

A STUDY ON MODEL SENSITIVITY FOR SINGLE-LINK FLEXIBLE ARM CONTROL

Bruno SICILIANO

Dipartimento di Informatica e Sistemistica
Università degli Studi di Napoli Federico II
Via Claudio 21, 80125 Napoli, Italy
E-mail: siciliano@na.infn.it

Abstract. Design of high-performance control systems for a robot manipulator having flexible links heavily relies on an accurate dynamic model of the system. Discretization of an inherently distributed-parameter system into a finite-dimensional model plays a relevant role both for control design and simulation of the system. This work is aimed at studying the sensitivity of inverse dynamics control laws for a single-link flexible arm to the number of modes included into the analysis. Simulation results are presented.

1. INTRODUCTION

The potential for successful utilization of robot manipulators having lightweight flexible links reposes trust in the effectiveness of controllers which are capable to reduce the vibrations naturally induced along the motion of such systems. Currently used flexible manipulators move at very low speeds to limit the excitement of vibration; for instance, the structure of the telemanipulator used on the Space Shuttle has very low resonant frequencies ($0.04 \div 0.35$ Hz) and operates at an average speed of 0.5 deg/s [1].

Mechanical flexibility becomes important for relatively fast speed motions. The design of enhanced controllers must consider the effects of flexibility and then it relies upon the availability of an accurate dynamic model of the system. Due to the distributed nature of flexibility, the approach used to discretize the system as well as the order of approximation plays a relevant role for control design. Nonetheless, also for testing the performance of the controlled system it is important to model the system as accurately as possible in order to simulate a situation as close as to physical reality.

This work reports a study on sensitivity of control laws for a single-link flexible arm to the number of modes included in the model of the system. This is obtained through the usual Lagrangian approach combined with the assumed modes method for modelling distributed flexibility [2,3]. Inverse dynamics control laws [4] are considered with both types of collocated and non-collocated outputs [5]. The numerical results obtained in simulation tests are presented and discussed.

2. DYNAMIC MODEL

A single-link flexible arm can be modelled as an Euler-Bernoulli beam. Under the usual assumptions that only planar bending occurs and deflections are small, the dynamic model can be derived using the Lagrangian approach combined with the assumed modes method [2,3] leading to

$$\begin{pmatrix} J & \mu^T \\ \mu & \rho \ell I \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{\delta} \end{pmatrix} + \begin{pmatrix} 0 & 0^T \\ 0 & D \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 & 0^T \\ 0 & K \end{pmatrix} \begin{pmatrix} \theta \\ \delta \end{pmatrix} + \begin{pmatrix} g_\theta(\theta, \delta) \\ g_\delta(\theta) \end{pmatrix} = \begin{pmatrix} u \\ 0 \end{pmatrix} \quad (1)$$

where θ is the joint angle, $\delta = (\delta_1 \dots \delta_N)$ is the vector of modal deflection variables, J is the total inertia at the joint, μ is a vector describing the inertial coupling between rigid body and flexible body motion and is a function of the mode shapes, ρ is the arm linear mass density, ℓ is the arm length, I is the

identity matrix of proper dimensions, D and K are respectively the link damping and stiffness diagonal matrices, g_θ and g_δ are the gravity torques, and u is the joint driving torque; it has been assumed that the arm is clamped at the joint location and then the joint torque enters directly only in the rigid-body equation [6]. As can be easily seen the only nonlinearity in the model (1) is due to gravity, and in particular the gravity torque in the flexible dynamics equations is only a function of the joint angle [7].

3. INVERSE DYNAMICS CONTROL

The equations of model (1) can be rewritten as

$$J\ddot{\theta} + \mu^T \ddot{\delta} + g_\theta(\theta, \delta) = u \quad (2)$$

$$\mu\ddot{\theta} + \rho\ell\ddot{\delta} + D\dot{\delta} + K\delta + g_\delta(\theta) = 0. \quad (3)$$

The flexible accelerations can be extracted from (3) as

$$\ddot{\delta} = -\frac{1}{\rho\ell} \left(\mu\ddot{\theta} + D\dot{\delta} + K\delta + g_\delta(\theta) \right) \quad (4)$$

and substituted in (2), yielding

$$\left(J - \frac{\mu^T \mu}{\rho\ell} \right) \ddot{\theta} - \frac{1}{\rho\ell} \mu^T (D\dot{\delta} + K\delta) + g_\theta(\theta, \delta) - \frac{\mu^T g_\delta(\theta)}{\rho\ell} = u. \quad (5)$$

An inverse dynamics control law can be designed as

$$u = \left(J - \frac{\mu^T \mu}{\rho\ell} \right) a - \frac{1}{\rho\ell} \mu^T (D\dot{\delta} + K\delta) + g_\theta(\theta, \delta) - \frac{\mu^T g_\delta(\theta)}{\rho\ell} \quad (6)$$

where a denotes a new input acceleration. Under control (6), the equations of the system become

$$\ddot{\theta} = a \quad (7)$$

$$\ddot{\delta} = -\frac{1}{\rho\ell} \left(\mu a + D\dot{\delta} + K\delta + g_\delta(\theta) \right) \quad (8)$$

where (7) is the linear system of a double integrator and (8) describes the internal zero dynamics left into the system. If $\theta_d(t)$ denotes a desired smooth joint trajectory, the well-known resolved acceleration linear control can be adopted

$$a = \ddot{\theta}_d + k_D(\dot{\theta}_d - \dot{\theta}) + k_P(\theta_d - \theta) \quad (9)$$

where k_D, k_P are suitable positive gains that shape the response of the system and guarantee that the joint trajectory is exactly reproduced. Regarding the internal dynamics, a simple Lyapunov argument can be used to show that the so-called zero dynamics ($\dot{\theta} \equiv 0$) is globally asymptotically stable [4] which is a sufficient condition for the overall system to be stable.

Notice that control (6) requires full state feedback. There is no problem to measure the joint position and velocity as well as to reconstruct the flexible variables from strain gauge measurements. In order to overcome the drawback of lack of flexible rates measurements, it is possible to implement the control law by resorting to a feedforward strategy. For the given joint trajectory $\theta_d(t)$, the dynamic equations (4) can be forward integrated over time with suitable initial conditions (usually null) to provide the time history of $\delta_d(t), \dot{\delta}_d(t)$. As a consequence, control (6),(9) can be modified into

$$u = u_d + k'_D(\dot{\theta}_d - \dot{\theta}) + k'_P(\theta_d - \theta) \quad (10)$$

where

$$u_d = \left(J - \frac{\mu^T \mu}{\rho\ell} \right) \ddot{\theta}_d - \frac{1}{\rho\ell} \mu^T (D\dot{\delta}_d + K\delta_d) + g_\theta(\theta_d, \delta_d) - \frac{\mu^T g_\delta(\theta_d)}{\rho\ell} \quad (11)$$

and

$$k'_P = \left(J - \frac{\mu^T \mu}{\rho\ell} \right) k_P \quad k'_D = \left(J - \frac{\mu^T \mu}{\rho\ell} \right) k_D. \quad (12)$$

It can be shown that robustness of the system to imperfect model compensation as in (11) is satisfactory in most practical cases [4].

The above inverse dynamics control law (6) globally linearizes the system with respect to a collocated output taken at the joint level. A more challenging task is to track a non-collocated output taken at the arm level. In detail consider a point at location x along the arm; the angle $\alpha = \theta + \arctg(y(x)/x)$ can be considered as a parametrized output, where $y(x) = \phi^T(x)\delta$ expresses the deflection as a function of the mode shapes at that point and the flexible variables. Proceeding as above, it is not difficult to show that the inverse dynamics control—in the case of zero gravity without loss of generality—

$$u = \left(J - \left(\mu^T - \frac{J}{x} \phi^T(x) \right) \left(\rho \ell I + \frac{1}{x} \mu \phi^T(x) \right)^{-1} \mu \right) a - \left(\mu^T - \frac{J}{x} \phi^T(x) \right) \left(\rho \ell I + \frac{1}{x} \mu \phi^T(x) \right)^{-1} (D\dot{\delta} + K\delta) \quad (13)$$

transforms the system into

$$\ddot{y} = a \quad (14)$$

$$\ddot{\delta} = - \left(\rho \ell I + \frac{1}{x} \mu \phi^T(x) \right)^{-1} (\mu a + D\dot{\delta} + K\delta), \quad (15)$$

where, similarly to (9), a can be chosen as

$$a = \ddot{y}_d + k_D(\dot{y}_d - \dot{y}) + k_P(y_d - y). \quad (16)$$

The stability of the zero dynamics that can be obtained from (15) by setting $\dot{y} \equiv 0$ depends on the sign of the function [5]

$$\Gamma(x) = 1 - \frac{\phi^T(x)\mu}{\rho \ell x}. \quad (17)$$

When $x = 0$ (joint output) it is $\Gamma(0) = 1$ and the system is always stable. At an x where $\Gamma(x)$ becomes negative the system goes unstable (equivalent phenomenon to nonminimum-phase linear systems).

4. MODEL SENSITIVITY

The actual arm considered in this work has the following data: length of 0.5 m, linear mass density of 0.2 kg/m, flexural rigidity of 1 Nm², joint+actuator inertia of 0.1083 kg/m², tip payload mass of 0.1 kg and inertia of 0.0005 kgm²; the arm inertia is negligible with respect to the joint+actuator inertia, so that constrained mode shapes can be used to model deflections [8]. With these data the first four natural frequencies are 2.1784, 15.9145, 40.1008, 92.6093 Hz which have been used in the simulated model of the flexible arm with a sampling time of 1 ms. The elements of the damping matrix D_i have been chosen as $0.1K_i$, $i = 1, 4$.

In the first case study, a joint trajectory of 90 deg has been assigned and three types of control laws have been simulated: • joint PD control + gravity compensation [PD+] with $k_P = 12, k_D = 1$, • full state inverse dynamics control [ID] with $k_P = 100, k_D = 20$, • inverse dynamics control with feedforward of flexible states [FID] and same gains. Notice that the first kind of control does not ensure exact tracking but at least guarantees globally asymptotic regulation of the final joint position [7]. The following table summarizes the results obtained with the above control laws and a variable number of modes in terms of the maximum joint and tip tracking errors.

control	# modes	max error [deg]	max error [mm]
PD+	0	3.0793	5.8811
ID	1	0.090461	5.7483
ID	2	0.089988	5.7452
ID	3	0.089953	5.7440
FID	1	0.092417	5.7484
FID	2	0.092057	5.7452
FID	3	0.092054	5.7440

In the second case study, a trajectory of 90 deg has been assigned to the angular output associated to an arm point. The full state inverse dynamics control has been simulated with the same gains as above. The following two tables summarize the results obtained with a variable distance from the joint location and a variable number of modes: in the left table the arm point x^* at which Γ changes sign is reported as a function of the number of modes, whereas in the right table the maximum joint and tip tracking errors are reported for stable outputs.

# modes	x^* [m]
1	0.480
2	0.352
3	0.212
4	0.148

x [m]	# modes	max error [deg]	max error [mm]
0.20	1	0.0922	0.6900
0.20	2	0.0910	0.6900
0.20	3	0.0902	0.6896
0.34	2	0.0900	0.6758
0.47	1	0.0901	0.6696

5. DISCUSSION

The foregoing numerical results indicate the following facts:

- As expected, the inverse dynamics control has better tracking performance than the PD control + gravity compensation.
- For the joint output tracking case, there is no appreciable reduction in the error tracking (both at joint and at tip level) as the number of modes used in the controller is increased.
- The inverse dynamics control with feedforward of flexible states behaves as well as the full state inverse dynamics control.
- The range of arm points for which stable tracking can be obtained decreases as the number of modes increases; if only one mode is used it is possible to control nearly all the points along the arm.
- Compared to the case of joint control, the angular tracking error is about the same but remarkably the tip tracking error is reduced by an order of magnitude.

ACKNOWLEDGEMENT

This work was supported by *Consiglio Nazionale delle Ricerche* under contract n. 93.00905.PF67.

REFERENCES

- [1] Nguyen, P.K., Ravindran, R., Carr, R., Gossain, D.M., Doetsch, K.H., "Structural flexibility of the Shuttle remote manipulator system mechanical arm," AIAA paper no. 82-1586, New York, NY, Aug. 1982.
- [2] Book, W.J., "Recursive Lagrangian dynamics of flexible manipulator arms," *International Journal of Robotics Research*, vol. 3, no. 3, 1984.
- [3] De Luca, A., Siciliano, B., "Closed-form dynamic model of planar multilink lightweight robots," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 21, no. 4, 1991.
- [4] De Luca, A., Siciliano, B., "Inversion-based nonlinear control of robot arms with flexible links," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 16, no. 6, 1993.
- [5] De Luca, A., Siciliano, B., "Trajectory control of a non-linear one-link flexible arm," *International Journal of Control*, vol. 50, no. 5, 1989.
- [6] Cetinkunt, S., Yu, W.L., "Closed-loop behavior of a feedback-controlled flexible arm: A comparative study," *International Journal of Robotics Research*, vol. 10, no. 3, 1991.
- [7] De Luca, A., Siciliano, B., "Regulation of flexible arms under gravity," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 3, 1993.
- [8] Barbieri, E., Özgüner, Ü., "Unconstrained and constrained mode expansions for a flexible slewing link," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 110, no. 4, 1988.