

A Grasping Force Optimization Algorithm for Dexterous Robotic Hands

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Abstract—The problem of grasping force optimization for a robotic system equipped with multi-fingered hands is considered in this paper. This problem is cast as a convex optimization problem, considering also joint torque constraints. A solution suitable for on-line implementation is proposed, which allows a substantial reduction of the computational load by dynamically decreasing the number of active torque constraints. Moreover, for the case of a bimanual manipulation system, a sub-optimal single-hand optimization algorithm is presented and compared with the optimal one. The effectiveness of the described methods has been tested in a simulation case study.

I. INTRODUCTION

The control of a robotic system equipped with multi-fingered hands involves several aspects ranging from the synthesis of the optimal grasping contact points and grasp planning to the load sharing and grasp control. In particular, the evaluation of the grasping forces able to guarantee the stability of the grasp and its feasibility, in the face of the external disturbances, represents a crucial problem. The complexity of the problem relies on the need for on-line resolution of an optimization problem where both constraints and objective functions are non-linear, the number of variable and constraints are relatively large, and the grasp configuration and load wrench may change with time. The grasping force optimization (GFO) problem has not yet been intensively investigated for the case of bimanual human-like robotic systems, for which the computational complexity becomes a major issue to be considered for an efficient on-line solution.

The study of frictional force-closure grasps has been addressed by linearizing the friction cone constraints and then applying linear programming (LP) techniques [1], [2], [3]. In [4] the friction cone constraints have been formulated in terms of linear matrix inequalities (LMIs) and the GFO is addressed as a convex optimization problem.

Starting from the observation that verifying the friction cone constraints is equivalent to testing the positive definiteness of certain symmetric matrices, in [5] the GFO has been formulated as a convex optimization problem on a Riemannian manifold with linear constraints. Several gradient flow type algorithms have been proposed to provide

solutions suitable for real-time applications [6], [7]. A further improvement has been presented in [8], consisting in a new compact representation of the friction cone constraints which allows a reduction of the optimization problem complexity.

In [9] and [10] a method based on the minimization of a cost function is presented, which gives an analytical solution but does not ensure the satisfaction of the friction constraints.

The method proposed in [5] requires the on-line inversion of a constrained matrix whose dimension linearly increases with the number of fingers of a factor that depends on the contact type. This factor can be reduced by adopting the constraint matrix representation proposed in [8]. However, if torque limits constraints are considered, the complexity of the problem increases more than quadratically with the number of joints, which is higher in a dual-hand system. Moreover, all the proposed solutions require, at each iteration, the evaluation of an initial point that satisfies all the constraints, e.g. as in [11] but with a significant computational effort.

The proposed algorithm is based on the compact formulation of [8] and on the solution of a convex optimization case as in [6], and it focalizes our previous work on single-hand manipulation to bimanual manipulations systems [12]. The method allows the consideration of joint torque constraints, with a minimum increase of computation complexity, compatible with real-time constraints. Moreover, the proposed iterative formulation does not require the evaluation at each step of a new initial point. Finally, a sub-optimal single-hand optimization algorithm is proposed to cope with very limited computational hardware availability, and compared with the optimal solution. In particular, a new criterion for the load sharing [13], [14], [15] between the hands is here introduced to improve the solution. The feasibility and the effectiveness of the proposed method have been tested in a simulation scenario where a robotic torso equipped with two dextrous hands is used to empty a bottle.

II. PROBLEM FORMULATION

Consider a bimanual robotic system equipped with two multi-fingered hands grasping an object with n contacts between the object and the fingertips, the links of the fingers and the palm. Denote the contact wrench of the grasp with $\mathbf{c} = [\mathbf{c}_r^T \ \mathbf{c}_l^T]^T = [\mathbf{c}_1^T \ \dots \ \mathbf{c}_n^T]^T \in \mathbb{R}^{nm}$, where $\mathbf{c}_i \in \mathbb{R}^m$ is the wrench vector of the i -th contact with dimension m depending on the adopted contact model, and $\mathbf{c}_r$ and $\mathbf{c}_l$ are the corresponding wrench vectors of all the contact points of the right and left hand, respectively.

The GFO problem consists in finding the set of contact wrenches balancing the generalized external force $\mathbf{h}_e \in \mathbb{R}^6$ acting on the object (including object inertia and weight),

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which are feasible with respect to the kinematics of the hands and the joint torque limits, and minimize the overall stress applied to the object, i.e., the internal forces. Moreover, to avoid the slippage of the fingers on the object surface, each contact wrench has to be confined within the friction cone.

The balance equation for the generalized forces applied to the object can be written in the form

$$\mathbf{h}_e = \mathbf{G}\mathbf{c}, \quad (1)$$

where $\mathbf{G} = [\mathbf{G}_r \ \mathbf{G}_l] \in \mathbb{R}^{6 \times nm}$ is the grasp map composed of the grasp matrices of the right and left hand, which is full-rank for force-closure grasps. It is assumed that the contact point configurations ensuring the force-closure constraints are assigned at each time by the planning system.

Although several contact models can be used, the two models usually adopted are the *point contact with friction* (PCWF) model and the *soft finger contact* (SFC) model.

In the PCWF case, the contact wrench has three components ($m = 3$): the normal component $c_{i,z}$ to the object surface and the two components $c_{i,x}$, $c_{i,y}$ on the tangent plane. The friction constraint is described as

$$\frac{1}{\mu_i^2} (c_{i,x}^2 + c_{i,y}^2) \leq c_{i,z}^2 \text{ and } c_{i,z} > 0, \quad (2)$$

where μ_i is the friction coefficient at the i -th contact point. The extension to the SFC case can be found in [12].

The balance equation for the torques applied to fingers joints of the hands can be written in the form

$$\mathbf{J}^T(\mathbf{q})\mathbf{c} + \boldsymbol{\tau}_e = \boldsymbol{\tau}, \quad (3)$$

where $\mathbf{J}(\mathbf{q}) = [\mathbf{J}_r^T \ \mathbf{J}_l^T]^T$ is the $(nm \times N)$ hands Jacobian matrix, depending on the N -vector $\mathbf{q}$ of the joint variables, being N the total number of the joints, $\boldsymbol{\tau}_e$ is the external torque, including gravity, Coriolis, centripetal and inertia terms, and $\boldsymbol{\tau}$ is the actuation torque.

Moreover, joint torque constraint have to be considered to ensure the motor actuation feasibility

$$\boldsymbol{\tau}_L \leq \boldsymbol{\tau} \leq \boldsymbol{\tau}_H, \quad (4)$$

where $\boldsymbol{\tau}_L$ ($\boldsymbol{\tau}_H$) is the lower (upper) joint torque limit.

The satisfaction of the force balance equation (1), with the friction constraints (2), and of the joint torque balance equation (3) with constraint (4), implies that the grasp is stable and feasible. The GFO problem considered here consists in finding the optimal grasp wrench that minimizes the internal forces acting on the object, under the above constraints. The internal forces are contact wrenches that satisfy the friction cone constraints and belong to the null space of the grasp matrix $\mathbf{G}$. These wrenches $\mathbf{c}_{int}$ do not contribute to the balance equation (1), being $\mathbf{G}\mathbf{c}_{int} = \mathbf{0}$, but are used to satisfy the friction cone constraints.

III. GRASPING CONSTRAINTS AND COST FUNCTION

A. Inequality constraints

The frictional inequalities (2) are equivalent to the positive definiteness of the block-diagonal matrix [8]

$$\mathbf{F}(\mathbf{c}) = \text{diag}(\mathbf{F}_1(\mathbf{c}_1), \dots, \mathbf{F}_n(\mathbf{c}_n)) > \mathbf{0}, \quad (5)$$

where $\mathbf{F}_i(\mathbf{c}_i)$ in the PCWF case is the symmetric matrix

$$\mathbf{F}_i(\mathbf{c}_i) = \begin{bmatrix} c_{i,z} + \frac{c_{i,x}^2}{\mu_i} & \frac{c_{i,y}}{\mu_i} \\ \frac{c_{i,y}}{\mu_i} & c_{i,z} - \frac{c_{i,x}^2}{\mu_i} \end{bmatrix}. \quad (6)$$

Similarly, the torque limit constraint (4), in view of the torque balance equation (3), is equivalent to the positive definiteness of the diagonal matrix

$$\mathbf{T}(\mathbf{c}, \mathbf{q}, \boldsymbol{\tau}_e) = \text{diag}(\boldsymbol{\tau}_B) > \mathbf{0}, \quad (7)$$

where

$$\boldsymbol{\tau}_B = \begin{bmatrix} \boldsymbol{\tau}_{B,L} \\ \boldsymbol{\tau}_{B,H} \end{bmatrix} = \begin{bmatrix} \mathbf{J}^T(\mathbf{q})\mathbf{c} - \boldsymbol{\tau}_L + \boldsymbol{\tau}_e \\ -\mathbf{J}^T(\mathbf{q})\mathbf{c} + \boldsymbol{\tau}_H - \boldsymbol{\tau}_e \end{bmatrix} \quad (8)$$

is the vector of the residual torques with respect to the lower ($\boldsymbol{\tau}_{B,L}$) and upper ($\boldsymbol{\tau}_{B,H}$) limit.

Hence, the simultaneous satisfaction of both frictional and joint torque constraints is equivalent to the positive definiteness of the linearly constrained block-diagonal matrix

$$\mathbf{P} = \text{diag}(\mathbf{F}, \mathbf{T}) > \mathbf{0}. \quad (9)$$

Notice that the elements of the matrices $\mathbf{F}$ and $\mathbf{T}$ are linearly dependent, because they both depend linearly on $\mathbf{c}$. Moreover, the force balance equation (1) and the torque balance equation (3) correspond to linear constraints imposed on matrix $\mathbf{P}$.

B. Linear constraints

By denoting with $\mathbf{c}(\mathbf{F})$ the contact wrench vector extracted from the frictional constraint matrix, with $\boldsymbol{\tau}_B(\mathbf{T})$ the vector composed by the diagonal elements of $\mathbf{T}$, and defining vector $\boldsymbol{\xi}(\mathbf{P}) = [\mathbf{c}(\mathbf{F})^T, \boldsymbol{\tau}_B(\mathbf{T})^T]^T$, the linear constraints on matrix $\mathbf{P}$ imposed by (1) and (3) can be represented in the following affine general form

$$\mathbf{A}\boldsymbol{\xi}(\mathbf{P}) = \mathbf{b} \quad (10)$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{G} & \mathbf{0}_{6 \times 2N} \\ \mathbf{A}_\tau \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{h}_e \\ \boldsymbol{\tau}_L - \boldsymbol{\tau}_e \\ \boldsymbol{\tau}_H - \boldsymbol{\tau}_e \end{bmatrix}, \quad (11)$$

where $\mathbf{A}_\tau$ is the $(2N \times nm + 2N)$ matrix defined as

$$\mathbf{A}_\tau = \begin{bmatrix} \mathbf{J}(\mathbf{q})^T & -\mathbf{I}_N & \mathbf{0}_N \\ \mathbf{J}(\mathbf{q})^T & \mathbf{0}_N & \mathbf{I}_N \end{bmatrix}, \quad (12)$$

being $\mathbf{0}_\times$ and $\mathbf{I}_\times$ the null and the identity matrix.

C. Cost function

The optimization is based on the minimization of the cost function $\Phi(\mathbf{P}) : \mathcal{P}(r) \rightarrow \mathbb{R}$, being $\mathcal{P}(r)$ the set of positive definite symmetric matrices $\mathbf{P} = \mathbf{P}^T > \mathbf{0}$, defined as

$$\Phi(\mathbf{P}) = \text{tr}(\mathbf{W}_p \mathbf{P} + \mathbf{W}_b \mathbf{P}^{-1}), \quad (13)$$

where $\text{tr}(\cdot)$ denotes the trace operator, and $\mathbf{W}_p$ and $\mathbf{W}_b$ are symmetric positive definite matrices. Notice that Φ is a strictly convex twice continuously differentiable function on $\mathcal{P}(r)$ and $\Phi(\mathbf{P}) \rightarrow +\infty$ for $\mathbf{P} \rightarrow \partial\mathcal{P}(r)$, being $\partial\mathcal{P}(r)$ the boundary of $\mathcal{P}(r)$.

By noting that the sum of the elements of $\mathbf{T}$ (i.e. of τ_B) is constant for each $\mathbf{c}$, because the sum of the two joint torque constraints for the i -th joint is constant and equal to $\tau_{r,i} = \tau_{H,i} - \tau_{L,i}$, the diagonal weighting matrix $\mathbf{W}_p = \text{diag}(w_p \mathbf{I}_6, \mathbf{0}_{2N})$, with $w_p > 0$, is considered. In this way, the term $\mathbf{W}_p \mathbf{P}$ weighs only the normal forces $\mathbf{c}_{i,z}$ at each contact point, i.e. the pressure forces on the object. If required, different weights can be used allowing higher contact forces for stronger fingers.

The second term $\mathbf{W}_b \mathbf{P}^{-1}$ represents a barrier function, which goes to infinity when $\mathbf{P}$ tends to a singularity, i.e. when friction or torque limits are approached. The barrier weight matrix is also chosen diagonal $\mathbf{W}_b = \text{diag}(\mathbf{W}_{b,F}, \mathbf{W}_{b,T})$, with

$$\begin{aligned} \mathbf{W}_{b,F} &= w_{b,F} \text{diag}(\mu_1, \dots, \mu_n) \\ \mathbf{W}_{b,T} &= w_{b,T} \text{diag}(\tau_{r,1}, \dots, \tau_{r,N}, \tau_{r,1}, \dots, \tau_{r,N}), \end{aligned} \quad (14)$$

being $w_{b,F} > 0$, $w_{b,T} > 0$.

Hence, the minimization of the cost function (13) with the linear constraint (10) corresponds to the minimization of the normal contact wrench components applied to the object while satisfying the friction and torque constraints.

IV. SEMIDEFINITE PROGRAMMING

The minimization problem can be solved using the linearly constrained gradient flow approach on the smooth manifold of positive definite matrices presented in [5]. In particular, it is possible to prove that $\Phi(\mathbf{P})$ presents a unique minimum that can be reached through the linear constrained exponentially convergent gradient flow

$$\dot{\mathbf{P}} = \mathbf{Q} \xi(\mathbf{P}^{-1} \mathbf{W}_b \mathbf{P}^{-1} - \mathbf{W}_p), \quad (15)$$

where $\mathbf{Q} = (\mathbf{I} - \mathbf{A}^\dagger \mathbf{A})$ is the linear projection operator onto the tangent space of $\mathbf{A}$, and $\mathbf{A}^\dagger = \mathbf{A}^T (\mathbf{A} \mathbf{A}^T)^{-1}$ is the pseudo-inverse of $\mathbf{A}$. Consequently, $\mathbf{A} \mathbf{Q} = \mathbf{0}$ and $\mathbf{A} \xi(\dot{\mathbf{P}}) = \mathbf{0}$; hence, if the solution satisfies the constraint (10) at $t = 0$, it will satisfy the constraint for all $t > 0$.

A discrete-time version of (15) based on the Euler numerical integration algorithm is

$$\xi(\mathbf{P}_{k+1}) = \xi(\mathbf{P}_k) + \alpha_k \mathbf{Q} \xi(\mathbf{P}_k^{-1} \mathbf{W}_b \mathbf{P}_k^{-1} - \mathbf{W}_p), \quad (16)$$

where the step-size α_k is chosen to ensure down-hill steps. Notice that the choice of α_k strongly affects the performance of the optimization algorithm. A wrong choice could determine a very slow convergence or the break of the barrier. Several strategies have been proposed for the self-tuning of α_k at each iteration (see [11] for details). The sensitivity to the step-size choice can be reduced by adopting a Dikin-type recursive algorithm [6], that leads to the discrete flow

$$\xi(\mathbf{P}_{k+1}) = \xi(\mathbf{P}_k) - \alpha_k \mathbf{Q} \frac{\xi(\mathbf{P}_k^{-1} \mathbf{W}_b \mathbf{P}_k^{-1} - \mathbf{W}_p)}{\|\mathbf{P}_k^{-1} \mathbf{W}_b \mathbf{P}_k^{-1} - \mathbf{W}_p\|_{\mathbf{P}_k}}, \quad (17)$$

where $\|\mathbf{X}\|_{\mathbf{Y}} = \text{tr}(\mathbf{Y}^{-1} \mathbf{X} \mathbf{Y}^{-1} \mathbf{X})$, and $0 \leq \alpha_k \leq 1$ can be evaluated with a bounded line search minimizing $\Phi(\mathbf{P}_{k+1})$.

V. COMPUTATIONAL IMPROVEMENTS

A. Affine Constraint Decomposition

The proposed algorithm requires the inversion of a $(6 + 2N)$ matrix $\mathbf{A} \mathbf{A}^T$ needed for the evaluation of $\mathbf{A}^\dagger$ at each iteration, also when the grasping configuration is unchanged, i.e. when $\mathbf{G}$ is constant, due to the variation of $\mathbf{J}(\mathbf{q})$.

Starting from the discrete version of the gradient flow (16), the following new formulation can be derived

$$\mathbf{c}_{k+1} = \mathbf{c}_k + \alpha_k \bar{\mathbf{Q}} \xi(\mathbf{P}^{-1}(\mathbf{c}_k) \mathbf{W}_b \mathbf{P}^{-1}(\mathbf{c}_k) - \mathbf{W}_p), \quad (18)$$

where $\bar{\mathbf{Q}} = (\mathbf{I} - \mathbf{G}^\dagger \mathbf{G})[\mathbf{I}_{nm} \mathbf{0}_{nm, 2N}](\mathbf{I} - \mathbf{A}_\tau^\dagger \mathbf{A}_\tau)$ is the result of the projection onto the null space of $\mathbf{A}_\tau$ in (12), which guarantees the coherence of the elements of matrix $\mathbf{P}$, and of the subsequent projection onto the null space of the grasp matrix, ensuring the force balance constraint (1). Therefore, the evaluation of the inverse of a $(6 + 2N)$ matrix is decomposed into the evaluation of the inverse of two matrices of lower dimensions (6 and $2N$, respectively). Moreover, if the grasp configuration remains unchanged, the projector depending on $\mathbf{G}$ can be evaluated off-line.

B. Initial Point Iterative Self-Evaluation

An iterative technique for the on-line evaluation of the initial point—the initial solution $\mathbf{P}_0$ for the optimization gradient flow algorithm—is proposed here, based on the previous optimal solution. The quantities that can vary between successive sampling times are the hand configuration $\mathbf{q}$, the external torque τ_e , and the grasp map $\mathbf{G}$, while they are taken constant during the iterations of the optimization algorithm between two consecutive sampling times.

To avoid the evaluation of an initial point at each sampling time, the following approach is proposed. Initially, at time t_0 , the method proposed in [11] (or a similar one) is used to evaluate off-line a first valid initial solution, which is employed for the first optimization cycle. For the next sampling times t_k , the initial point is evaluated from the previous optimal solution $\mathbf{c}_{k-1}$, through the iterative algorithm

$$\begin{aligned} \bar{\mathbf{c}}_j &= (\mathbf{I} - \mathbf{G}_k^\dagger \mathbf{G}_k) \bar{\mathbf{c}}_{j-1} + \gamma_j \mathbf{G}_k^\dagger \mathbf{h}_{e,k} + \bar{\gamma}_j \mathbf{G}_{k-1}^\dagger \mathbf{h}_{e,k-1} \\ \bar{\tau}_j &= \mathbf{J}^T(\gamma_j \mathbf{q}_k + \bar{\gamma}_j \mathbf{q}_{k-1}) \bar{\mathbf{c}}_j + \gamma_j \tau_{e,k} + \bar{\gamma}_j \tau_{e,k-1}, \end{aligned} \quad (19)$$

with $\bar{\mathbf{c}}_0 = \mathbf{c}_{k-1}$, where the subscript k is referred to the current optimization cycle and $\bar{\gamma}_j = 1 - \gamma_j$, while the subscript j and the variables with the bar are referred to the iterations within the cycle. The coefficient $\gamma_j \in (0, 1]$ is chosen at each iteration according to a monotone sequence, using a simple linear search algorithm, as the maximum value that does not produce invalid solutions ($\mathbf{P}_0 \leq 0$).

In detail, at each step of the optimization cycle, the first equation of (19) gradually modifies the external wrench component of the current solution until the full external wrench $\mathbf{h}_{e,k}$ is balanced (i.e., $\gamma_j = 1$). If the solution evaluated at the previous sampling time ($\mathbf{c}_{k-1}$) is sufficiently far from the boundaries—the distance depends also on the weights assigned to $\mathbf{W}_b$ —, γ_0 can be set to 1 at the first iteration, and thus the initial point has the same internal wrench component of the previous optimal solution. On

the other hand, when $\gamma_0 < 1$, the effect of the barrier function produces a new solution that, at each iteration of the optimization cycle, goes away from the boundaries; this guarantees that γ_j increases at each step, until $\gamma_j = 1$. The second equation is required to modify the joint torque with the same rationale of the first equation.

C. Dynamic Joint-Torque Constraints Selection

By assuming that the solutions of the GFO evaluated at successive sampling times are close, the joint torque constraints can be simplified by observing that not all the joint torque constraints can be effective simultaneously. For example, if for the current optimal solution the actuator of joint i provides a torque close to the upper bound $\tau_{H,i}$, the constraint on the lower bound $\tau_{L,i}$ can be deactivated at the next sampling time, being negligible the corresponding barrier term in the cost function. More in general, if for a grasp configuration it is required a given contact force along a certain direction, it is reasonable to assume that the corresponding joint torques will not change significantly at the next sampling time. Starting from this observation, the number of joint torque constraints can be dynamically set at each sampling time, by using the residual torques evaluated at the previous sampling time with respect to the lower and upper bounds as the criterion for selecting the constraint (the lower or the upper one) that needs to be activated. Only those constraints with a distance higher than a torque limit threshold, that can be chosen as a fraction $\sigma_\tau \geq 0$ of the corresponding torque limit, will be activated. Notice that the case $\sigma_\tau = 0$ corresponds to the selection of the closest torque limits, i.e. the half of the total number of constraints.

D. Iterative single-hand grasp force optimization

For applications with limited computational resource, a further simplification in the algorithm can be introduced by splitting the bimanual optimization problem into two simpler single-hand problems. In this case, the initial point iterative self-evaluation algorithm presented above can be employed to find the initial common solution. Then two independent optimization procedures can be started separately for each hand, and the corresponding solutions are composed only at the end to achieve a unique wrench vector solution. The price to pay with the simplified algorithm is that the solution is not the optimal in a global sense and it is possible that the single-hand optimization problems are not feasible.

The problem feasibility can be improved by considering a weighted pseudo-inverse of the grasp matrix in the equation (19), with the goal of achieving a load sharing between the hands in the face of the actual load of the hand actuators. In detail, at each sampling time, the minimum distance of the joint torques with respect the corresponding limits is evaluated for each hand, namely $\delta_{\tau,r}$ and $\delta_{\tau,l}$ for the right and for the left hand, respectively. Then a weighting matrix

$$W_G = \text{diag} \left(\frac{\delta_{\tau,r} + \delta_{\tau,l}}{\delta_{\tau,r}} \mathbf{I}_{n_r}, \frac{\delta_{\tau,r} + \delta_{\tau,l}}{\delta_{\tau,l}} \mathbf{I}_{n_l} \right), \quad (20)$$

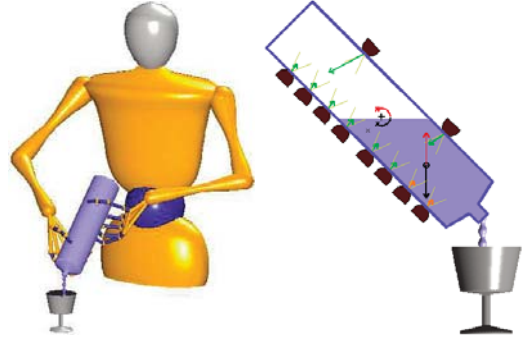


Fig. 1. *Left*: anthropomorphic torso with two DEXMART hands grasping a bottle (www.dexmart.eu). *Right*: section of the grasped bottle with graphical representation of the gravity force and torque (black arrows), of the resultant force and torque applied by the fingers (red arrows), of the optimal contact forces (green arrows if not affected by joint torque constraints, orange arrow otherwise), and of the friction cones (yellow triangles).

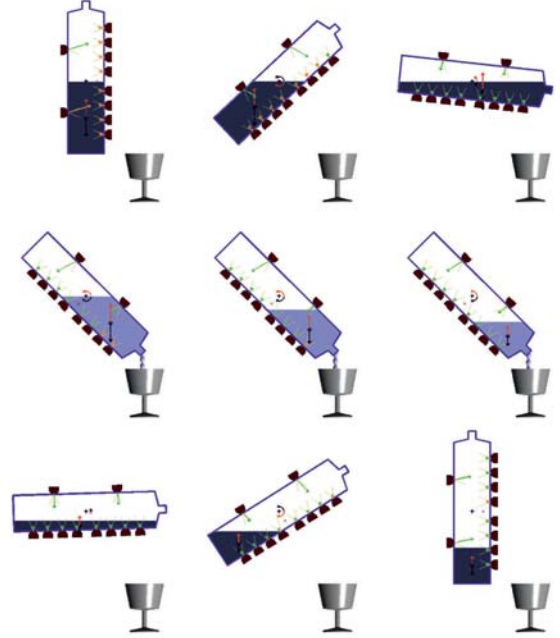


Fig. 2. Sequence of significant configurations of the bottle and of the forces during task execution.

with n_r and n_l the number of contact points for the right and for the left hand, is adopted for the evaluation of the weighed pseudo-inverse of the grasp matrix

$$G^\# = W_G^{-1} G (G W_G^{-1} G^T)^{-1}. \quad (21)$$

The corresponding quadratic form $\mathbf{c}^T W_G \mathbf{c}$ is thus minimized, reducing the load requirement on the hand closest to its torque limits.

VI. CASE STUDY

The proposed GFO algorithm has been tested in simulation using two DEXMART hands mounted on an torso as shown in Fig. 1. It is assumed that the hands grasp a bottle filled with water; the task consists in pouring water by reorienting

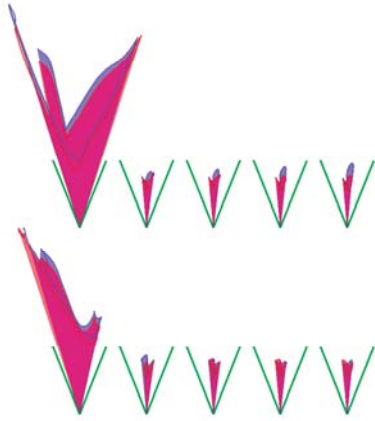


Fig. 3. Areas covered by the contact forces of each finger (*Top*: right hand, *Bottom*: left hand; from the left side: from the thumb to the little finger) without (blue color) and with (red color) torque constraints with respect to the friction cones (represented with green lines).

the bottle, which is initially vertically grasped, through three steps: 1) a rotation of 135 deg; 2) some water comes out from the bottle (the mass and inertia of the bottle change accordingly); 3) the bottle is brought back in the initial pose.

A simulation has been performed considering the variation of the position of the center of mass of the water and that of its weight. Figure 1 shows on the right a section of the bottle half filled with water, where the intensity of the gravity force (torque) is proportional to the black vertical (circular) arrow applied to the instantaneous center of mass. The red arrows represent the external force and torque balancing the gravity effects and resulting from the contact forces applied by the fingers, represented by green arrows if not affected by joint torque constraints, orange arrow otherwise. The sections of the friction cones in the contact points are colored in yellow. A sequence of significant configurations of the bottle during task execution is shown in Fig. 2.

A. Joint torque limits constraints

The effectiveness of the friction and of joint torque limits constraints is shown considering two different simulations: in the first one only the friction constraint is considered, without any constraint on the joint torque limits, while in the second one different torque limits are set for the fingers. In particular, the thumb actuators are considered stronger than the corresponding actuators of the other fingers of the hand (± 0.5 vs. ± 0.075 Nm), like for a human hand.

In Fig. 3 the trajectory and the areas covered by the contact force vectors during the bottle motion are shown, in blue (red) color for the case without (with) torque constraints. The frictional constraints are respected in both cases accordingly to the barrier function of the cost function (13).

In Fig. 4 a comparison of the norm of the contact wrenches and of the joint torques is shown. The differences for the norm of the torque between the two simulation cases is limited, while the contact wrenches are improved (smaller norm), due to the balancing of the load between the fingers.

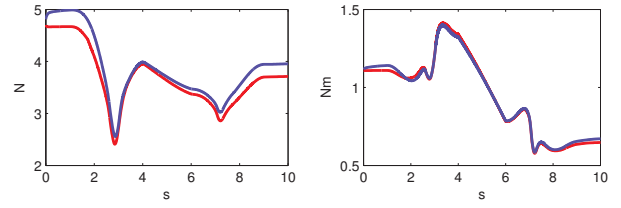


Fig. 4. Time history of the norm of the contact wrenches (on the left) and of the joint torques (on the right), in red (blue) color for the case with (without) torque constraints.

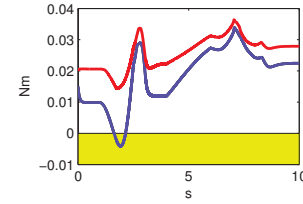


Fig. 5. Time history of the minimum distance of the joint torques from the corresponding limits for all the actuators, in red (blue) color for the case with (without) torque constraints. Negative values (the yellow area) correspond to the violation of one or more joint torque limits.

The time history of the minimum distance of the joint torques from the constraints is shown in Fig. 5. The effect of the barrier function acting also on the torques allows full respect of the adopted limits, without affecting significantly the contact wrenches as shown in Fig. 4.

B. Online joint torque constraints selection

The benefits of the on-line joint-torque constraints selection are shown in Fig. 6, where the time history of the computational time effort and the number of employed constraints are represented. It is assumed that to achieve a good grasp-force feedback control, a real-time constraint of less than 10 ms is required, while 2 ms is here considered the ideal case. To remove the dependence from the employed hardware, all the considered cases are normalized with respect to the maximum value of the fully constrained case to the value 100 (23 ms on an Intel P.IV at 2.8 Ghz). Four different cases are compared: all constraints, $\sigma_\tau = 0$ (red), $\sigma_\tau = 0.5$, $\sigma_\tau = 0.8$, and unconstrained, where σ_τ is the threshold for the activation of the joint torque constraints. The achieved reduction of the mean of the computational time varies between a minimum of about 30% for $\sigma_\tau = 0$ to a maximum of 90% for $\sigma_\tau = 0.8$. Notice that, for $\sigma_\tau = 0.5$ the less restrictive constraint of 10 ms is guaranteed, while the most restrictive constraint for the real-time of 2 ms is verified for $\sigma_\tau = 0.8$.

C. Iterative single-hand grasp force optimization

The adoption of the sub-optimal single-hand GFO algorithm can provide a significant reduction on the computational time with respect to the optimal one, as shown in Fig. 6 for the case of all joint torque constraint simultaneously active. However, the drawback is that the sub-optimal

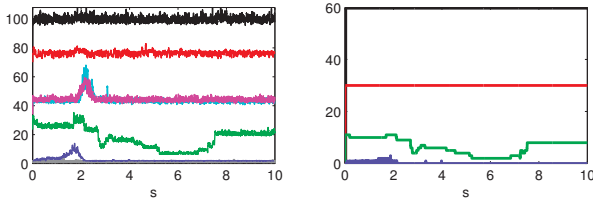


Fig. 6. Time history of the normalized computational time effort (left), where the value 100 corresponds to 23 ms on an Intel Pentium IV at 2.8 Ghz, and of the number of employed joint torque constraints (right) for the cases with all constraints (black), $\sigma_\tau = 0$ (red), $\sigma_\tau = 0.5$ (green), $\sigma_\tau = 0.8$ (blue), unconstrained (gray, close to 1.3 %), and single-hand local optimization with (violet) and without (cyan) weighted pseudo-inverse of the grasp matrix.

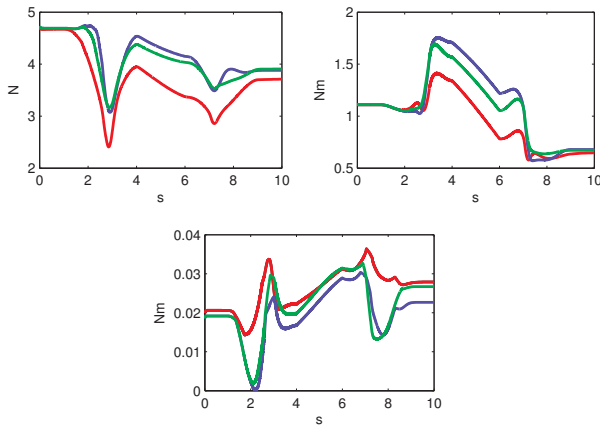


Fig. 7. Time history of the norm of the contact wrenches (on the top-left), of the joint torques (on the top-right), and of the minimum distance of the joint torques from the corresponding limits (on the bottom) for the cases of local single-hand optimization without (blue) and with (green) weighted pseudo-inverse of the grasp matrix, and global optimization (red).

solution has reduced performance in terms of both the norm of contact wrenches and of joint torque. Consequently, also the distance with respect to the joint torque limits reduces significantly, as shown in Fig. 7, but without violating the imposed constrained. As shown in these figures, the adoption of the weighted pseudo-inverse of the grasp matrix in (21) can improve the achieved solution resulting in a well-shared load between the two hands.

The time history of the norm of the load force and moments for the right (red) and left (blue) hand in the cases of sub-optimal (continuous lines) and optimal (dashed lines) method are shown in Fig. 8. As expected, the whole balancing of effort between the hands is degraded with respect to the optimal solution (dashed lines), but the adopted load sharing method allows an on-line load share in respect of the current load capability of each hand.

VII. CONCLUSION

An on-line GFO algorithm for bimanual robotic systems has been presented, considering also joint torque limits. The computational load of the algorithm has been reduced by adopting an iterative formulation based on a dynamic set of

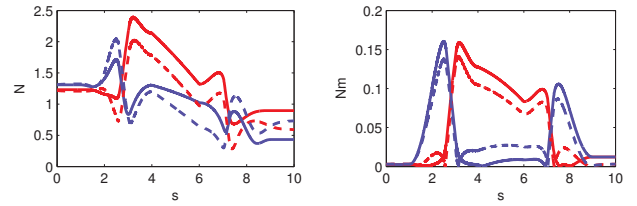


Fig. 8. Time history of the norm of the load force (on the left) and moments (on the right) for the right (red) and left (blue) hand in the cases of local (continuous lines) and global (dashed lines) method.

active constraints and by avoiding the evaluation of the initial point at the each iteration. Moreover, a sub-optimal solution has been proposed to further reduce the complexity of the problem in the case of bimanual grasping tasks. Simulations have shown the effectiveness of the proposed technique.

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