

A Unified Framework for Design of Interaction Control Schemes for Robot Manipulators

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Abstract: A unified framework for design of control schemes for robot manipulators interacting with compliant environments is presented in this paper. Compliance and impedance control schemes are logically derived from operational space motion control schemes, with integration of contact force measurements. Force control schemes are then presented which are obtained by closing an outer force feedback loop around an inner position or velocity feedback loop. Numerical examples are illustrated.

1 Introduction

One of the fundamental requirements for the success of a manipulation task is the capability to handle *interaction* between the robot manipulator and the environment. The quantity that describes the state of interaction more effectively is the *contact force* at the manipulator's end effector. High values of contact force are generally undesirable since they may stress both the manipulator and the manipulated object [1].

This paper is aimed at establishing a unified framework for design of interaction controllers where contact forces can be controlled with two different strategies; namely, an *indirect* strategy via a suitable use of motion control laws or a *direct* strategy via true force control laws [2].

The performance of operational space motion control schemes [3] is analyzed first. The concepts of mechanical *compliance* [4] and *impedance* [5] are presented, with special regard to the problem of integrating contact force measurements into the control strategy [6]. An interesting feature of compliance and impedance control schemes is that the interaction force can be indirectly controlled by acting on the reference position of manipulator motion control [7].

On the other hand, if it is wished to accurately control the contact force, it is necessary to devise control schemes that allow directly specifying the desired interaction force. The realization of a *force control* scheme can be entrusted to the closure of an outer force regulation feedback loop generating the control input for the position control scheme the manipulator is usually endowed with [8]. Three different schemes are presented; namely,

a force control scheme with inner position loop, a force control scheme with inner velocity loop, and a parallel force/position control scheme [9,10].

Numerical examples are presented throughout the paper to illustrate the performance of the various interaction control schemes for a simple planar arm in contact with an elastically compliant plane.

2 Compliance Control

For a detailed analysis of interaction between the manipulator and environment it is worth considering the behavior of the system under a position control scheme when contact forces arise. Since these are naturally described in the operational space, it is convenient to refer to *operational space control* schemes.

Consider the manipulator dynamic model which can be written as

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = u - J^T(q)h, \quad (1)$$

where q is the $(n \times 1)$ vector of joint variables, B is the $(n \times n)$ inertia matrix, $C(q, \dot{q})\dot{q}$ is the $(n \times 1)$ vector of Coriolis and centrifugal terms, F is the $(n \times n)$ matrix of joint viscous friction, g is the $(n \times 1)$ vector of gravity terms, u is the $(n \times 1)$ vector of input torques, $J(q)$ is the end-effector geometric Jacobian, and h is the vector of contact forces exerted by the manipulator's end effector on the environment.

Let x denote the $(m \times 1)$ vector of end-effector location. Let also $\tilde{x}$ be the operational space error between a *constant* desired end-effector location x_d and x . Choose the input torque as a PD operational space control with joint space gravity compensation of the type

$$u = J_A^T(q)(K_P\tilde{x} - K_D\dot{\tilde{x}}) + g(q) \quad (2)$$

where J_A indicates the analytical Jacobian; this is related to the above geometric Jacobian by

$$J = T_A(x)J_A \quad (3)$$

where $T_A(x)$ is a transformation matrix which depends on the set of Euler angles chosen to describe end-effector orientation.

The control law (2), in the absence of contact forces, ensures global asymptotic stability of the desired posture x_d as long as the analytical Jacobian is nonsingular (no kinematic or representation singularities). On the other hand, in the case $h \neq 0$, the control law (2) no longer ensures that the end effector reaches x_d . In fact, at the equilibrium it is

$$J_A^T(q)K_P\tilde{x} = J^T(q)h. \quad (4)$$

On the assumption of a full-rank Jacobian, one has

$$\tilde{x} = K_P^{-1}T_A^T(x)h = K_P^{-1}h_A, \quad (5)$$

where h_A is the vector of equivalent generalized forces that can be related to h by

$$T_A^T(x)h = h_A. \quad (6)$$

Eq. (5) shows that at the equilibrium the manipulator, under a position control action, behaves as a generalized spring in the operational space with compliance K_P^{-1} in respect of force h_A . Since the transformation matrix T_A is nondiagonal while matrix K_P is typically diagonal, it can be recognized that linear compliance (due to force components) is independent of the posture while torsional compliance (due to moment components) does depend on the current manipulator configuration through the matrix T_A .

On the other hand, if $h \in \mathcal{N}(J^T)$, one has $\tilde{x} = 0$ with $h \neq 0$, i.e. contact forces are completely balanced by the manipulator mechanical structure; for instance, an anthropomorphic manipulator at a shoulder singularity does not react to any force orthogonal to the plane of the structure.

A detailed description of the contact is demanding from a modeling viewpoint. To point out the fundamental aspects of interaction control, it is convenient to resort to a simple but significant model of contact. To this purpose, a decoupled *elastically compliant environment* is considered which is described by the model

$$h = \begin{bmatrix} f \\ \mu \end{bmatrix} = \begin{bmatrix} K_f & O \\ O & K_m \end{bmatrix} \begin{bmatrix} dp \\ \omega dt \end{bmatrix} = K \begin{bmatrix} dp \\ \omega dt \end{bmatrix}, \quad (7)$$

where dp is the vector of translation along the reference frame axes and ωdt is the vector of small rotation about the axes of such frame. Hence the vector $[dp^T \ \omega^T dt]^T$ describes a generalized displacement from the environment rest position. The *stiffness matrix* K is typically *positive semi-definite*. In fact, the environment does not generate reaction forces along those directions where unconstrained end-effector motion is allowed.

In view of (3), Eq. (7) can be written in terms of operational space variables as

$$h = KT_A(x)dx \quad (8)$$

where dx denotes the operational space generalized displacement with respect to the undeformed environment rest position x_e

$$dx = x - x_e. \quad (9)$$

Resorting to (6),(9) gives

$$h_A = T_A^T(x)KT_A(x)dx = K_A(x)(x - x_e) \quad (10)$$

that allows relating the equivalent forces on the manipulator with the environment deformation through the matrix K_A , i.e. the environment stiffness matrix. The matrix K_A^{-1} , if it can be defined, is the environment *compliance* matrix. It represents a *passive* compliance since it describes an inherent property of the environment in the operational space chosen to express manipulator end-effector position and orientation. By observing that

K_A is only positive semi-definite, the concept of compliance cannot be globally defined in all operational space but only along those directions, spanning $\mathcal{R}(K_A)$, along which end-effector motion is constrained by the environment.

On the other hand, notice that the matrix K_P^{-1} in (5) represents an *active compliance* since it is performed on the manipulator by a suitable position control action. With the environment model (10), Eq. (5) becomes

$$\tilde{x} = K_P^{-1} K_A(x)(x - x_e); \quad (11)$$

at the equilibrium, the end-effector location is given by

$$x_\infty = (I + K_P^{-1} K_A(x))^{-1} (x_d + K_P^{-1} K_A x_e), \quad (12)$$

while the contact force can be shown to be

$$h_{A\infty} = (I + K_A(x) K_P^{-1})^{-1} K_A(x)(x_d - x_e). \quad (13)$$

Analysis of (12) shows that the equilibrium position depends on the environment rest position as well as on the desired position imposed by the control system to the manipulator. The interaction of the two systems (environment and manipulator) is influenced by the mutual weight of the respective compliance features. It is then possible to increase the active compliance so that the manipulator dominates the environment and vice versa. Such a dominance can be specified with reference to the single directions of the operational space. For a given environment stiffness, according to the given interaction task, one may choose large values of the elements of K_P for those directions along which the environment has to comply and small values of the elements of K_P for those directions along which the manipulator has to comply.

Expression (13) gives the value of the contact force at the equilibrium which reveals that it may be appropriate to tune manipulator compliance with environment compliance along certain directions of the operational space. In fact, along a direction with high environment stiffness, it is better to have a compliant manipulator so that it can taper the intensity of interaction through a suitable choice of the desired position. In this case the end-effector equilibrium position x_∞ practically coincides with the environment rest position x_e , and the manipulator generates an interaction force, depending on the corresponding element of K_P , that is determined by the choice of the component of $(x_d - x_e)$ along the relative direction.

In the dual case of high environment compliance, if the manipulator is made stiff, the end-effector equilibrium position x_∞ is very close to the desired position x_d , and it is the environment to generate the elastic force along the constrained directions of interest.

In certain cases, it is possible to employ mechanical devices interposed between the manipulator's end effector and the environment so as to change passive compliance along particular directions of the operational space. For instance, in a peg-in-hole insertion task, the gripper is provided with a device ensuring high stiffness along the insertion direction and high compliance along the other directions (*remote center of compliance*). Therefore,

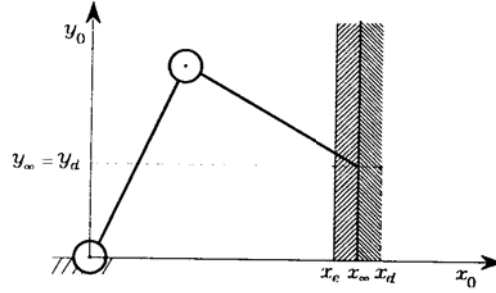


Figure 1: Two-link planar arm in contact with an elastically compliant plane.

in the presence of unavoidable position displacements from the planned insertion, trajectory contact forces and moments arise which modify the peg position so as to facilitate insertion.

The inconvenience of such devices is their low versatility to different operating conditions and generic interaction tasks, i.e. whenever a modification of the compliant mechanical hardware is required. On the other hand, with active compliant actions the control software can be easily modified so as to satisfy the requirements of different interaction tasks.

Example 1. Consider the two-link planar arm whose tip is in contact with a purely frictionless elastic plane; let x_e be the equilibrium position of the plane, which is assumed to be orthogonal to axis x (Fig. 1). The environment stiffness matrix is

$$K_A = K_f = \text{diag}\{k_x, 0\},$$

corresponding to the absence of interaction forces along the vertical direction ($f_y = 0$). Let $\mathbf{p}_d = [x_d \ y_d]^T$ be the desired tip position, which is located beyond the contact plane. The proportional control action on the arm is characterized by

$$K_P = \text{diag}\{k_{Px}, k_{Py}\}.$$

The equilibrium equations for position and force (12),(13) give

$$\mathbf{p}_\infty = \begin{bmatrix} \frac{k_{Px}x_d + k_x x_e}{k_{Px} + k_x} \\ y_d \end{bmatrix} \quad \mathbf{f}_\infty = \begin{bmatrix} \frac{k_{Px}k_x}{k_{Px} + k_x}(x_d - x_e) \\ 0 \end{bmatrix}.$$

With reference to positioning accuracy, the arm tip reaches the vertical coordinate y_d since the vertical motion direction is not constrained. As for the horizontal direction, the presence of the elastic plane imposes that the arm can move as far as it reaches the coordinate x_∞ . The value of the horizontal contact force at the equilibrium is related to the difference between x_e and x_d by an equivalent stiffness coefficient which is given by the parallel composition of the stiffness coefficients of the two interacting systems.

Hence, the arm stiffness and environment stiffness influence the resulting equilibrium configuration. In the case when

$$k_{Px}/k_x \gg 1,$$

it is

$$x_\infty \approx x_d \quad f_{x\infty} \approx k_x(x_d - x_e)$$

and thus the arm prevails over the environment, in that the plane complies almost up to x_d and the elastic force is essentially generated by the environment (passive compliance). In the opposite case

$$k_{Px}/k_x \ll 1,$$

it is

$$x_\infty \approx x_e \quad f_{x\infty} \approx k_{Px}(x_d - x_e)$$

and thus the environment prevails over the arm which complies up to the equilibrium x_e , and the elastic force is generated by the arm (active compliance).

3 Impedance Control

It is now desired to analyze the interaction of manipulator with environment under the action of an inverse dynamics control in the operational space. With reference to model (1), consider the inverse dynamics control law

$$u = B(q)y + n(q, \dot{q}), \quad (14)$$

with $n = C\dot{q} + F\dot{q} + g$. In the presence of end-effector forces, the controlled manipulator is described by

$$\ddot{q} = y - B^{-1}(q)J^T(q)h \quad (15)$$

that reveals the existence of a nonlinear coupling term due to contact forces. Choose y as

$$y = J_A^{-1}(q)M_d^{-1}(M_d\ddot{x}_d + K_D\dot{\tilde{x}} + K_P\tilde{x} - M_d\dot{J}_A(q, \dot{q})\dot{q}), \quad (16)$$

where M_d is a positive definite diagonal matrix. Substituting (16) into (15) and accounting for second-order differential kinematics in the form

$$\ddot{x} = J_A(q)\ddot{q} + \dot{J}_A(q, \dot{q})\dot{q} \quad (17)$$

yields

$$M_d\ddot{\tilde{x}} + K_D\dot{\tilde{x}} + K_P\tilde{x} = M_dB_A^{-1}(q)h_A, \quad (18)$$

where

$$B_A(q) = J_A^{-T}(q)B(q)J_A^{-1}(q) \quad (19)$$

is the inertia matrix of the manipulator in the operational space; this matrix is configuration-dependent and is positive definite if J_A has full rank.

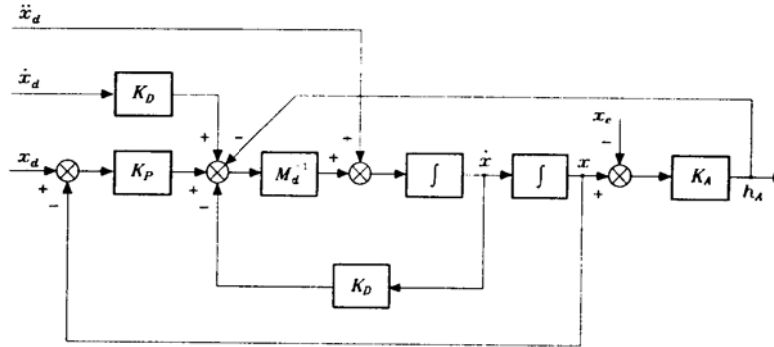


Figure 2: Equivalent block scheme of a manipulator in contact with an elastic environment under impedance control.

Eq. (18) establishes a relationship through a generalized *mechanical impedance* between the vector of resulting forces $M_d B_A^{-1} h_A$ and the vector of displacements $\tilde{x}$ in the operational space. This impedance can be attributed to a mechanical system characterized by a mass matrix M_d , a damping matrix K_D and a stiffness matrix K_P which allow specifying the dynamic behavior along the operational space directions.

The presence of B_A^{-1} makes the system coupled. If it is wished to keep linearity and decoupling during interaction with the environment, it is then necessary to *measure* the generalized contact force; this can be achieved by means of appropriate force sensors which are usually mounted on the manipulator wrist. Choosing

$$u = B(q)y + n(q, \dot{q}) + J^T(q)h \quad (20)$$

with

$$y = J_A^{-1}(q)M_d^{-1}(M_d\ddot{x}_d + K_D\dot{\tilde{x}} + K_P\tilde{x} - M_d\dot{J}_A(q, \dot{q})\dot{q} - h_A), \quad (21)$$

on the assumption of error-free force measurements, yields

$$M_d\ddot{\tilde{x}} + K_D\dot{\tilde{x}} + K_P\tilde{x} = h_A. \quad (22)$$

It is worth noticing that the addition of the term $J^T h$ in (20) exactly compensates the contact forces and then it renders the manipulator infinitely stiff with respect to external stress. In order to confer a compliant behavior to the manipulator, the term $-J_A^{-1}M_d^{-1}h_A$ has been introduced in (21) which allows characterizing the manipulator as a *linear impedance* with regard to the equivalent forces h_A , as shown in (22). The resulting block scheme of a manipulator in contact with an elastic environment under impedance control is illustrated in Fig. 2.

The behavior of system (22) at the equilibrium is analogous to that described by Eq. (4); nonetheless, compared to a compliance control specified by K_P , Eq. (22) allows a complete characterization of system dynamics through an *active impedance* specified by

matrices M_d , K_D , K_P . These matrices are usually taken as diagonal; also in this case it is not difficult to recognize that impedance is configuration-independent as regards the force components, while it depends on the current manipulator configuration as regards the moment components through the matrix T_A .

Furthermore, similarly to active and passive compliance, the concept of *passive impedance* can be introduced if the interaction force h_A is generated at the contact with an environment of proper mass, damping and stiffness. In this case the system of manipulator with environment can be regarded as a mechanical system constituted by the parallel of the two impedances, and then its dynamic behavior is conditioned by the relative weight between them. As pointed out above, one may think about constructing a mechanical device with proper passive impedance that allows the manipulator to better cope with the given interaction task.

Example 2. Consider the planar arm in contact with an elastically compliant plane of the previous example. Apply the impedance control with force measurement (20),(21) characterized by:

$$M_d = \text{diag}\{m_{dx}, m_{dy}\} \quad K_D = \text{diag}\{k_{Dx}, k_{Dy}\} \quad K_P = \text{diag}\{k_{Px}, k_{Py}\}.$$

If x_d is constant, the dynamics of the manipulator and environment system along the two directions of the operational space is described by

$$\begin{aligned} m_{dx}\ddot{x} + k_{Dx}\dot{x} + (k_{Px} + k_x)x &= k_x x_e + k_{Px}x_d \\ m_{dy}\ddot{y} + k_{Dy}\dot{y} + k_{Py}y &= k_{Py}y_d. \end{aligned}$$

Along the vertical direction one has an unconstrained motion whose time behavior is determined by the following natural frequency and damping factor:

$$\omega_{ny} = \sqrt{\frac{k_{Py}}{m_{dy}}} \quad \zeta_y = \frac{k_{Dy}}{2\sqrt{m_{dy}k_{Py}}},$$

while along the horizontal direction the behavior of the contact force $f_x = k_x(x - x_e)$ is determined by

$$\omega_{nx} = \sqrt{\frac{k_{Px} + k_x}{m_{dx}}} \quad \zeta_x = \frac{k_{Dx}}{2\sqrt{m_{dx}(k_{Px} + k_x)}}.$$

Below the dynamic behavior of the system is analyzed for two different values of environment compliance: $k_x = 10^3$ N/m and $k_x = 10^4$ N/m. The arm is characterized by the following data:

$$a_1 = a_2 = 1 \text{ m} \quad \ell_1 = \ell_2 = 0.5 \text{ m} \quad m_{\ell_1} = m_{\ell_2} = 50 \text{ kg} \quad I_{\ell_1} = I_{\ell_2} = 10 \text{ kg} \cdot \text{m}^2$$

where a_i is the link length, ℓ_i is the distance of center of mass from joint axis, m_{ℓ_i} is the link mass, I_{ℓ_i} is the link moment of inertia about joint axis, for $i = 1, 2$, and

$$k_{r1} = k_{r2} = 100 \quad m_{m1} = m_{m2} = 5 \text{ kg}$$

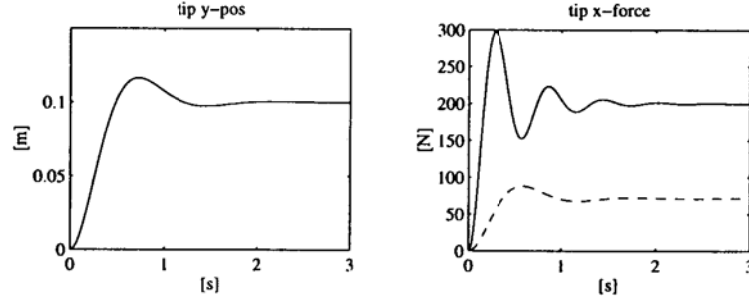


Figure 3: Time history of the tip position along vertical direction and of the contact force along horizontal direction with impedance control scheme for environments of different compliance.

$$I_{m_1} = I_{m_2} = 0.01 \text{ kg} \cdot \text{m}^2 \quad F_{m_1} = F_{m_2} = 0.01 \text{ N} \cdot \text{m} \cdot \text{s/rad}$$

where k_{r_i} is the gear reduction, m_{m_i} is the mass of joint motor, I_{m_i} is the rotor moment of inertia about joint axis, F_{m_i} is the viscous friction, for $i = 1, 2$; motors are located at the joint axes.

The impedance control with force measurements (20),(21) is chosen as

$$m_{dx} = m_{dy} = 100 \quad k_{Dx} = k_{Dy} = 500 \quad k_{Px} = k_{Py} = 2500.$$

For these values it is

$$\omega_{ny} = 5 \text{ rad/s} \quad \zeta_y = 0.5.$$

Then, for the more compliant environment it is

$$\omega_{nx} \approx 5.9 \text{ rad/s} \quad \zeta_x \approx 0.42,$$

while for the less compliant environment it is

$$\omega_{nx} \approx 11.2 \text{ rad/s} \quad \zeta_x \approx 0.22.$$

Let the arm tip be in contact with the environment at position $\mathbf{p} = [1 \ 0]^T$; it is desired to take it to position $\mathbf{p}_d = [1.1 \ 0.1]^T$.

The results in Fig. 3 show that motion dynamics along the vertical direction is the same in the two cases. As regards the contact force along the horizontal direction, for the more compliant environment (*dashed line*) a well-damped behavior is obtained, while for the less compliant environment (*solid line*) the resulting behavior is less damped. Further, at the equilibrium, in the first case a displacement of about 7.1 cm with a contact force of about 71.4 N are observed, while in the second case a displacement of 2 cm with a contact force of 200 N are observed.

4 Force Control

In the above schemes the interaction force could be indirectly controlled by acting on the reference value x_d of the manipulator motion control system. Interaction between manipulator and environment is anyhow directly influenced by compliance of the environment and by either compliance or impedance of the manipulator.

If it is desired to accurately control the contact force, it is necessary to devise control schemes that allow directly specifying the desired interaction force. The development of a *force control* system, in analogy to a motion control system, would require the adoption of a stabilizing PD control action on the force error besides the usual nonlinear compensation actions. Force measurements are typically corrupted by noise and then a derivative action cannot be implemented in practice. The stabilizing action is then to be provided by suitable damping of velocity terms. As a consequence, a force control system typically features a control law based not only on force measurements but also on velocity measurements, and eventually position measurements too.

The realization of a force control scheme can be entrusted to the closure of an *outer force regulation feedback loop* generating the control input for the position control scheme the manipulator is usually endowed with. Therefore, force control schemes are presented below which are based on the use of an inverse dynamics position control. Nevertheless, notice that a force control strategy is meaningful only for those directions of the operational space along which interaction forces between manipulator and environment may arise.

4.1 Force Control with Inner Position Loop

With reference to the inverse dynamics law with force measurement (20), choose in lieu of (21) the control

$$y = J_A^{-1}(q)M_d^{-1}(-K_D\dot{x} + K_P(x_F - x) - M_d\ddot{J}_A(q, \dot{q})\dot{q}) \quad (23)$$

where x_F is a suitable reference to be related to a force error. Notice that the control law (23) does not foresee the adoption of compensating actions relative to $\dot{x}_F$ and $\ddot{x}_F$. Substituting (23) into (20) leads, after similar algebraic manipulation, to the system described by

$$M_d\ddot{x} + K_D\dot{x} + K_Px = K_Px_F, \quad (24)$$

which shows how Eqs. (20),(23) perform a position control taking x to x_F with a dynamics specified by the choice of matrices M_d , K_D , K_P .

Let h_{Ad} denote the desired *constant* force reference; the relation between x_F and the force error can be symbolically expressed as

$$x_F = C_F(h_{Ad} - h_A), \quad (25)$$

where C_F is a diagonal matrix whose elements give the control actions to perform along the operational space directions of interest. Eqs. (24),(25) reveal that force control is developed on the basis of a preexisting position control loop.

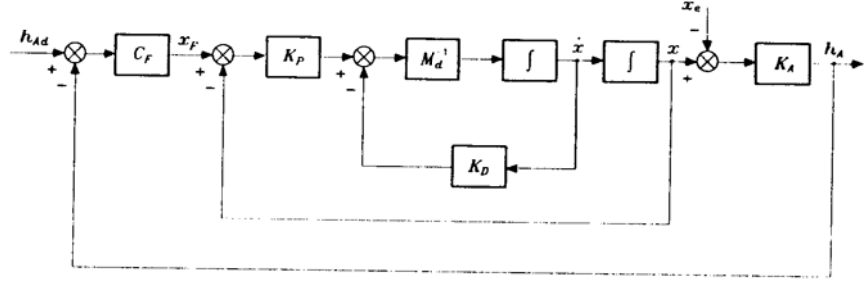


Figure 4: Block scheme of force control with inner position loop.

On the assumption of the elastically compliant environment described by (10), Eq. (24) with (25) becomes

$$M_d \ddot{x} + K_D \dot{x} + K_P(I + C_F K_A)x = K_P C_F(K_A x_e + h_{Ad}). \quad (26)$$

To decide about the kind of control action to specify with C_F it is worth representing Eqs. (10), (24), (25) in terms of the block scheme in Fig. 4, which is logically derived from the scheme in Fig. 2. This scheme suggests that if C_F has a purely proportional control action, then h_A cannot reach h_{Ad} and x_e influences the interaction force also at steady state.

If C_F has also an integral control action on the components of generalized force, then it is possible to achieve $h_A = h_{Ad}$ at steady state and at the same time to reject the effect of x_e on h_A . Hence, a convenient choice for C_F is a *proportional-integral (PI) action*

$$C_F = K_F + K_I \int_0^t (\cdot) d\zeta. \quad (27)$$

The dynamic system resulting from (26), (27) is of third order, and then it is necessary to adequately choose the matrices K_D , K_P , K_F , K_I in respect of the characteristics of the environment. Since the values of environment stiffness are typically high, the weight of the proportional and integral actions shall be contained; the choice of K_F and K_I influences the stability margins and the bandwidth of the system under force control. On the assumption that a stable equilibrium is reached, it is $h_{A,\infty} = h_{Ad}$ and then

$$K_A x_\infty = K_A x_e + h_{Ad}. \quad (28)$$

4.2 Force Control with Inner Velocity Loop

From the block scheme of Fig. 4 it can be observed that, if the position feedback loop is opened, x_F represents a velocity reference and then an integration relationship exists between x_F and x . This leads to recognize that in this case the interaction force with

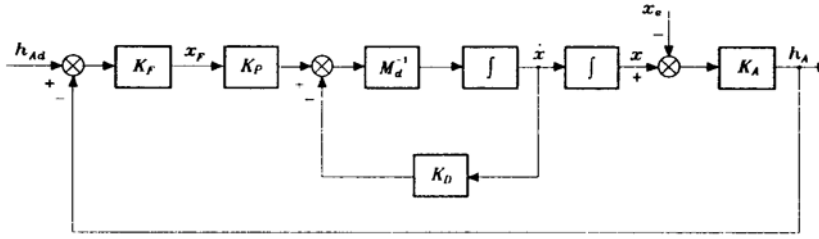


Figure 5: Block scheme of force control with inner velocity loop.

the environment coincides with the desired value at steady state, even with a proportional force controller C_F . In fact, choosing

$$y = J_A^{-1}(q)M_d^{-1}(-K_D\dot{x} + K_P x_F - M_d \dot{J}_A(q, \dot{q})\dot{q}) \quad (29)$$

with a purely proportional control structure ($C_F = K_F$) on the force error yields

$$x_F = K_F(h_{Ad} - h_A), \quad (30)$$

and then system dynamics is described by

$$M_d \ddot{x} + K_D \dot{x} + K_P K_F K_A x = K_P K_F (K_A x_e + h_{Ad}). \quad (31)$$

The relationship between position and contact force at the equilibrium is given by (28). The corresponding block scheme is reported in Fig. 5. It is worth emphasizing that control design is simplified, since the resulting system now is of second order.

4.3 Parallel Force/Position Control

The presented force control schemes require the force reference to be consistent with the geometrical features of the environment. In fact, if h_{Ad} has components on $\mathcal{R}(K_A)$, both Eq. (26) (in case of an integral action in C_F) and Eq. (31) show that, along the corresponding operational space directions, the components of h_{Ad} are interpreted as velocity references which cause a drift of the end-effector position. If h_{Ad} is correctly planned along the directions outside $\mathcal{R}(K_A)$, the resulting motion governed by the position control action tends to take the end-effector position to zero in the case of (26), and the end-effector velocity to zero in the case of (31). Hence, the above control schemes do not allow position control even along the admissible motion directions.

If it is desired to specify a desired end-effector location x_d like in pure position control schemes, the scheme of Fig. 4 can be modified by adding the reference x_d to the input where positions are summed. This corresponds to choosing

$$y = J_A^{-1}(q)M_d^{-1}(-K_D\dot{x} + K_P(\tilde{x} + x_F) - M_d \dot{J}_A(q, \dot{q})\dot{q}) \quad (32)$$

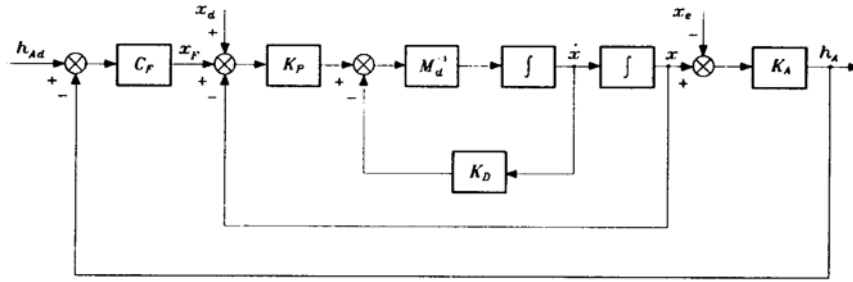


Figure 6: Block scheme of parallel force/position control.

where $\tilde{x} = x_d - x$. The resulting scheme (Fig. 6) is termed *parallel force/position control* in view of the presence of a position control action $K_P \tilde{x}$ in parallel to a force control action $K_P C_F (h_{Ad} - h_A)$. It is easy to verify that in this case the equilibrium position satisfies the equation

$$x_\infty = x_d + C_F (K_A (x_e - x_\infty) + h_{Ad}). \quad (33)$$

Therefore, along those directions outside $\mathcal{R}(K_A)$ where motion is unconstrained, the position reference x_d is reached by x . Vice versa, along those directions in $\mathcal{R}(K_A)$ where motion is constrained, x_d is treated as an additional disturbance; the adoption of an integral action in C_F as for the scheme of Fig. 4 ensures that the force reference h_{Ad} is reached at steady state at the expense of a position error on x depending on environment compliance.

Example 3. Consider again the planar arm in contact with the elastically compliant plane of the above examples; let the initial contact position be the same as that of Example 2. Performance of the various force control schemes is analyzed; as in Example 2, a more compliant ($k_x = 10^3$ N/m) and a less compliant ($k_x = 10^4$ N/m) environment are considered. The position control actions M_d , K_D , K_P are chosen as in Example 2; a force control action is added along the horizontal direction, i.e.

$$C_F = \text{diag}\{c_{Fx}, 0\}.$$

The reference for the contact force is chosen as $h_{Ad} = [10 \ 0]^T$; the position reference—meaningful only for the parallel control—is taken as $p_d = [1.015 \ 0.1]^T$.

With regard to the scheme with inner position loop of Fig. 4, a PI control action c_{Fx} is chosen with parameters:

$$k_{Fx} = 0.00064 \quad k_{Ix} = 0.0016.$$

This confers two complex poles $(-1.96, \pm j5.74)$, a real pole (-1.09) and a real zero (-2.5) to the overall system, for the more compliant environment.

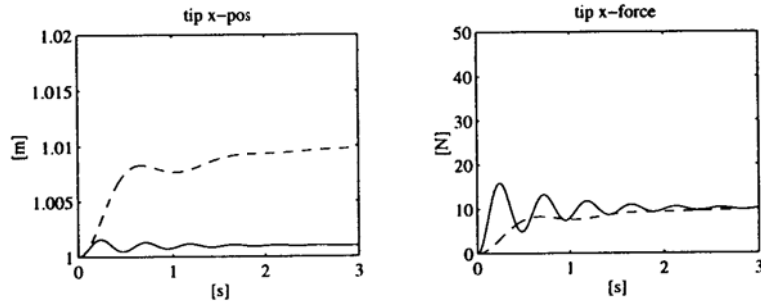


Figure 7: Time history of the tip position and of the contact force along horizontal direction with force control scheme with inner position loop for two environments of different compliance.

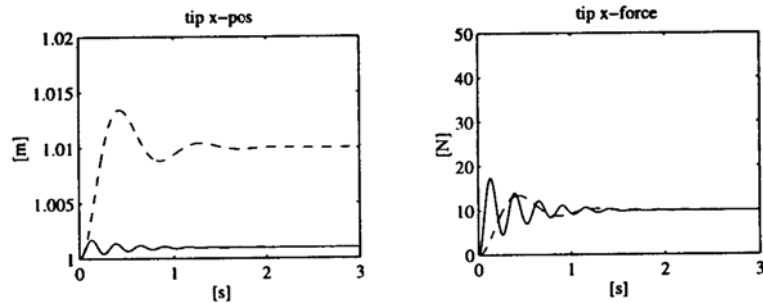


Figure 8: Time history of the tip position and of the contact force along horizontal direction with force control scheme with inner velocity loop for two environments of different compliance.

With regard to the scheme with inner velocity loop of Fig. 5, the proportional control action in c_{Fx} is

$$k_{Fx} = 0.0024$$

so that the overall system, for the more compliant environment, has two complex poles $(-2.5, \pm j7.34)$.

With regard to the parallel control scheme of Fig. 6, the PI control action c_{Fx} is chosen with the same parameters as for the first control scheme.

Figs. 7,8,9 report the time history of the tip position and contact force along axis x for the three considered schemes. A comparison between the various cases shows what follows.

- All control laws guarantee a steady-state value of contact forces equal to the desired one for both the more compliant (*dashed line*) and the less compliant (*continuous line*) environment.
- For given position control actions (M_d, K_D, K_P) , the force control with inner velocity

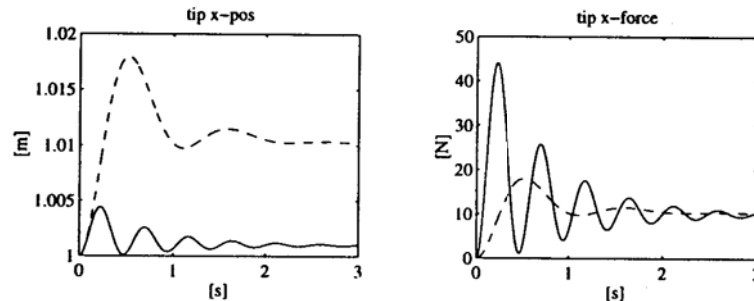


Figure 9: Time history of tip position and of the contact force along horizontal direction with parallel force/position control scheme for two environments of different compliance.

loop presents a faster dynamics than that of the force control with inner position loop.

- The dynamic response with the parallel force/position control shows how the imposition of a position reference along the horizontal direction degrades the transient behavior, even though it does not influence the steady-state contact force. This effect can be justified by observing that a step position input is equivalent to a properly filtered impulse force input.
- The reference position along axis y is obviously reached by the arm tip according to dynamics of position control; the relative time history is not reported.

5 Conclusions

The problem of interaction between manipulator and environment has been tackled by resorting to control schemes which make use of contact force measurements. Force can be controlled either indirectly through an active compliance or impedance, or directly through an outer force feedback loop which generates the reference to the inner motion feedback loop. The performance of the various schemes has been illustrated by means of numerical examples for an arm in contact with an elastic environment. It is understood that the interaction control schemes presented can be integrated in the hybrid control framework which is typically used to control force along constrained motion directions and position along unconstrained motion directions.

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