

Robot Control in Singular Configurations — Analysis and Experimental Results

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Abstract

This paper describes analytical and experimental work for controlling robotic manipulators in the neighbourhood of kinematically singular configurations. The proposed method is based on a damped least-squares solution with user-defined accuracy. Results are given for a five-joint industrial robot.

Introduction

It has been long demonstrated that singular configurations constitute a serious problem in sensory control of robotic manipulators. Close to kinematic singularities the usual inverse differential kinematics solutions based on Jacobian (pseudo-)inverse become ill-conditioned, and this is experienced in the form of very high joint velocities and large control deviations.

When a preprogrammed reference end-effector trajectory is to be tracked, it is possible to interpolate in joint coordinates close to singular configurations, or to plan motions so that singularities are avoided. Instead, this is not possible in sensory control where the reference trajectory is not known a priori. The same problem is encountered in joy-stick control of a robot if the operator attempts to lead the robot through a singularity using end-effector motion increments.

Several research efforts have been devoted to devise control schemes that allow the robot to handle the occurrence of singularities. Some schemes identify the degenerate

directions in the task space associated with a given singularity and eliminate the task velocity components along those directions in a suitable neighbourhood of the singularity [1–4]. Other schemes modify the exact inverse differential kinematic mapping by resorting to approximate mappings that offer robustness to singularities at the expense of reduced tracking accuracy [5–7]. A recent overview of control techniques of robotic systems through singularities can be found in [8].

In this paper we investigate the performance of a scheme for control in singular configurations based on the damped least-squares solution [5,6]. Implementation issues are discussed regarding the tuning of the damping factor for high accuracy with feasible velocities [9,10]. In particular, the use of weighted damped least-squares to achieve user-defined accuracy along given end-effector space directions is presented [11].

The scheme is applied in experiments to the five-joint ABB Trallfa TR400 robot. The results demonstrate the good performance of the system in crossing a singularity, and the influence of the weighting parameters is tested out extensively.

Damped least-squares solution

A robotic manipulator is naturally actuated in the joint space, whereas its end-effector motions are planned in the operational space. Therefore, in order to design effective control schemes in the joint or in the operational space, it is necessary to take into account the differential kinematic mapping between a motion increment $\delta \mathbf{q} \in \mathbb{R}^n$ in the joint space and the corresponding motion increment $\delta \mathbf{x} \in \mathbb{R}^m$ in the operational space, i.e.

$$\delta \mathbf{x} = \mathbf{J}(\mathbf{q})\delta \mathbf{q}, \quad (1)$$

where $\mathbf{J}(\mathbf{q})$ is the $(m \times n)$ Jacobian matrix of the manipulator considered. In practical cases, it is $m \leq n$; when $m < n$ the manipulator is said to be redundant and there exists an $(n - m)$ -dimensional subspace of $\mathbb{R}^n$ in which differential joint-space motions give null differential operational space motions.

A configuration $\hat{\mathbf{q}}$ is said to be *singular* if $\text{rank}(\mathbf{J}(\hat{\mathbf{q}})) = r < m$. At a singular configuration the subspace of the differential joint motion space which maps into the null differential motion vector in the operational space increases its dimension, as $\dim(\mathcal{N}(\mathbf{J}(\hat{\mathbf{q}}))) = n - r > n - m$. On the other hand, since $\dim(\mathcal{R}(\mathbf{J}(\hat{\mathbf{q}}))) = r < m$, only an r -dimensional subspace of differential motion in the operational space can be spanned at a singularity; this subspace is the space of feasible motion for the manipulator. In general, at a singular configuration a differential motion vector in the operational space may have both *feasible components*, lying in $\mathcal{R}(\mathbf{J}(\hat{\mathbf{q}}))$, and *degenerate components*, belonging to $\mathcal{R}^\perp(\mathbf{J}(\hat{\mathbf{q}}))$.

The linear mapping (1) can be effectively analyzed in terms of the singular value decomposition of the Jacobian matrix, that is

$$\mathbf{J} = \mathbf{U}\Sigma\mathbf{V}^T = \sum_{i=1}^m \sigma_i \mathbf{u}_i \mathbf{v}_i^T \quad (2)$$

where $\mathbf{U}$ is the $(m \times m)$ matrix of the output singular vectors $\mathbf{u}_i$, $\mathbf{V}$ is the $(n \times n)$ matrix of the input singular vectors $\mathbf{v}_i$, and $\Sigma = (\mathbf{S} \quad \mathbf{O})$ is the $m \times n$ matrix whose $(m \times m)$

diagonal submatrix $\mathbf{S}$ contains the singular values σ_i of the matrix $\mathbf{J}$. If r denotes the rank of $\mathbf{J}$, the following hold:

- a) $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r > \sigma_{r+1} = \dots = \sigma_m = 0$
- b) $\mathcal{R}(\mathbf{J}) = \text{span}\{\mathbf{u}_1, \dots, \mathbf{u}_r\}$
- c) $\mathcal{N}(\mathbf{J}) = \text{span}\{\mathbf{v}_{r+1}, \dots, \mathbf{v}_n\}$.

Notice that the $m - r$ output singular vectors associated to the null singular values represent the degenerate directions in the given configuration. The singular value decomposition is continuous and well-behaved not only in singular values but also in the direction of the singular vectors; thus, the $\mathbf{u}_i$ and $\mathbf{v}_i$ vectors will not change appreciably in the neighbourhood of a singularity.

Upon these premises, it can be recognized that the control system of a robotic manipulator should be provided with the capability of handling singularities. Usually, control operates in the joint space generating the driving torques at the joint actuators. The reference trajectory for the joint control servos is to be generated via kinematic inversion of the given end-effector trajectory, and then the occurrence of a singularity may prevent motion along degenerate components. On the other hand, even in the case the robot control system is designed directly in the operational space, still the problem of motion through and close to singular configurations exists since any controller of that kind is based on the use of the manipulator Jacobian.

An effective strategy that permits control of robotic manipulators in the neighbourhood of kinematic singularities is the *damped least-squares* technique originally proposed in [5,6]. The method corresponds to solving the equation

$$\mathbf{J}^T(\mathbf{q})\delta\mathbf{x} = (\mathbf{J}^T(\mathbf{q})\mathbf{J}(\mathbf{q}) + \lambda^2\mathbf{I})\delta\mathbf{q} \quad (3)$$

in lieu of eq. (1); in (3) $\lambda \in \mathbb{R}$ is the damping factor and $\mathbf{I}$ is the identity matrix of proper dimension. It can be easily shown that the solution to (3) can be cast in the form

$$\delta\mathbf{q} = \mathbf{J}^T(\mathbf{q}) (\mathbf{J}(\mathbf{q})\mathbf{J}^T(\mathbf{q}) + \lambda^2\mathbf{I})^{-1} \delta\mathbf{x} \quad (4)$$

which remarkably requires the inverse of an $(m \times m)$ matrix instead of an $(n \times n)$ matrix. Notice that when λ is zero eq. (1) becomes identical to (3) and the damped least-squares solution reduces to the well-known Moore-Penrose pseudo-inverse solution.

It is important to point out that solutions of (3) satisfy the condition

$$\min_{\delta\mathbf{q}} \|\delta\mathbf{x} - \mathbf{J}(\mathbf{q})\delta\mathbf{q}\|^2 + \lambda^2 \|\delta\mathbf{q}\|^2 \quad (5)$$

which evidences the possibility of trading off accuracy against feasibility of the joint space motion increment required to match up with the given operational space motion. Therefore, it is essential to select suitable values for the damping factor: Small values of λ give accurate solution but low robustness to the occurrence of singular and near-singular configurations. High values of λ result in low tracking accuracy even where a feasible and accurate solution would be possible.

To gain more insight into the features of solution (4), the above singular value decomposition (2) is helpful as it allows to rewrite the damped least-squares solution (4) as

$$\delta \mathbf{q} = \sum_{i=1}^m \frac{\sigma_i}{\sigma_i^2 + \lambda^2} \mathbf{v}_i \mathbf{u}_i^T \delta \mathbf{x}. \quad (6)$$

It is clear that the components for which $\sigma_i \gg \lambda$ are little influenced by the damping factor, being

$$\frac{\sigma_i}{\sigma_i^2 + \lambda^2} \approx \frac{1}{\sigma_i} \quad (7)$$

On the other hand, when a singularity is approached the smallest singular value tends to zero while the associated component of the solution is driven to zero by the factor σ_i/λ^2 ; this progressively reduces the joint motion required to achieve near-degenerate components of the commanded $\delta \mathbf{x}$.

The value selected for the damping factor represents the index which allows to decide about the closeness of the current configuration to a singularity; moreover, λ determines the degree of approximation introduced with respect to the pure least-squares (pseudo-inverse) solution.

An optimal choice for λ requires consideration of the smallest non-null singular value experienced along the whole trajectory and of the minimum damping needed to ensure feasible joint motion. To achieve good performance in the entire manipulator's workspace the use of a configuration-varying damping factor was proposed in [5]. The natural choice is to adjust λ as a function of some measure of closeness to the singularity at the current configuration of the robot arm; to this purpose manipulability measures [5] or estimates of the smallest singular value [9] can be adopted. A varying damping factor is in general computationally expensive; simplified computation exploiting knowledge of the kinematical structure has been recently proposed in [12] and in [13,10] for the manipulability and for the smallest singular value, respectively.

User-defined accuracy

The above damped least-squares method achieves a compromise between accuracy and robustness of the solution. This is performed without specific regard to the components of the particular task assigned to the manipulator's end effector. Consequently, it would be nice to think of a strategy that allows to discriminate between directions in the operational space where higher accuracy is desired and directions where lower accuracy can be tolerated. This is the case, for instance, of spot welding or spray painting in which the tool angle around the approach direction is not essential to the fulfilment of the task. Below is illustrated a *user-defined accuracy* technique [11] that meets the above requirements.

Let a weighted task increment be defined as

$$\delta \tilde{\mathbf{x}} = \mathbf{W} \delta \mathbf{x}, \quad (8)$$

where $\mathbf{W}$ is the $(m \times m)$ task-dependent weighting matrix that takes into account the anisotropy of the task requirements. The differential mapping relating the motion increment in joint coordinates to the weighted task increment is then given by

$$\delta \tilde{\mathbf{x}} = \tilde{\mathbf{J}}(\mathbf{q})\delta \mathbf{q} \quad (9)$$

where $\tilde{\mathbf{J}} = \mathbf{W}\mathbf{J}$. It is worth noticing that if $\mathbf{W}$ is full-rank, solving (1) is equivalent to solving (9), but with different conditioning in the matrix to be inverted. This suggests to select only the strictly necessary weighting action in order to avoid undesired ill-conditioning of $\tilde{\mathbf{J}}$.

A solution to the inverse kinematics problem for eq. (9) is obtained using the weighted damped least-squares technique [5]. This corresponds to solving the equation

$$\tilde{\mathbf{J}}^T(\mathbf{q})\delta \tilde{\mathbf{x}} = (\tilde{\mathbf{J}}^T(\mathbf{q})\tilde{\mathbf{J}}(\mathbf{q}) + \lambda^2 \mathbf{I})\delta \mathbf{q} \quad (10)$$

which in turn implies the fulfilment of the condition

$$\min_{\delta \mathbf{q}} \|\delta \tilde{\mathbf{x}} - \tilde{\mathbf{J}}(\mathbf{q})\delta \mathbf{q}\|^2 + \lambda^2 \|\delta \mathbf{q}\|^2. \quad (11)$$

Again, the singular value decomposition of the matrix $\tilde{\mathbf{J}}$ comes in support, i.e.

$$\tilde{\mathbf{J}} = \sum_{i=1}^m \tilde{\sigma}_i \tilde{\mathbf{u}}_i \tilde{\mathbf{v}}_i^T, \quad (12)$$

and the solution to (10) can be written as

$$\delta \mathbf{q} = \sum_{i=1}^m \frac{\tilde{\sigma}_i}{\tilde{\sigma}_i^2 + \lambda^2} \tilde{\mathbf{v}}_i \tilde{\mathbf{u}}_i^T \delta \tilde{\mathbf{x}}. \quad (13)$$

It is clear that the singular values $\tilde{\sigma}_i$ and the singular vectors $\tilde{\mathbf{u}}_i$ and $\tilde{\mathbf{v}}_i$ depend on the choice of the weighting matrix $\mathbf{W}$. While this has no effect on the solution $\delta \mathbf{q}$ as long as $\tilde{\sigma}_m \gg \lambda$, close to singularities where $\tilde{\sigma}_r \ll \lambda$ for some $r < m$ the solution can be shaped by properly selecting the matrix $\mathbf{W}$. To this purpose, two major issues must be considered:

- a) The matrix $\mathbf{W}$ modifies the singular values σ_i into $\tilde{\sigma}_i$, and then the relative weight of each singular value is changed with respect to the selected λ ; this differently modulates the damping of the near-degenerate components in comparison to the basic solution.
- b) The matrix $\mathbf{W}$ modifies the singular vectors; in particular, this can be exploited to obtain that the output singular vectors $\tilde{\mathbf{u}}_i$ associated to degenerate directions be aligned to the task-space components which can tolerate loss of tracking accuracy.

Experimental results

The weighted damped least-squares method was applied in experiments to an ABB Trallfa TR400 industrial robot with hydraulic actuators. The computer system had a VME interface to the robot which was developed in cooperation with ABB Trallfa. The

interface runs at 100 Hz. The inverse kinematic solutions were executed on a Motorola 68020/68881 VME board. On the same processor, independent joint control servos were implemented using proportional feedback which is the conventional type of controller for hydraulic drives; a bandwidth of 18 rad/sec was assigned.

As for the kinematic description, the manipulator has five joints with a vertical first axis, horizontal and parallel second, third and fourth axes, and a fifth axis which is orthogonal to the fourth axis; Figure 1 shows the manipulator with its Denavit-Hartenberg parameters.

The workspace of this manipulator is limited by the linear actuators of the rotary joints 2 and 3, so that the elbow and shoulder singularities are outside the workspace. Then, the only internal singularity of the manipulator is associated with the wrist.

The actual configuration of the wrist singularity will depend on the definition of the end-effector coordinates. It is clear that at most five end-effector coordinates can be specified. The orientation around the tool axis was left undefined; thus, the end-effector motion is described by the velocity vector

$$\dot{\mathbf{x}} = (\dot{x}_0 \quad \dot{y}_0 \quad \dot{z}_0 \quad \omega_y \quad \omega_z)^T$$

where $\dot{x}_0$, $\dot{y}_0$, $\dot{z}_0$ are the end-effector velocity components in the base frame 0, and ω_y , ω_z are the angular velocity components along the tool axes y and z . The end-effector motion increment to be used in connection with (1) is $\delta\mathbf{x} = \dot{\mathbf{x}}\delta t$, where δt is a time increment.

With this end-effector coordinate vector, the singularity occurs for $\cos q_5 = 0$. In the neighbourhood of the singularity, it can be shown that the output singular vector corresponding to the smallest singular value is [13]

$$\mathbf{u}_5 = (0 \quad \alpha \quad 0 \quad 1 - \alpha^2 \quad 0)^T, \quad (14)$$

where $\alpha \ll 1$ depends both on the weighting of the matrix $\mathbf{W}$ in (8) and on the actual configuration of the arm. From the above expression, it follows that the effect of damped least-squares is to eliminate a linear combination of the commanded motion in end-effector coordinates 2 and 4, which correspond to a translation in the y_0 direction in the base frame and a rotation around the same axis in the tool frame.

As emphasized in the previous section, the adoption of the weighting matrix $\mathbf{W}$ modifies the singular values and the singular vectors of $\mathbf{J}$ into the ones characterizing $\tilde{\mathbf{J}}$. According to the expression in (14), different weights were assigned to the second and fourth components of the task space end-effector vector. The choice

$$\mathbf{W}_1 = \text{diag}(1, w, 1, 1, 1) \quad (15)$$

provides low task space sensitivity for a translation along y_0 in the base frame, while the choice

$$\mathbf{W}_2 = \text{diag}(1, 1, 1, w, 1) \quad (16)$$

provides low task space sensitivity for a rotation along y in the tool frame; obviously, a scalar $0 < w < 1$ is to be chosen.

The other crucial point for a damped least-squares solution is the selection of the damping factor. To this purpose, a singular region can be defined on the basis of the

estimate of the smallest singular value of $\mathbf{J}$; outside the region the exact solution is used, while inside the region configuration-varying damping and weighting factors are introduced to obtain the desired approximate solution. Both factors must be chosen so that continuity of the joint space motion increment $\delta\mathbf{q}$ is ensured in the transition between the solutions. Following the guidelines of [11], the damping and weighting factors take on the following expressions:

$$\lambda^2 = \begin{cases} 0 & \text{when } \hat{\sigma}_5 \geq \epsilon \\ \left(1 - \left(\frac{\hat{\sigma}_5}{\epsilon}\right)^2\right) \lambda_{max}^2 & \text{otherwise,} \end{cases}$$

$$(w-1)^2 = \begin{cases} 0 & \text{when } \hat{\sigma}_5 \geq \epsilon \\ \left(1 - \left(\frac{\hat{\sigma}_5}{\epsilon}\right)^2\right) (w_{min}-1)^2 & \text{otherwise,} \end{cases}$$

where $\hat{\sigma}_5$ is the estimate of the smallest singular value, which can be computed as

$$\hat{\sigma}_5 = \left| |q_5| - \frac{\pi}{2} \right|,$$

and ϵ defines the size of the singular region. The numerical values used in the experiments were $\epsilon = 0.2$, corresponding to a singular region of about 20 degrees wide in q_5 , and $\lambda_{max} = 0.15$.

The initial configuration of the arm was

$$\mathbf{q}(0) = (0 \quad -0.1 \quad 0.1 \quad -0.1 \quad \pi/2 - 0.4)^T,$$

and the commanded step velocity was

$$\dot{\mathbf{x}} = (0 \quad 0 \quad 0 \quad 0.07 \quad 0.17)^T$$

It can be easily recognized that the resulting motion trajectory passes through the singularity $q_5 = \pi/2$ with significant degenerate components.

Experimental results were obtained adopting four different solutions; namely,

1. pure resolved-rate inverse Jacobian,
2. usual damped least-squares without weighting ($\mathbf{W} = \mathbf{I}$),
3. weighted damped least-squares with $\mathbf{W}_1$ as in (15) and $w_{min} = 0.1$,
4. weighted damped least-squares with $\mathbf{W}_2$ as in (16) and $w_{min} = 0.01$.

This was intended to analyze the performance of the proposed method in comparison to the standard methods.

With the first solution, the motion of the arm in the neighbourhood of the singularity becomes unacceptable; large joint velocities are generated which reach the mechanical limits and the arm breaks down.

With the second solution, a significant deviation in y_0 was observed due to the damping of the motion in the $\mathbf{u}_5$ task space direction. The nonzero $\alpha = 0.064$ in (14) gives a kinematic coupling between the second and the fourth component of the task velocity.

With the third solution, a very large deviation from the given trajectory along y_0 was generated (Fig. 2) due to the fact that the weighting is applied along that same

direction (resulting in $\alpha = 0.54$); a maximum error of 0.54 m occurred after 5 sec of motion.

With the fourth solution, the performance along y_0 was much improved at the expense of that along ω_y ($\alpha = 0.0006$). The arm tracked satisfactorily the translational part of the trajectory with a maximum error of 0.03 m (Fig. 2).

Notice that both the resulting joint trajectories in the last two cases (Fig. 3) pass through the singular configuration without any discontinuity. Obviously, the joint motion is different in the two cases.

The motion of the manipulator in the experiments is illustrated in a section of the video proceedings of this conference.

Conclusions

Analytical and experimental results have been presented for a damped least-squares solution with user-defined accuracy. Satisfactory performance was achieved for controlling the motion of a five joint industrial robot manipulator through the wrist singularity. It was shown that suitable weighting of task space directions can be exploited to achieve accurate tracking along specified axes of motion. Smoothness and continuity of the solution was ensured by defining a proper singular region in the neighbourhood of the singularity.

Future developments of this joint research project will be devoted to applying the proposed method to the case of the six joint ABB IRB 2000 robot with both shoulder and wrist singularities.

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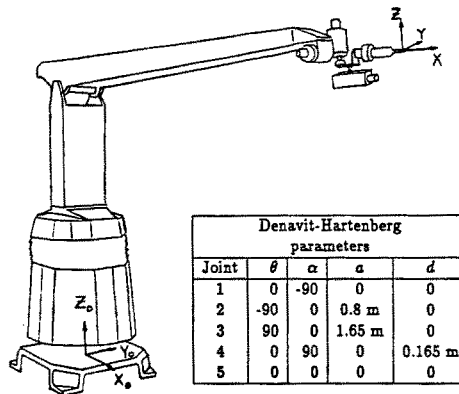


Fig. 1. The ABB Trallfa TR 400 industrial robot manipulator.

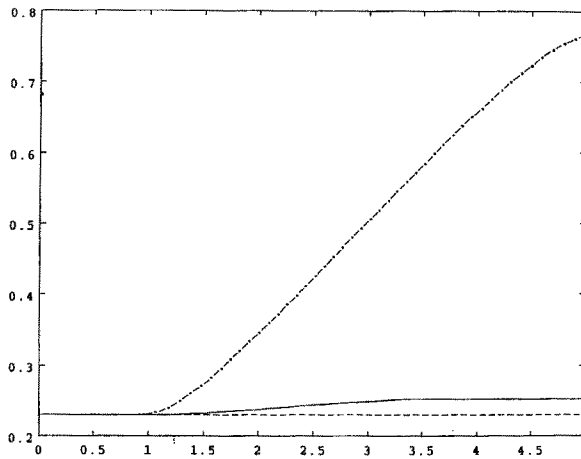


Fig. 2. Motion in y_0 due to commanded motion in the ω_y for the weighted damped least-squares solution. The y_0 reference is constant (dashed curve). The motion in y_0 and ω_y becomes linearly dependent in the singularity, and an error in y_0 is observed. The error in y_0 depends strongly on the weighting. With a weighting of 0.01 on ω_y the maximum error is 0.03 m (solid curve), while with a weighting of 0.1 on y_0 the maximum error is 0.54 m (curve with dots and dashes).

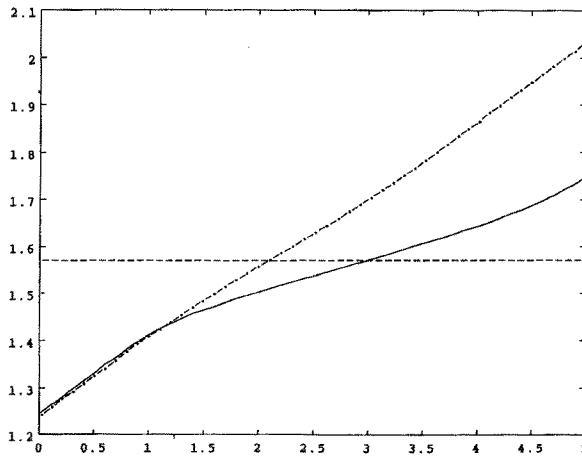


Fig. 3. Motion in q_5 for the weighted damped least-squares solution. The singularity occurs for $q_5 = \pi/2$.