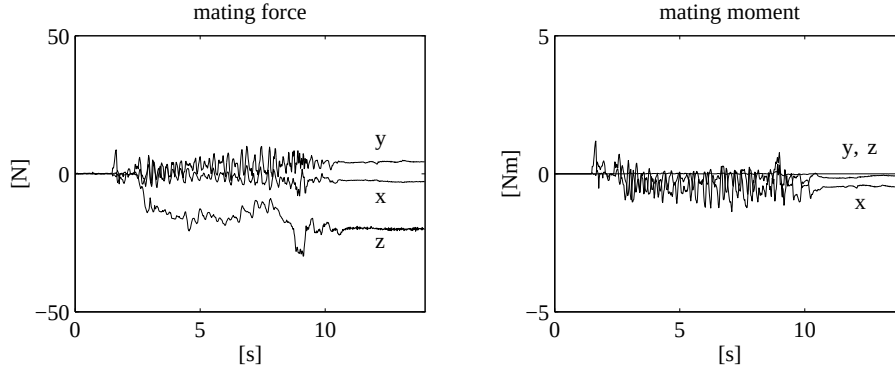


Figure 5.12. Force and moment in the peg-in-hole task

The results are illustrated in Figure 5.12 in terms of the time history of the three components of mating force and moment. Interestingly enough, the values of force and moment keep limited despite of the incorrect task planning; moreover at steady state, nonnull values of force and moment can be observed which are obviously caused by the planned misalignment and overshoot.

5.3 Tight Cooperation

In the experiment devoted to testing the proposed tight cooperative control strategy, the two robot end effectors tightly grasp the ends of a wooden bar of 1 m length. At the center of the bar is fixed a steel stick with a wooden disk of 5.5 cm radius at its tip.

The environment is constituted by a cardboard box; the translational stiffness at the contact between the disk and the surface is of the order of 5000 N/m, while the rotational stiffness for small angles is of the order of 15 Nm/rad.

The task in the experiment consists in taking the disk in contact with the surface; that is placed at an unknown distance with an angle of unknown magnitude. The origin of Σ_o is required to make a desired motion along a straight line with a vertical displacement of -0.275 m along the Z_b -axis of Σ_b . The trajectory along the path is generated according to a 5th-order interpolating polynomial with null initial and final velocities and accelerations, and a duration of 6 s. The desired orientation of the object frame is required to remain constant.

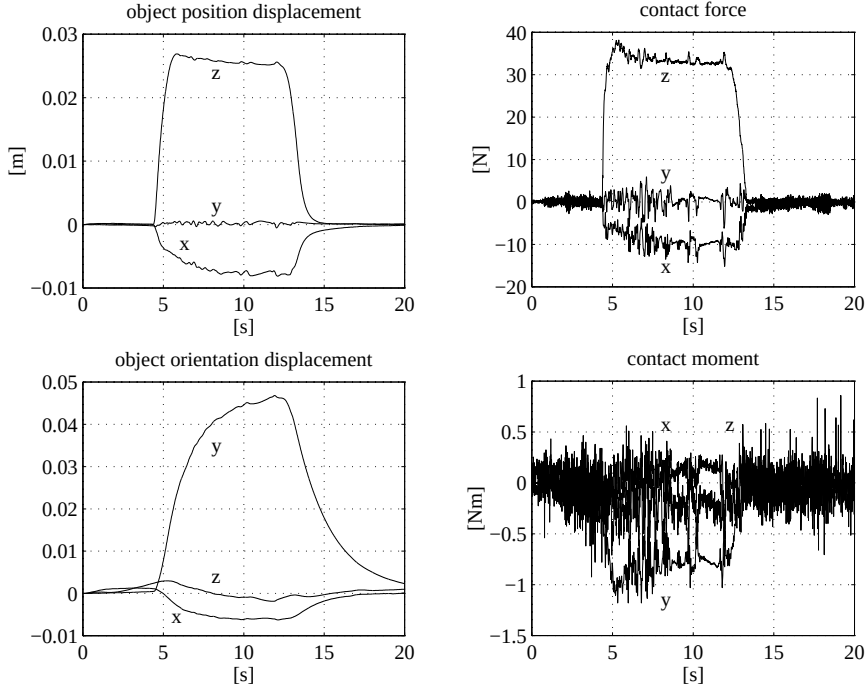


Figure 5.13. Experiment of tight cooperative control: *Left*: Object position and orientation displacement between Σ_d and Σ_o . *Right*: Contact force and moment acting on the object

The parameters of the translational part of the impedance equation (3.1) have been set to $\mathbf{M}_p = 30\mathbf{I}$, $\mathbf{D}_p = 555\mathbf{I}$, $\mathbf{K}_p = 1300\mathbf{I}$, while the parameters of the rotational part of the impedance equation (3.30) have been set to $\mathbf{M}_o = \text{diag}\{10, 2, 10\}$, $\mathbf{D}_o = \text{diag}\{35, 20, 35\}$, $\mathbf{K}_o = \text{diag}\{20, 8, 20\}$. Notice that the stiffness matrices have been chosen so as to ensure a compliant behaviour (limited values of contact force and moment) during the contact, while the damping matrices have been chosen so as to guarantee a well-damped behaviour. The feedback gains in (2.9), (2.15) have been set to $k_{D_p} = 65$, $k_{P_p} = 1800$ for the position loop, and $k_{D_o} = 65$, $k_{P_o} = 3600$ for the orientation loop, respectively.

From Figure 5.13 (*left*), after the contact, the component along the Z_b -axis of the position displacement between the desired frame Σ_d and the object frame Σ_o , expressed in Σ_b , significantly deviates from zero, as expected; a smaller displacement can also be seen for the component along the X_b -axis, due to contact friction. As for the orientation displacement between Σ_o and Σ_d , expressed in Σ_d , only the component along the Y_d -axis significantly deviates from zero since the object frame has to rotate about the Y_d -axis of Σ_d in order to comply with the surface after the contact.

From Figure 5.13 (*right*), in view of the imposed task, a prevailing component of the contact force can be observed along the Z_b -axis after the contact, while a significant component along the X_b -axis arises, corresponding to the above position displacement. As for the contact moment, the only nonnegligible component is that along the Y_d -axis of Σ_d , which corresponds to the above orientation displacement. It can be recognised that all the above quantities reach constant steady-state values after the desired motion is stopped. The oscillations on the force and moment can be ascribed to the flexibility effects of the commonly held object.

6 Conclusion and Future Directions

An impedance control strategy has been presented for six-DOF tasks. By considering an angle/axis representation of the end-effector orientation displacement, a six-DOF impedance equation has been derived, which exhibits a physically meaningful behavior and task geometric consistency properties. Among the different angle/axis representations, the unit quaternion has been chosen which avoids the occurrence of representation singularities. The superior performance of the proposed angle/axis-based impedance controller over two different impedance controllers based on Euler angles has been shown both in theory and in practice.

For the case of redundant manipulators, the dynamically consistent pseudoinverse of the manipulator Jacobian has been adopted to decouple the dynamics of the end-effector motion from the null-space motion. Moreover, redundancy is exploited to stabilise null-space joint velocities and optimise an additional task function.

Finally, the problem of two cooperative robots manipulating a common object has been considered and both loose and tight cooperation have been addressed. In the case of loose cooperation, limited interaction forces have been achieved by adopting the geometrically consistent impedance control strategy for one of the two manipulators. In the case of tight cooperation, interaction of the grasped object with the environment has been managed by enforcing a geometrically consistent impedance behaviour at the object level.

The proposed control strategies have been experimentally tested on a set-up composed by two industrial robots Comau SMART-3S with two force/torque sensors ATI FT-30/100 mounted at the wrist.

Future work will be devoted to extending the use of geometrically consistent impedance for cooperative manipulators. Namely, the adoption of an impedance control strategy for controlling the internal forces at the grasped object is currently under investigation. Under this regard, an open problem is the cooperative manipulation of flexible objects. It is expected that the adoption of impedance control may provide a simple and robust way to cope with manipulation of flexible objects or manipulation of rigid objects via elastic grasp.

Promising future research will be directed towards the integration of the interaction control strategies, based on the use of position and force/torque sensors, with information coming from vision sensors. Indeed, the adoption of sensory feedback coming from cameras is expected to enhance the robustness of the control loop as well as may represent a decisive step towards robotic systems capable of autonomous interaction with the surrounding environment, even partially.

Last but not least, it should be mentioned that the problem of ensuring a safe interaction between the robot the environment can be considered as a major objective to be pursued in the next few years. In the case of interaction with an artificial environment, suitable strategies must be adopted aimed at preventing and/or managing robot failures and abrupt changes of the environment conditions, so as to avoid damage for the various devices involved in the contact and ensure task completion. On the other hand, when the human/robot interaction is of concern (e.g. in anthropic and service robotics), safety of the human subject becomes the primary objective to be achieved, even at the expense of the task assigned to the robot.

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