

Closed-Loop Inverse Kinematics Algorithms for Redundant Spacecraft/Manipulator Systems

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Abstract

This paper is aimed at solving inverse kinematics of space manipulators mounted on a free-floating spacecraft. The approach followed is based on the generalized Jacobian notion that allows to describe the reaction effects of manipulator motion on the spacecraft. Redundancy of the system with respect to the number of task variables for spacecraft attitude and manipulator end-effector pose is considered. Depending on the nature of the task for the spacecraft/manipulator system, a number of closed-loop inverse kinematics algorithms are proposed which are conceptually derived from previously developed algorithms with task space augmentation and task priority strategy for ground-fixed manipulators. In particular, if it is desired to keep spacecraft attitude constant during end-effector task execution, the so-called fixed-attitude-restricted Jacobian is employed. The nice feature of the proposed algorithms is that the transpose of the projected constraint Jacobian is utilized resulting in computationally lighter algorithms than those based on Jacobian pseudoinversion. Case studies are developed for a planar system of a spacecraft with a four-link arm attached.

1. Introduction

The last decade has witnessed an increasing interest towards robotic applications in space. Space robotics is expected to allow for experiment-handling, material processing, assembly and servicing with a limited amount of highly expensive manned missions. Entrusting extravehicular activities to a mechanical robot manipulator rather than to an astronaut is foreseen to reduce the danger of space servicing jobs [1].

Coordination of motion between the spacecraft and the manipulator becomes crucial for successful task execution. In fact, due to the lack of a stationary base, the free-floating spacecraft will react to the motions of the attached manipulator. If the inertia of the manipulator with respect to that of the spacecraft is non-negligible, the application of conventional modeling and control techniques for ground-fixed manipulators becomes unfeasible and different strategies have to be pursued.

Several approaches to solving the above problem were proposed in the literature. A method for controlling spacecraft attitude by means of reaction wheels was suggested in [2]; this apparatus is operated to compensate for the reaction forces and moments on the spacecraft and thus keep its attitude constant during manipulator motion. The use of wheels, however, entails fuel consumption that may have a significant effect on mission costs. The so-called virtual manipulator concept introduced in [3] allows the end-effector of such manipulator to track a desired trajectory, while spacecraft attitude changes are compensated through suitable cyclic motions of the manipulator. An interesting approach was suggested in [4] as that to regard the spacecraft/manipulator system as a macro/micro manipulator; the end-effector motion is decoupled from the spacecraft motion via a fuel-efficient controller. On the other hand, no attitude control was performed in [5], where jet thrusters are used to keep the workspace within reach of the manipulator.

An effective tool based on a general description of the reaction effects of manipulator motion on the free-floating spacecraft is the generalized Jacobian notion established in [6]; the ordinary manipulator Jacobian is modified by the addition of a term accounting for the spacecraft Jacobian and the relative weight between the spacecraft and manipulator inertia. This discloses the problem of coordinated motion between manipulator and spacecraft to a potential of inverse kinematics solutions that resemble those developed for ground-fixed manipulator. Among them, resolved motion rate control can be carried out as well as exploitation of redundant degrees of freedom with respect to tasks defined for either the spacecraft only or the overall spacecraft/manipulator system [7].

The contribution of this paper is to present a number of solution algorithms that are logically derived from closed-loop inverse kinematics schemes for constrained redundant manipulators in the framework of task space augmentation with task priority [8]. Compared to the usual inverse kinematics techniques for redundant systems based on matrix pseudoinversion, the proposed algorithms are computationally less demanding and more robust to occurrence of algorithmic singularities induced by conflicting task situations. As usual, the lower-priority task Jacobian

is projected onto the null space of the higher-priority task but only the transpose thereof is taken, thanks to the introduction of a suitable feedback correction term. In this context, the nature of the required task plays a significant role; in particular, if it is desired to keep spacecraft attitude constant during end-effector task execution without any additional constraint task, the matrix to be transposed turns out to be the so-called fixed-attitude-restricted Jacobian [9].

A number of case studies are developed by simulating the motion of a planar system composed by a free-floating spacecraft with a four-link arm attached.

2. Modeling of Spacecraft/Manipulator System

Object of the investigation is a system composed by an n -degree-of-freedom manipulator with rigid links mounted on a rigid body spacecraft free-floating in a zero-gravity environment. Let q denote the $(n \times 1)$ vector of joint variables and ϕ the $(m_1 \times 1)$ vector of variables expressing spacecraft attitude—at most, it is $m_1 = 3$ with three orientation angles, e.g. roll, pitch and yaw; spacecraft position is of no concern. Let also the manipulator end-effector pose be described by an $(m_2 \times 1)$ vector x_E of task variables—at most, it is $m_2 = 6$ with three position coordinates and three orientation angles.

With reference to an inertial frame, the kinematic equation relating the joint and spacecraft variables to the end-effector pose can be written as

$$x_E = k_E(\phi, q), \quad (1)$$

where k_E denotes a direct kinematics function formally analogous to the case of a ground-fixed manipulator.

In view of solving inverse kinematics for such a system, it is convenient to consider differential kinematics in lieu of Eq. (1). To the purpose, if J_S denotes the $(m_2 \times m_1)$ spacecraft Jacobian relating the spacecraft angular velocity ω_S to the end-effector velocity v_E and J_M denotes the usual $(m_2 \times n)$ manipulator geometric Jacobian relating the joint velocities $\dot{q}$ to the end-effector velocity v_E , it is possible to write

$$v_E = J_S \omega_S + J_M \dot{q}. \quad (2)$$

It has been assumed that the initial velocity of the free-floating system is null. Notice also that in general it is not rigorous to assert that ω_S is the time derivative of ϕ , since the integral of ω_S has no physical meaning; it can be shown that a transformation matrix exists which relates $\dot{\phi}$ to ω_S . In the same fashion, v_E coincides with the time derivative of x_E only for what regards the position components, not for the orientation components.

It is assumed that no external forces or torques act on the center of mass of the system. Thus, there are no devices intended to change spacecraft attitude, e.g. reaction wheels.

In this case, momentum conservation dictates that

$$\sum_{i=0}^n M_i \dot{r}_i = 0 \quad (3)$$

for the translational momentum and

$$\sum_{i=0}^n (I_i \omega_i + M_i r_i \times \dot{r}_i) = 0 \quad (4)$$

for the rotational momentum, where M_i is the mass of body i , r_i is the position vector of the center of mass of body i , ω_i is its angular velocity, and I_i is the inertia matrix of body i around its center of mass. It has been assumed that the initial momentum of the free-floating system is null, without loss of generality.

The quantities r_i , $\dot{r}_i$, ω_i and I_i are referred to the inertial frame, and it is not difficult to compute their expressions as a function of ϕ_S and q [10]. Substituting such expressions into Eqs. (3,4), momentum conservation can be compactly expressed as

$$I_S \omega_S + I_M \dot{q} = 0, \quad (5)$$

where I_S is an $(m_1 \times m_1)$ matrix related to the spacecraft inertia and I_M is an $(m_1 \times n)$ matrix related to the manipulator inertia.

Equations (2,5) are the fundamental equations for analyzing the motion of the system composed by a robotic manipulator mounted on a free-floating spacecraft. Since $I_S \omega_S$ represents the spacecraft rotational momentum, I_S is an invertible matrix; then, solving (5) for ω_S and substituting in (2) allows to eliminate the dependence on the spacecraft attitude changes, i.e.

$$v_E = J_G \dot{q} \quad (6)$$

where the matrix

$$J_G = J_M - J_S I_S^{-1} I_M \quad (7)$$

is termed the *generalized Jacobian* for the spacecraft/manipulator system [6].

The attractive feature of eq. (6) is its formal analogy with the well-known differential kinematics equation for ground-fixed manipulators. The manipulator Jacobian J_M is modified by the presence of a term accounting for the relative inertial weight between the spacecraft and the manipulator. The larger the spacecraft inertia, the smaller the reaction caused by the manipulator motion; in the limit of a very massive spacecraft, the generalized Jacobian will tend to the manipulator Jacobian.

3. Closed-Loop Inverse Kinematics Algorithms

If it is desired to control the free-floating system by acting only on the manipulator joint actuators, an inverse kinematics problem has to be solved first which can be formulated as follows. For a given end-effector motion trajectory, find the corresponding joint trajectories compatible

with a desired spacecraft attitude. In this framework, it is important to recognize the presence of redundant degrees of freedom of the system with respect to the required task. As pointed out in [7], three cases of redundancy can be distinguished in connection with the number of joint variables n versus the number of system task space variables $m_1 + m_2$. If $n < m_1 + m_2$, the manipulator can be redundant with respect either to the spacecraft task ($n > m_1$) or to the end-effector task ($n > m_2$), but redundancy will not allow effective coordination of spacecraft and end-effector motion. If $n = m_1 + m_2$, the available redundancy can be exploited to coordinate the motion of the spacecraft with that of the end-effector. If $n > m_1 + m_2$, it is possible to introduce additional constraints to be satisfied along with spacecraft/manipulator motion coordination. In the following it is assumed that $n \geq m_1 + m_2$.

By applying the well-known Jacobian transpose algorithm [11] to the robotic system described by eq. (6), the joint velocity solution can be computed as

$$\dot{q} = J_G^T K_E e_E, \quad (8)$$

where e_E denotes the error between the desired x_{Ed} and the actual x_E end-effector trajectory computed via (1), and K_E is a positive definite (diagonal) matrix. The typical name of *closed-loop inverse kinematics* algorithms given to this class of algorithms is due to the presence of the feedback correction term $K_E e_E$, differently from the usual resolved motion rate algorithms. The tracking error is upper-bounded—the larger the elements of K the smaller the error norm—and asymptotic convergence at steady-state is ensured by the update law (8). The essential advantage of this algorithm resides in the avoidance of pseudoinversion of J_G , which is more time-consuming and may be computationally ill-posed.

With the above solution, however, the resulting spacecraft attitude varies as the manipulator end-effector moves along the trajectory. It is then advisable to exploit the redundant degrees of freedom $n - m_2 \geq m_1$ to impose a desired time evolution of spacecraft attitude ϕ_{sd} . This is a typical case of task space augmentation for redundant manipulators [12]. Since a conflict may arise between the end-effector and the constraint (spacecraft) task, an order of priority should be assigned [13]. By revisiting the solution algorithm proposed in [8], the joint velocity solution can be computed as

$$\dot{q} = J_G^\dagger (v_{Ed} + K_E e_E) + (I - J_G^\dagger J_G) J_C^T K_C e_C \quad (9)$$

where the matrix $(I - J_G^\dagger J_G)$ projects the additional joint velocity contribution on the null space of the generalized Jacobian so as to avoid interference of the constraint task with the end-effector task, which has then been given higher priority. Further, e_C is the error for the constraint task and J_C is the associated Jacobian, while K_C is a positive definite (diagonal) matrix playing the same role as

K_E . Algorithm convergence properties are based on Lyapunov arguments and are fully discussed in [8]. Here it is worth noticing that the first term of solution (9) is based on the pseudoinverse and not on the transpose of J_G , since $J_G^\dagger$ is to be computed anyhow for the second term; to the purpose, it is assumed that J_G is non-singular. Note also that a feedforward velocity term is added like in conventional resolved motion rate algorithms and the feedback term serves as a correction term to avoid numerical drift or deviations along the trajectory due to discrete-time implementation of the algorithm.

Regarding the constraint task, if $n - m_2 = m_1$, then $e_C = \phi_{sd} - \phi_s$ and

$$J_C = -I_S^{-1} I_M \quad (10)$$

from (5); in this case, manipulator redundancy is exploited to reach the desired spacecraft attitude ϕ_{sd} while tracking the desired end-effector trajectory. On the other hand, if $n - m_2 > m_1$, then the constraint task can encompass also an additional constraint, such as mechanical joint range, obstacle avoidance etc.

With solution (9), the constraint task is not guaranteed to be satisfied along the whole motion execution. Therefore, for those applications where the constraint task is judged to be more important than the end-effector task, the order of priority can be switched with obvious transposition of subscripts in (9).

An interesting case is that when it is desired to keep the spacecraft attitude constant during manipulator motion ($\omega_{sd} = 0$), e.g. not disturbing the orientation of some antenna for communication between spacecraft and earth. According to the above technique, the joint velocity solution can be computed as

$$\dot{q} = J_C^\dagger K_C e_C + (I - J_C^\dagger J_C) J_G^T K_E e_E. \quad (11)$$

If no other constraint is imposed and $n - m_2 = m_1$, then J_C is given as in (10); it is assumed that J_C is non-singular, otherwise a transpose should be employed. By using the expression for J_G given in (7), one can write

$$J_G = J_M + J_S J_C; \quad (12)$$

then computing the matrix $J_G(I - J_C^\dagger J_C)$ with J_C as in (10) and J_G as in (12) leads to—after ordinary algebraic manipulation—

$$J_G(I - J_C^\dagger J_C) = J_M(I - I_M^\dagger I_M), \quad (13)$$

which coincides with the so-called *fixed-attitude-restricted* Jacobian introduced in [9].

As a consequence, solution (11) provides end-effector trajectories not changing spacecraft attitude which are computed via the transpose of the fixed-attitude-restricted Jacobian—the projector $(I - J_C^\dagger J_C)$ is symmetric—and then can be simplified into

$$\dot{q} = J_C^\dagger K_C e_C + (I - I_M^\dagger I_M) J_M^T K_E e_E. \quad (14)$$

This result is argued to be quite important in view of occurrence of algorithmic singularities of the above Jacobian [7], whereas an algorithm based on a pseudoinverse of such Jacobian might suffer from numerical problems.

4. Case Studies

In order to study the performance of the above inverse kinematics algorithms, a number of case studies were developed for a planar system composed by a free-floating spacecraft with a four-link arm attached. The geometry of the system indicates that $m_1 = 1$, since the spacecraft attitude is determined by the angle of rotation about the axis orthogonal to the plane. Also, it is $n = 4$, while it is chosen to be $m_2 = 2$, i.e. only the arm tip position is of concern. The arm manipulator then possesses one degree of redundancy with respect to the entire system; this implies that in principle it is possible to obtain not only spacecraft/arm motion coordination but also fulfilment of an additional scalar constraint task.

The dimensions and inertia properties of the system are indicated in the table below in standard SI units, where ℓ denotes link length, I_x, I_y, I_z are moments of inertia about link coordinate axes, and 0 refers to the spacecraft.

	0	1	2	3	4
M	2000	50	50	50	50
ℓ	2.5	2.5	2.5	2.5	2.5
I_x	2083	0.25	0.25	0.25	0.25
I_y	2083	26	26	26	26
I_z	2083	26	26	26	26

The first case study is aimed at showing the reaction effects induced by the motion of the arm on the spacecraft. The arm tip is required to track a short rectilinear path; the trajectory is computed by assigning a 5th-order smooth polynomial with zero initial and final velocities and accelerations and a motion duration of 2 sec. The inverse kinematics algorithm based on solution (8) is used with a suitable choice of the feedback gains. The resulting motion of the system in Fig. 1 confirms that the spacecraft attitude varies while the arm tip follows the imposed trajectory with limited tracking errors. In particular, it was observed that the errors were dominant along the (horizontal) direction of the path, i.e. a time lag that is acceptable in practice; this is typical with a Jacobian transpose algorithm [11].

In the second case study not only is it desired to keep a constant spacecraft attitude during the motion but also a constraint is added, i.e. the arm tip is required to reach a horizontal orientation; the path is rectilinear again but longer than above and the trajectory is generated through the same kind of polynomial with a motion duration of 2 sec. The inverse kinematics algorithm based on solution (9) is employed with a suitable choice of the feedback gains; this corresponds to giving higher priority to the

arm tip task. The resulting motion of the system in Fig. 2 reveals the following facts: The tip trajectory is satisfactorily followed; actually, tracking errors are smaller than above thanks to the use of the pseudoinverse in lieu of the transpose. The spacecraft attitude is maintained practically constant along the task. Remarkably, system redundancy is exploited to drive the arm tip orientation to the final horizontal orientation.

The order of priority between the arm tip position task and the constraint task is switched in the third case study along the same trajectory as for the previous case study; this time, no additional constraint on the arm tip orientation is introduced. This corresponds to choosing the inverse kinematics algorithm based on solution (14). Differently from above, now the spacecraft attitude is kept constant at the expense of limited arm tip tracking errors; these are due to the use of the transpose of the manipulator Jacobian and, as for the first case study, do not constitute a serious inconvenience since basically they are time lags and not deviations from the path.

5. Conclusions

Closed-loop inverse kinematics algorithms for spacecraft/manipulator systems have been illustrated in this paper. The presence of redundancy in the system has been conveniently exploited to accomplish additional constraint tasks with respect to the usual manipulator end-effector task, including the typical requirement of maintaining constant spacecraft attitude along the motion. The algorithms have been derived in the framework of the task space augmentation approach with task priority previously developed for ground-fixed manipulator, by virtue of the adoption of the generalized Jacobian concept that allows to describe the effects of the manipulator motion on the free-floating spacecraft. Differently from earlier inverse kinematics solution of open-loop resolved motion rate type, the proposed algorithms are of closed-loop type and enjoy the computationally advantageous feature of using the transpose of the proper Jacobians in place of the pseudoinverse. The numerical case studies have confirmed the results anticipated in theory.

It is clear that other aspects are significant for effective operation of spacecraft/manipulator systems, such as path planning [10] and inclusion of dynamic effects for control purposes [14–16]. Future research efforts will be likely dedicated to these and other aspects of space robotics manipulation.

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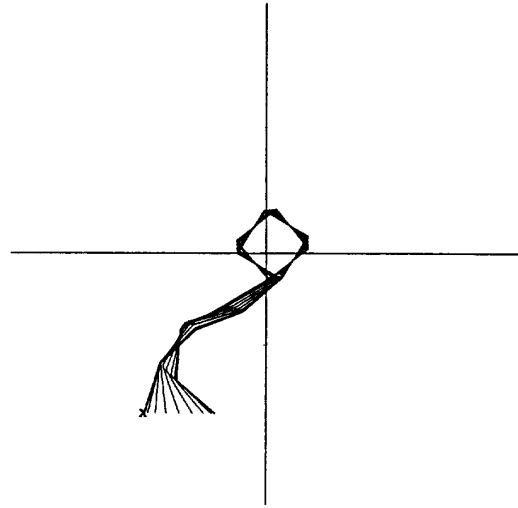


Fig. 1 — Motion of the spacecraft/arm system for a given arm tip position path with solution (8).

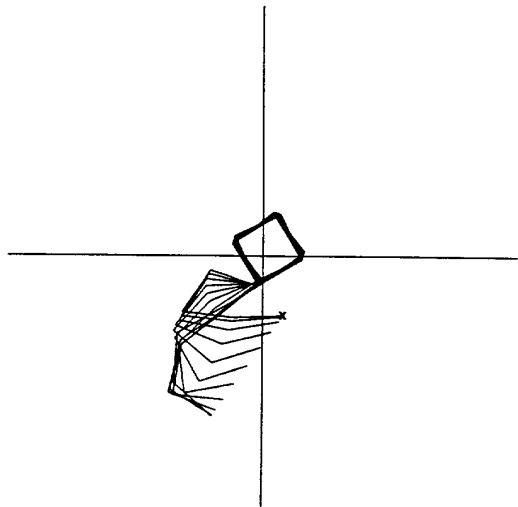


Fig. 2 — Motion of the spacecraft/arm system for a given arm tip position path and constraints on constant spacecraft attitude and final arm tip orientation with solution (9).

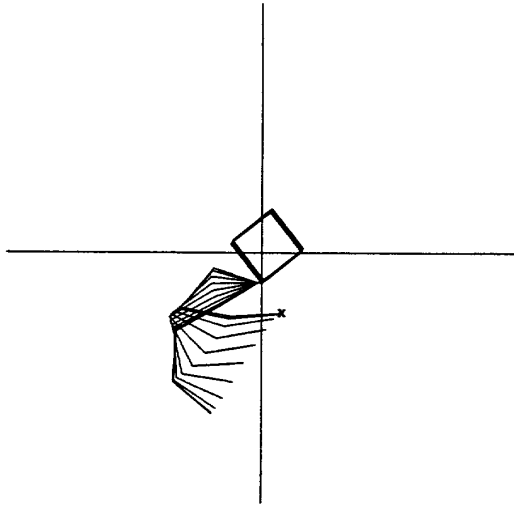


Fig. 3 — Motion of the spacecraft/arm system for a given arm tip position path and a constraint on constant spacecraft attitude with solution (14).