

Online Dextrous-Hand Grasping Force Optimization with Dynamic Torque Constraints Selection

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Abstract—In this paper, a new algorithm for online grasping force optimization (GFO) of a dextrous robotic hand is presented. The GFO problem is cast in a convex optimization problem, considering also torque joint constraints. The proposed formulation allows to simplify the computational complexity of the problem by dynamically reducing the number of active torque constraints. Moreover, differently from other approaches, it does not require the evaluation of a new initial point at the beginning of each iteration. The effectiveness and the performance of the proposed method have been tested in a simulation case study where the hand manipulates a load with time-varying mass.

I. INTRODUCTION

The control of a multi-fingered hand involves several aspects which range from the synthesis of the optimal grasping contact points, to grasp planning, and grasp control. With respect to this last issue, the evaluation of the grasping forces able to guarantee the stability of the grasp and its feasibility, in reason of the external disturbances and of the task, represents a crucial problem. The complexity of the problem relies on the necessity of resolving online an optimization problem where both constraints and objective functions are nonlinear, and the grasp configuration or the load wrench may change with time (as an example, when pouring water from a bottle into a glass).

Notice that the force closure problem [1] and, more in general, the problem of computing the optimal grasp configurations, are not considered here because it is assumed that the contact points, which guarantees the force closure property, are assigned by the path planner.

The nonlinearity of the contact friction models (point contact with friction or soft-finger contact) represents the major obstacle to the solution of the optimal contact force distribution problem. Although a number of general and specialized theorems exist to address this problem, due to its computational complexity, the analysis and synthesis of frictional force-closure grasps has been initially studied by linearizing the friction cone constraints and then applying linear programming techniques [2], [3], [4]. The corresponding problems, however, are ill-conditioned. Nonlinear

programming techniques have been also investigated [5], but with results not suitable for real-time applications.

Starting from the observation that verifying the friction cone constraints is equivalent to testing the positive definiteness of certain symmetric matrices, in [6] the grasp force optimization has been formulated as a convex optimization problem on a Riemannian manifold with linear constraints. Several gradient flow type algorithms have been proposed to provide solutions suitable for real-time applications [7]; to reduce the complexity of the matrix inversion, the computation of the solution can be split into one on-line and one off-line phase and sparse matrix techniques can be adopted [8]. This technique has been employed and experientially tested with an impedance control approach addressing the regrasping problem for dextrous manipulation tasks [9].

In [10] the friction cone constraints have been formulated in terms of linear matrix inequalities (LMIs), and the grasping optimization problem is addressed as a convex optimization problem involving LMIs with the max-det function as objective function. This problem can be efficiently solved with the interior point algorithm for a small number of fingers. Moreover, joint torque limits can be considered and formulated in the same framework as LMIs.

A further improvement has been presented in [11], consisting in a new compact semidefinite representation of the friction cone constraints which allows a significant reduction of the dimension of the optimization problem. Moreover, an estimation technique and a recursion method for selecting the step size in the gradient algorithm are proposed, together with the proof of the quadratic convergence of the algorithm.

An interesting comparison of the methods developed in [6], [10], [11] is proposed in [12], [13], where a common framework has been developed to test the previous competing algorithms for grasping force optimization, and the issue of the step size selection for each method is addressed. Moreover, a new solution for the initial point problem, i.e. an initial solution that satisfies the friction cone constraints and the force equilibrium equation, has been provided.

The method proposed in [6] has the main disadvantage that it requires the on-line pseudo-inversion of a structurally constrained matrix whose dimension linearly increases with the number of fingers, of a factor that depends on the contact type. By adopting the frictional cone constraint matrix representation proposed in [11], the dimension of the problem decreases considerably so that the solution can be computed in real time. However, if torque limits constraints are considered, the complexity of the problem increases more than quadratically with the number of joints, making

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it unsuitable for real-time applications. Moreover, all the proposed solutions require, at each iteration, the evaluation of an initial point that satisfies the frictional cone constraints and the joint torque limits. The initial point can be computed with the method proposed in [12], but at the expense of a significant computational effort.

The algorithm proposed here, based on the compact formulation of [11] and on the solution of a convex optimization problem as in [7], allows to consider also torque joint constraints, with a minimum increase of computation complexity, compatible with real-time constraints. Moreover, the proposed iterative formulation fully exploits the results of the previous optimization step, without requiring the evaluation at each step of a new initial point. The feasibility and the effectiveness of the proposed method have been tested in a simulation scenario where a robotic hand is used to empty a bottle.

II. PROBLEM FORMULATION

Consider a multi-fingered robotic hand grasping an object with n contacts between the object and the fingertips, the links of the fingers and the palm. Denote the contact wrench of the grasp with $c = [c_1^T \dots c_n^T]^T \in \mathbb{R}^{nm}$, where $c_i \in \mathbb{R}^m$ is the wrench vector of the i -th contact with dimension m depending from the adopted contact model.

The grasping force optimization problem (GFO) consists in finding the set of contact wrenches balancing the generalized external force $h_e \in \mathbb{R}^6$ acting on the object (including object inertia and weight), which are feasible with respect to the kinematic structure of the hand and to the corresponding joint torque limits, and minimize the overall stress applied to the object, i.e., the internal forces. Moreover, to avoid the slippage of the fingers on the object surface, each contact wrench has to be confined within the friction cone.

The balance equation for the generalized forces applied to the object can be written in the form

$$h_e = Gc, \quad (1)$$

where $G \in \mathbb{R}^{6 \times nm}$ is the grasp map, which is full rank for force-closure grasps [1]. It is assumed that the contact point configurations ensuring the force-closure constraint are assigned at each time by the planning system.

Although several contact models can be used, the two usually adopted are the *point contact with friction* (PCWF) model and the *soft finger contact* (SFC) model.

In the PCWF case, the contact wrench has three degree-of-freedom ($m = 3$): the normal component $c_{i,z}$ to the object surface and the two components $c_{i,x}$, $c_{i,y}$ on the tangent plane. The friction constraint is represented by the law

$$\frac{1}{\mu_i} \sqrt{c_{i,x}^2 + c_{i,y}^2} \leq c_{i,z}, \quad (2)$$

where μ_i denotes the tangential friction coefficient at the i -th contact point.

In the SFC case, the contact wrench has an additional degree-of-freedom $c_{i,t}$ ($m = 4$), corresponding to the torsional component of the moment about the contact normal.

In this case, the friction constraint can be expressed in the form

$$\frac{1}{\mu_i} \sqrt{c_{i,x}^2 + c_{i,y}^2} + \frac{1}{\mu_{t,i}} |c_{i,t}| \leq c_{i,z}, \quad (3)$$

where $\mu_{t,i}$ denotes the torsion friction coefficient at the i -th contact point.

The balance equation for the torques applied to fingers joints of the hand can be written in the form

$$J^T(q)c + \tau_e = \tau, \quad (4)$$

where τ_e is the external torque, including gravity, Coriolis, centripetal and inertia effects at the fingers joints, τ is the torque provided by the actuators, and $J(q)$ is the $(nm \times l)$ hand Jacobian matrix, depending on the $(l$ -dimensional) vector q of the joint variables, being l the total number of the hand joints. For simplicity, it is assumed that $\mathcal{N}(J^T) = \emptyset$, meaning the absence of structurally dependent forces, namely, contact forces not caused by joint torques but depending on hand mechanics (see, e.g., [1]).

To ensure that the joint actuators are able to provide the required torques, a joint torque constraint must also be considered

$$\tau_L \leq \tau \leq \tau_U, \quad (5)$$

where τ_L and τ_U denote the lower and upper joint torque bound, respectively.

The simultaneous satisfaction of the force balance equation (1), with the friction constraints (2) and (3), and of the joint torque balance equation (4) with constraint (5), implies that the grasp is stable and feasible.

The GFO problem considered here consists in finding the optimal grasp wrench that minimize the internal forces acting on the object, under the above constraints. The internal forces are contact wrenches that satisfy the friction cone constraints and belong to the null space of the grasp matrix G . These wrenches c_{int} do not contribute to the balance equation (1), being $Gc_{int} = 0$, but are used to satisfy the friction cone constraints at the contact points.

III. GRASPING CONSTRAINTS AND SEMIDEFINITE PROGRAMMING

As shown in [11], the frictional inequalities (2) and (3) are equivalent to the positive definiteness of the block-diagonal matrix

$$F(c) = \text{diag}(F_1(c_1), \dots, F_n(c_n)) > 0, \quad (6)$$

where $F_i(c_i)$ is the symmetric (2×2) matrix

$$F_i(c_i) = \begin{bmatrix} c_{i,z} + \frac{c_{i,x}}{\mu_i} & \frac{c_{i,y}}{\mu_i} \\ \frac{c_{i,y}}{\mu_i} & c_{i,z} - \frac{c_{i,x}}{\mu_i} \end{bmatrix} \quad (7)$$

in the PCWF case, while it is the Hermitian (2×2) matrix

$$F_i(c_i) = \begin{bmatrix} c_{i,z} + \frac{c_{i,x}}{\sqrt{\mu_i}} & \frac{c_{i,y}}{\sqrt{\mu_i}} - j \frac{c_{i,t}}{\sqrt{\mu_{t,i}}} \\ \frac{c_{i,y}}{\sqrt{\mu_i}} + j \frac{c_{i,t}}{\sqrt{\mu_{t,i}}} & c_{i,z} - \frac{c_{i,x}}{\sqrt{\mu_i}} \end{bmatrix}, \quad (8)$$

in the SFC case.

Similarly, the torque limit constraint (5), in view of the torque balance equation (4), is equivalent to the positive definiteness of the diagonal matrix

$$\mathbf{T}(\mathbf{c}, \mathbf{q}, \boldsymbol{\tau}_e) = \text{diag}(\boldsymbol{\tau}_B) > 0, \quad (9)$$

where

$$\boldsymbol{\tau}_B = \begin{bmatrix} \boldsymbol{\tau}_{B,L} \\ \boldsymbol{\tau}_{B,H} \end{bmatrix} = \begin{bmatrix} \mathbf{J}^T(\mathbf{q})\mathbf{c} - \boldsymbol{\tau}_L + \boldsymbol{\tau}_e \\ -\mathbf{J}^T(\mathbf{q})\mathbf{c} + \boldsymbol{\tau}_H - \boldsymbol{\tau}_e \end{bmatrix} \quad (10)$$

contains the gaps of actuator torques from the lower ($\boldsymbol{\tau}_{B,L}$) and upper ($\boldsymbol{\tau}_{B,H}$) limit, respectively.

Hence, the simultaneous satisfaction of both frictional and joint torque constraints is equivalent to the positive definiteness of the linearly constrained block-diagonal matrix

$$\mathbf{P} = \text{diag}(\mathbf{F}, \mathbf{T}) > 0. \quad (11)$$

Notice that the elements of the matrices $\mathbf{F}$ and $\mathbf{T}$ are linearly dependent, because both depend on $\mathbf{c}$. Moreover, the force balance equation (1) and the torque balance equation (4) corresponds to linear constraints imposed on matrix $\mathbf{P}$.

By denoting with $\mathbf{c}(\mathbf{F})$ the contact wrench vector extracted from the frictional constraint matrix, with $\boldsymbol{\tau}_B(\mathbf{T})$ the vector composed by the diagonal elements of $\mathbf{T}$, and defining vector $\boldsymbol{\xi}(\mathbf{P}) = [\mathbf{c}(\mathbf{F})^T, \boldsymbol{\tau}_B(\mathbf{T})^T]^T$, the linear constraints on matrix $\mathbf{P}$ imposed by (1) and (4) can be represented in the following affine general form

$$\mathbf{A}\boldsymbol{\xi}(\mathbf{P}) = \mathbf{b} \quad (12)$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{A}_\tau \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} h_e \\ \boldsymbol{\tau}_L - \boldsymbol{\tau}_e \\ \boldsymbol{\tau}_H - \boldsymbol{\tau}_e \end{bmatrix}, \quad (13)$$

where

$$\mathbf{A}_\tau = \begin{bmatrix} \mathbf{J}(\mathbf{q})^T & -\mathbf{I} & \mathbf{0} \\ \mathbf{J}(\mathbf{q})^T & \mathbf{0} & \mathbf{I} \end{bmatrix}. \quad (14)$$

being $\mathbf{0}$ the null matrix and $\mathbf{I}$ the identity matrix of proper dimension.

The optimization procedure is based on the minimization of the cost function $\Phi(\mathbf{P}) : \mathcal{P}(r) \rightarrow \mathbb{R}$, being $\mathcal{P}(r)$ the set of positive definite symmetric ($r \times r$) matrices $\mathbf{P} = \mathbf{P}^T > 0$, $r = 2(n + l)$, defined as

$$\Phi(\mathbf{P}) = \text{tr}(\mathbf{W}_p \mathbf{P} + \mathbf{W}_b \mathbf{P}^{-1}), \quad (15)$$

where $\text{tr}(\cdot)$ denotes the trace operator, $\mathbf{W}_p$ and $\mathbf{W}_b$ are symmetric positive definite matrices. Notice that Φ is a strictly convex twice continuously differentiable function on $\mathcal{P}(r)$ and $\Phi(\mathbf{P}) \rightarrow +\infty$ for $\mathbf{P} \rightarrow \partial\mathcal{P}(r)$, being $\partial\mathcal{P}(r)$ the boundary of $\mathcal{P}(r)$.

In the case of diagonal weighting matrices, the term $\mathbf{W}_p \mathbf{P}$ weights the normal forces $\mathbf{c}_{i,z}$ at each contact point, i.e. the pressure forces on the object. Notice that the elements of $\mathbf{T}$ are irrelevant because the sum of the two joint torque constraints for joint i is constant and equal to $\tau_{H,i} - \tau_{L,i}$. Therefore, the linear weighting matrix is chosen equal to $\mathbf{W}_p = \text{diag}(w_p \mathbf{I}_{2n}, \mathbf{0}_{2l})$, with $w_p > 0$, considering only the diagonal elements of $\mathbf{F}$ in the first term of the cost function.

Obviously, if required, different weights can be used for each finger, e.g., to permit higher contact forces to some fingers.

The second term $\mathbf{W}_b \mathbf{P}^{-1}$ represents a barrier function, which goes to infinity when $\mathbf{P}$ tends to a singularity, i.e. when friction or torque limits are approached. The barrier weight matrix is also chosen diagonal $\mathbf{W}_b = \text{diag}(\mathbf{W}_{b,F}, \mathbf{W}_{b,T})$, with

$$\begin{aligned} \mathbf{W}_{b,F} &= w_{b,F} \text{diag}(\mu_1, \dots, \mu_n) \\ \mathbf{W}_{b,T} &= w_{b,T} \text{diag}(\tau_{H,1} - \tau_{L,1}, \dots, \tau_{H,l} - \tau_{L,l}, \\ &\quad \tau_{H,1} - \tau_{L,1}, \dots, \tau_{H,l} - \tau_{L,l}), \end{aligned} \quad (16)$$

being $w_{b,F} > 0$ and $w_{b,T} > 0$.

Hence, the minimization of the cost function (15) with the linear constraint (12), corresponds to the minimization of the normal contact wrench components applied to the object while satisfying the friction and torque constraints. This problem can be solved using the linearly constrained gradient flow approach on the smooth manifold of positive definite matrices presented in [14], and applied in [6] to a GFO problem. In particular, it is possible to prove that $\Phi(\mathbf{P})$ presents a unique minimum that can be reached through the linear constrained exponentially convergent gradient flow

$$\dot{\boldsymbol{\xi}}(\mathbf{P}) = \mathbf{Q}\boldsymbol{\xi}(\mathbf{P}^{-1}\mathbf{W}_b\mathbf{P}^{-1} - \mathbf{W}_p), \quad (17)$$

where $\mathbf{Q} = (\mathbf{I} - \mathbf{A}^\dagger \mathbf{A})$ is the linear projection operator onto the tangent space of $\mathbf{A}$, and $\mathbf{A}^\dagger = \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}$ is the pseudo-inverse of $\mathbf{A}$. Consequently, $\mathbf{A}\mathbf{Q} = \mathbf{0}$ and $\mathbf{A}\dot{\boldsymbol{\xi}}(\mathbf{P}) = \mathbf{0}$; hence, if the solution satisfies the constraint (12) at $t = 0$, it will satisfy the constraint for all $t > 0$.

A discrete-time version of (17) based on the Euler numerical integration algorithm is

$$\boldsymbol{\xi}(\mathbf{P}_{k+1}) = \boldsymbol{\xi}(\mathbf{P}_k) + \alpha_k \mathbf{Q}\boldsymbol{\xi}(\mathbf{P}_k^{-1}\mathbf{W}_b\mathbf{P}_k^{-1} - \mathbf{W}_p), \quad (18)$$

where the step-size α_k is chosen to ensure down hill steps. Notice that the choice of α_k strongly affects the performance of the optimization algorithm. A wrong choice could determine a very slow convergence or the break of the barrier. Several strategies have been proposed for the self-tuning of α_k at each iteration (see [12] for details). The sensitivity to the step-size choice can be reduced by adopting a Dikin-type recursive algorithm [7], [15], that leads to the discrete flow

$$\boldsymbol{\xi}(\mathbf{P}_{k+1}) = \boldsymbol{\xi}(\mathbf{P}_k) - \alpha_k \mathbf{Q} \frac{\boldsymbol{\xi}(\mathbf{P}_k^{-1}\mathbf{W}_b\mathbf{P}_k^{-1} - \mathbf{W}_p)}{\|\mathbf{P}_k^{-1}\mathbf{W}_b\mathbf{P}_k^{-1} - \mathbf{W}_p\|_{\mathbf{P}_k}}, \quad (19)$$

where $\|\mathbf{X}\|_Y = \text{tr}(\mathbf{Y}^{-1}\mathbf{X}\mathbf{Y}^{-1}\mathbf{X})$, and $0 \leq \alpha_k \leq 1$ can be evaluated with a bounded line search minimizing $\Phi(\mathbf{P}_{k+1})$.

IV. ALGORITHMIC IMPROVEMENTS

The online implementation of the proposed algorithm presents the disadvantage of requiring the inversion of a $(6 + 2l)$ square matrix $\mathbf{A}\mathbf{A}^T$ needed for the evaluation of $\mathbf{A}^\dagger$ at each iteration, also when the grasping configuration is unchanged, i.e. when $\mathbf{G}$ is constant, due to the variation of $\mathbf{J}(\mathbf{q})$. Moreover, at each iteration, the evaluation of a valid initial point is required for the correct initialization of the gradient flow algorithm.

In the following, some improvements are proposed to cope with the previous issues for an efficient recursive implementation of the proposed algorithm.

A. Affine Constraint Decomposition

Starting from the discrete version of the gradient flow (18), the following new formulation can be derived

$$\mathbf{c}_{k+1} = \mathbf{c}_k + \alpha_k \bar{\mathbf{Q}} \xi(\mathbf{P}^{-1}(\mathbf{c}_k) \mathbf{W}_b \mathbf{P}^{-1}(\mathbf{c}_k) - \mathbf{W}_p), \quad (20)$$

where $\bar{\mathbf{Q}} = (\mathbf{I} - \mathbf{G}^\dagger \mathbf{G})[\mathbf{I}_{nm} \mathbf{0}_{2l}](\mathbf{I} - \mathbf{A}_\tau^\dagger \mathbf{A}_\tau)$ is the result of the projection onto the tangent space of matrix $\mathbf{A}_\tau$ in (14), which guarantees the coherence of the elements of matrix $\mathbf{P}$, and of the subsequent projection onto the tangent space of the grasp matrix, ensuring the force balance constraint (1). Therefore, the evaluation of the inverse of a $6 + 2l$ matrix is decomposed into the evaluation of the inverse of two matrices of lower dimensions (6 and $2l$, respectively). Moreover, if the grasp configuration remains unchanged, the projector depending on $\mathbf{G}$ can be evaluated off-line.

A similar decomposition can be easily achieved for the gradient flow (19).

B. Dynamic Joint-Torque Constraints Selection

In the reasonable hypothesis that the solutions of the recursive algorithm evaluated at successive sampling times are quite close, the above problem can be further simplified observing that not all the joint torque constraints can be effective simultaneously. For example, if, for the current solution, the actuator of joint i provides a torque close to the upper bound $\tau_{H,i}$, the constraint on the lower bound $\tau_{L,i}$ can be deactivated at the next sampling time, the corresponding barrier term in the cost function being negligible. Similarly, if, for a grasp configuration, it is required a given contact force along a certain direction, it is reasonable to assume that the corresponding joint torques will not change sign at the next sampling time, and thus the constraints on the bounds of opposite sign can be deactivated.

Starting from this observation, the number of the joint torque constraints can be dynamically reduced from $2l$ to l at each sampling time, by using the distance of the torque evaluated at the previous sampling time from the lower and upper bounds as the criterion for selecting the constraint (the lower or the upper one) that needs to be activated. A further simplification can be achieved by activating a constraint only if the corresponding distance is lower than a threshold $\sigma_\tau > 0$.

C. Initial Point Self-Evaluation

A new technique for the on-line evaluation, at each sampling time, of the initial point—the initial solution $\mathbf{P}_0$ for the optimization gradient flow algorithm—is proposed here, based on the optimal solution at the previous sampling time. The quantities that can vary between successive sampling times are the hand configuration $\mathbf{q}$, the external torque $\boldsymbol{\tau}_e$, and the grasp map $\mathbf{G}$, while they are taken constant during the iterations of the optimization algorithm between two consecutive sampling times (optimization cycle).

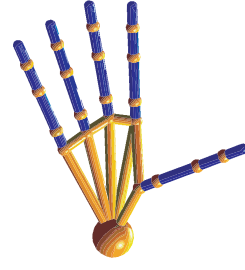


Fig. 1. DEXMART hand skeleton.

To avoid the evaluation of an initial point at each sampling time, the following approach is proposed. Initially, at time t_0 , the method proposed in [12] (or an equivalent one) is used to evaluate off-line a first valid initial solution, which is employed for the first optimization cycle. For the next sampling times t_k , the initial point is computed from the optimal solution $\mathbf{c}_{k-1}$ computed at the end of the previous optimization cycle, through the iterative algorithm

$$\begin{aligned} \bar{\mathbf{c}}_j &= (\mathbf{I} - \mathbf{G}_k^\dagger \mathbf{G}_k) \bar{\mathbf{c}}_{j-1} + \gamma_j \mathbf{G}_k^\dagger \mathbf{h}_{e,k} + (1 - \gamma_j) \mathbf{G}_{k-1}^\dagger \mathbf{h}_{e,k-1} \\ \bar{\boldsymbol{\tau}}_j &= \mathbf{J}^T (\gamma_j \mathbf{q}_k + (1 - \gamma_j) \mathbf{q}_{k-1}) \bar{\mathbf{c}}_j + \gamma_j \boldsymbol{\tau}_{e,k} + (1 - \gamma_j) \boldsymbol{\tau}_{e,k-1}, \end{aligned} \quad (21)$$

with initial condition $\bar{\mathbf{c}}_0 = \mathbf{c}_{k-1}$, where the subscript k is referred to the sampling time (i.e., to the current optimization cycle) while the subscript j and the variables with the bar are referred to the iterations within the cycle. The coefficient $\gamma_j \in (0, 1]$ is chosen at each iteration according to a monotone sequence, using a simple linear search algorithm, as the maximum value that does not produce invalid solutions ($\mathbf{P}_0 \leq 0$). In the worst case, γ_0 must be set to a value close to zero.

In detail, at each step of the optimization cycle, the first equation of (21) gradually modifies the external wrench component of the current solution until the full external wrench $\mathbf{h}_{e,k}$ is balanced (i.e., $\gamma_j = 1$). Obviously, the optimization cycle cannot be terminated until γ_j does not reach 1. If the solution evaluated at the previous sampling time ($\mathbf{c}_{k-1}$) is sufficiently far from the boundaries (the distance depends also from the weights assigned to $\mathbf{W}_b$), γ_0 can be set to 1 at the first iteration, and thus the initial point has the same internal wrench component of the previous optimal solution. On the other hand, when $\gamma_0 < 1$, the effect of the barrier function produces a new solution that, at each iteration of the optimization cycle, goes away from the boundaries; this guarantees that γ_j increases at each step, until $\gamma_j = 1$. The second equation is required to modify the joint torque with the same rationale of the first equation.

V. CASE STUDY

The proposed GFO algorithm has been tested in simulation using the DEXMART [16] anthropomorphic hand shown in Fig. 1, mounted on a 7-dof anthropomorphic arm.

It is assumed that the hand grasps a cylinder representing a bottle half filled with water (see Fig. 2) and that the task consists in pouring water by reorienting the bottle using

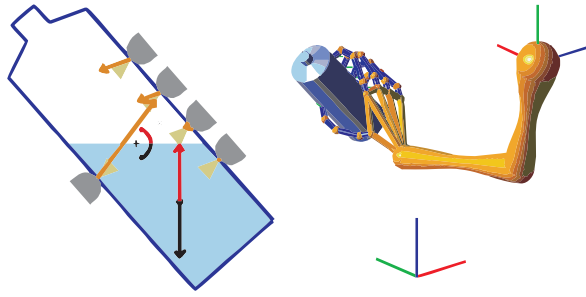


Fig. 2. Left: section of the bottle with graphical representation of the gravity force and torque (black arrows), of the resultant force and torque applied by the fingers (red arrows), of the optimal contact forces (orange arrows), and of the friction cones (yellow triangles); the intensity of the forces and torques are proportional to the arrow lengths. Right: arm and hand grasping the bottle.

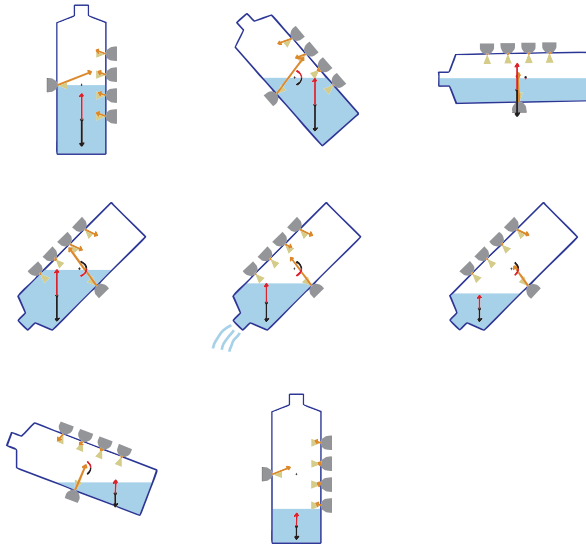


Fig. 3. Sequence of significant configurations of the bottle and of the forces during task execution.

only the robot wrist. The bottle is initially grasped with the main axis aligned to the vertical direction. The task can be decomposed into three steps:

- a rotation of 135 deg about the horizontal axis through the geometric center of the cylinder is commanded, using only the robot wrist;
- the hand is stopped while some water comes out from the bottle;
- the opposite rotation is commanded to the wrist, to put back the bottle in the initial pose.

A dynamic simulation has been performed using Matlab/Simulink, where the variation of the position of the center of mass of the water and that of its weight have been considered, by neglecting, for simplicity, the inertial effects. Figure 2 shows on the left a section of the bottle half filled with water. In the figure, the intensity of the gravity force is proportional to the black vertical arrow applied to the instantaneous center of mass, while the intensity of the gravity torque with respect to the center of the bottle is proportional to the black circular arrow. The red arrows represent the

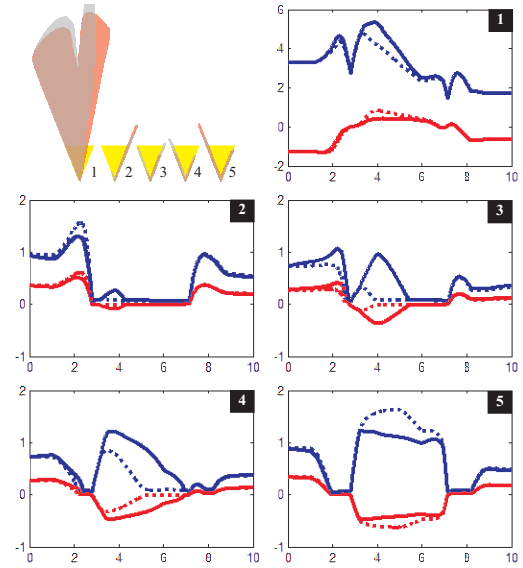


Fig. 4. Time histories of the contact forces ([N]) for the 5 fingers of the hand without (dotted lines) and with (continuous lines) torque constraints. Top left: Areas covered by the contact forces within the friction cones without (gray color) and with (red color) torque constraints; the friction cones are represented in yellow.

external force and torque balancing the gravity effects and resulting from the contact forces applied by the fingers, represented by orange arrows (of length proportional to the intensity of the force). The sections of the friction cones in the contact points are colored in yellow. A sequence of significant configurations of the bottle during task execution is shown in Fig. 3.

Two different simulations have been done. In the first simulation only the friction constraint is considered, without any constraint on the joint torque limits. In the second simulation, instead, different torque limits are set for the fingers, considering the thumb actuators 5 times stronger than the actuators of the other fingers (± 0.5 vs ± 0.1 Nm), like for human hand.

In Fig. 4 the time histories of the two significant contact force components for each finger are shown, represented in a frame fixed to the bottle and labeled from 1 to 5 from the thumb to the little finger. The dotted lines correspond to the first simulation, while the solid lines correspond to the second simulation. At the top left of the figure, the areas covered by the contact force vector of each contact point during the bottle motion is shown, in gray color for the first simulation and in red color for the second simulation.

The time history of the corresponding joint torques is shown in Fig. 5, where the dotted lines correspond to the first simulation, the solid lines correspond to the second simulation, and the gray horizontal lines represent the joint limits. The torques of the same finger are grouped together. Notice that there are 3 actuators for each fingers, being the last two joints of each finger coupled.

When only the friction constraints are imposed (first simulation), fingers 2 (index) and 5 (little finger) are loaded more than the others by the GFO algorithm during the bottle

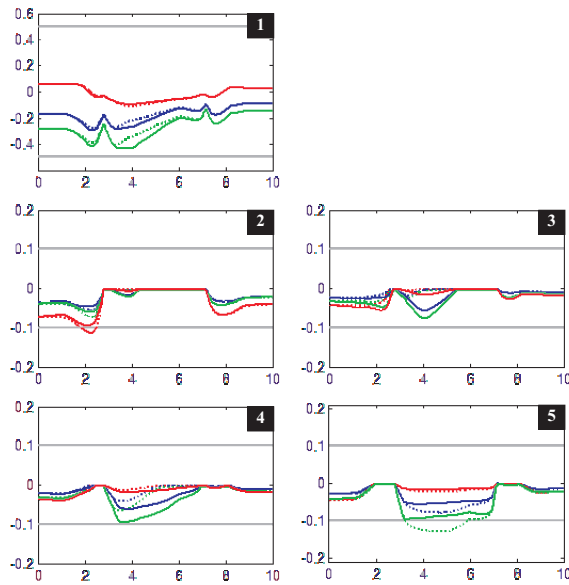


Fig. 5. Time history of the joint torques ([Nm]), for the 5 fingers of the hand without (dotted lines) and with (continuous lines) torque constraints. The gray horizontal lines represent the joint limits.

rotation, while fingers 3 and 4 remain almost inactive. This can be easily explained because fingers 2 and 5, in some crucial configurations, have moment arms with respect to the center of mass larger than the two central fingers, and are able to produce a large moment with a reduced contact force to balance the gravity moment. However, the joint torque limits are exceeded for both the fingers 2 and 5, resulting in an unfeasible grasp. When torque constraints are considered, fingers 3 and 4 are used more actively to sustain part of the load in charge to the adjacent fingers when the torque limits are approached, and thus all the torque constraints can be respected.

The benefits resulting from the adoption of the dynamic joint-torque constraints selection are showed in Fig. 6, where the time history of the computational time effort are represented on the left and the number of employed constraints are represented on the right. To remove the dependence from the employed hardware, all the considered cases are normalized with respect to the maximum value of the fully constrained case (black line) to the value 100. In particular, four different cases are compared: all constraints (black lines), $\sigma_\tau = 0$ (red lines), $\sigma_\tau = 0.5$ (green lines), $\sigma_\tau = 0.8$ (blue lines), and unconstrained (gray lines), where σ_τ is the threshold for the activation of the joint torque constraints. The achieved reduction of the mean of the computational time varies between a minimum of about 30% for $\sigma_\tau = 0$ to a maximum of about 90% for $\sigma_\tau = 0.8$.

VI. CONCLUSION

A new algorithm for online dextrous-hand grasping force optimization has been presented in this paper, considering also joint torque limits. The computational charge of the algorithm has been reduced by adopting an iterative formulation based on a dynamic set of active constraints and by

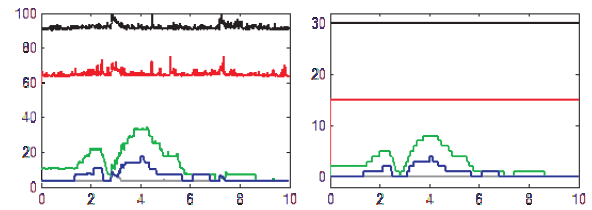


Fig. 6. Time history of the normalized computational-time effort (left) and of the number of employed joint torque constraints (right) for the cases of all constraints (black), $\sigma_\tau = 0$ (red), $\sigma_\tau = 0.5$ (green), $\sigma_\tau = 0.8$ (blue), and unconstrained (gray).

avoiding the evaluation of the initial point at the beginning of each iteration, as required by alternative approaches. A simulation case study has been proposed to show the feasibility and the effectiveness of the proposed technique.

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