

A Stable Force/Position Controller for Robot Manipulators

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Abstract

A force/position controller for robot manipulators in contact with an elastically compliant surface is presented in this work. The controller consists of a PD action on the position loop, a PI action on the force loop, together with gravity compensation and desired contact force feedforward. Asymptotic stability of the system in the neighborhood of the equilibrium state is proven via the classical Lyapunov method with LaSalle invariance set theorem. A modification of the Lyapunov function leads to deriving an exponential stability result. Numerical case studies are developed.

1. Introduction

For typical robotic tasks that require interaction with the environment, contact forces must properly be handled by the robot controller. In such cases, a pure motion controller usually gives degraded performance and can even cause instability.

If force sensor information is not available for control purposes, one can assign a suitable dynamic behavior between position and force variables at the contact (e.g. impedance control) [1,2].

On the other hand, several schemes can be devised which attempt to control both end-effector position and contact force by embedding force measurements in the controller. Hybrid control is perhaps the most widely adopted strategy to force/position control of robot manipulators [3-5]. The basic idea is the possibility to choose whether to control position or force along each task space direction through the use of proper selection matrices. Stability of hybrid control was addressed in [6].

A conceptually different approach to force/position control of robot manipulators is the parallel control strategy [7]. As opposed to the hybrid control strategy, both force and position variables are used along the same task space direction without any selection mechanism. The effectiveness of the scheme is ensured by the dominance of the force control loop over the position control loop along the task directions where interaction occurs. This makes the scheme suitable to manage contacts with unstructured environment and unplanned collisions, which are known to represent a drawback for hybrid controllers. Extensive description of the parallel approach and performance of a control scheme with full dynamic compensation in the case of contact with an elastically compliant frictionless surface can be found in [8].

In view of real-time implementation, a new parallel control scheme was recently proposed [9] which is based on simple position PD control + gravity compensation

+ desired force feedforward + force PI control. A preliminary analysis, inspired by the work in [10], showed asymptotic stability of the system around an equilibrium state but was based on a too restrictive assumption.

This work presents a complete proof of asymptotic stability based on the Lyapunov direct method with use of LaSalle invariance set theorem; only boundedness of the initial error is required. It is shown that, for given force and position set points, the force error is asymptotically driven to zero at the expense of a steady-state position error.

The Lyapunov function is further modified to prove exponential stability of the scheme yielding a new set of conditions to be satisfied for the feedback gains.

The proposed control scheme is tested in simulation on an industrial robot. The numerical case study confirms the results anticipated in theory.

2. Modeling

The dynamic model of a robot manipulator constrained by the environment can be effectively written in the operational space [4] in the form

$$B(x)\ddot{x} + C(x, \dot{x})\dot{x} + g(x) = u - f, \quad (1)$$

where x is the $(m \times 1)$ vector of operational variables (usually end-effector location), B is the $(m \times m)$ inertia matrix, $C\dot{x}$ is the $(m \times 1)$ vector of Coriolis and centrifugal generalized forces, g is the $(m \times 1)$ vector of gravitational generalized forces, u is the $(m \times 1)$ vector of driving generalized forces, and f is the $(m \times 1)$ vector of contact generalized forces exerted by the manipulator on the environment; all operational space quantities are expressed in a common reference frame.

When m is equal to the number of joints n and the manipulator moves in a singularity-free region of the workspace, the vector of operational variables constitutes a set of Lagrangian generalized coordinates and B assumes the meaning of a true inertia matrix. Instead, in the case of kinematically redundant manipulators ($m < n$), B is only a pseudo inertia matrix [4].

The $(n \times 1)$ vector τ of joint actuating generalized forces is computed as

$$\tau = J^T u, \quad (2)$$

where J is the $(m \times n)$ manipulator Jacobian matrix.

Two notable properties of the dynamic model (1) can be established:

1. The matrix B is symmetric and positive definite. If λ_m (λ_M) denotes the minimum (maximum) eigenvalue of B , then

$$0 < \lambda_m I \leq B \leq \lambda_M I, \quad (3)$$

where I is the $(m \times m)$ identity matrix; in the case of all revolute joints, it is $\lambda_M < \infty$ [11].

2. There exists a choice of the matrix C such that the matrix

$$S(\mathbf{x}, \dot{\mathbf{x}}) = \dot{B}(\mathbf{x}) - 2C(\mathbf{x}, \dot{\mathbf{x}}) \quad (4)$$

is skew-symmetric [12]. This also implies that

$$\dot{B}(\mathbf{x}) = C(\mathbf{x}, \dot{\mathbf{x}}) + C^T(\mathbf{x}, \dot{\mathbf{x}}). \quad (5)$$

Further, the matrix C is upper-bounded in $\mathbf{x}$ and linear in $\dot{\mathbf{x}}$ [13]; hence, a constant $k_C > 0$ exists such that

$$\|C(\mathbf{x}, \dot{\mathbf{x}})\| \leq k_C \|\dot{\mathbf{x}}\|. \quad (6)$$

For the purpose of the present work, the attention is restricted to the case of non-redundant non-singular manipulators with $m = n = 3$, i.e. only translational motion and force components are considered. Then, $\mathbf{x}$ denotes the end-effector position.

Accurate modeling of the contact between the manipulator and the environment is usually difficult to obtain in analytic form, due to the complexity of the physical phenomena involved during the interaction. It is then reasonable to resort to a simple but significant model, relying on the robustness of the control system in order to absorb the effects of inaccurate modeling.

The case of a rigid frictionless and elastically compliant plane is analyzed [7,8]. The model of the contact force takes on the simple form

$$\mathbf{f} = K(\mathbf{x} - \mathbf{x}_0), \quad (7)$$

where $\mathbf{x}$ is the position of the contact point, $\mathbf{x}_0$ is a point of the plane at rest, and K is the (3×3) constant symmetric *stiffness matrix* that establishes a linear mapping between $(\mathbf{x} - \mathbf{x}_0)$ and $\mathbf{f}$; notice that Eq. (7) holds only when the manipulator is in contact with the environment and all quantities are expressed in the common reference frame.

If $\mathbf{n}$ denotes the unit vector along the normal to the contact plane, the matrix K can be decomposed as [9]

$$K = k \mathbf{n} \mathbf{n}^T \quad (8)$$

where $k > 0$ is the stiffness coefficient.

3. Force/Position Controller

A typical task for a robot manipulator in contact with the environment can be prescribed in terms of a position set point $\mathbf{x}_d$ and a force set point $\mathbf{f}_d$. It can be recognized that simultaneous achievement of both set points is not guaranteed to be compatible with the task geometry.

A viable strategy is to adopt the parallel control approach [7]; this is especially effective in the case of inaccurate contact modeling. The key feature is to have a force control loop working in parallel to a position control loop along each task space direction. The logical conflict between the two loops is managed by imposing dominance of the force control action over the position one. The potential offered by this technique compared to conventional controllers also using force feedback sensory information is extensively discussed in [8].

A force/position parallel controller for the system (1) was proposed in [9], based on position *PD* control + gravity compensation + desired force feedforward + force *PI* control, i.e.

$$\mathbf{u} = -K_D \dot{\mathbf{x}} + K_P \Delta \mathbf{x} + \mathbf{g}(\mathbf{x}) + \mathbf{f}_d + K_F \Delta \mathbf{f} + K_I \int_0^t \Delta \mathbf{f} d\sigma, \quad (9)$$

where $\Delta \mathbf{x} = \mathbf{x}_d - \mathbf{x}$ is the position error, $\Delta \mathbf{f} = \mathbf{f}_d - \mathbf{f}$ is the force error, K_P, K_D, K_F, K_I are suitable gain matrices. These matrices are typically chosen as scalar matrices, i.e. $K_P = k_P I, K_D = k_D I, K_F = k_F I, K_I = k_I I$, with $k_P, k_D, k_F, k_I > 0$. It is important to remark that no exact knowledge of the stiffness matrix K is required by the control (9).

Looking at the properties of the elastic contact model (7), the only possibility to obtain a null force error is to assign a set point $\mathbf{f}_d \in \mathcal{R}(K)$. On the other hand, this is consistent with the fact that the considered environment can generate reaction forces only along the direction of $\mathbf{n}$. If no information about environment geometry is available, i.e. $\mathbf{n}$ is unknown, the null vector can be assigned for $\mathbf{f}_d$ that is anyhow in the range space of any matrix K . Thus, in the remainder, it is assumed that $\mathbf{f}_d \in \mathcal{R}(K)$. Analogously, it can be recognized that null position errors can be obtained only in the contact plane, while the component of $\mathbf{x}$ along $\mathbf{n}$ has to accommodate the force requirement specified by $\mathbf{f}_d$; thus, $\mathbf{x}_d$ can be freely reached only in $\mathcal{N}(K)$.

As a further assumption, it is supposed that the contact between the manipulator and the environment is not lost after the impact.

As demonstrated in [9], an equilibrium point for system (2) under control (9) is

$$\mathbf{x}_\infty = K^-(\mathbf{f}_d + K\mathbf{x}_0) + (I - K^-K)\mathbf{x}_d \quad (10)$$

$$\mathbf{f}_\infty = K(\mathbf{x}_\infty - \mathbf{x}_0) = \mathbf{f}_d. \quad (11)$$

The matrix K^- in (10) indicates a generalized inverse of the matrix K ; in view of (8), K^- can be written in the simple form

$$K^- = (1/k) \mathbf{n} \mathbf{n}^T. \quad (12)$$

4. Stability

Asymptotic stability of the equilibrium (10,11) was proven in [9] under a somewhat restricting assumption. In the following, a complete proof is given that leads to a set of simple conditions for the feedback gains.

Define

$$\mathbf{e} = \mathbf{x}_\infty - \mathbf{x} \quad (13)$$

which, by virtue of (7,10), can be also written as

$$\mathbf{e} = (I - K^-K)\Delta \mathbf{x} + K^- \Delta \mathbf{f} = \Delta \mathbf{x} + K_P^{-1} \mathbf{d}, \quad (14)$$

where

$$\mathbf{d} = K_P K^- (\mathbf{f}_d + K(\mathbf{x}_0 - \mathbf{x}_d)) \quad (15)$$

is a constant vector. For later use, notice that

$$K^- K \mathbf{e} = K^- \Delta \mathbf{f}. \quad (16)$$

From (13) it is

$$\dot{e} = -\dot{x}. \quad (17)$$

Further, define

$$s = n^T \left(\int_0^t K^- \Delta f d\sigma - K_I^{-1} K^- d \right). \quad (18)$$

Deriving (18) with respect to time and accounting for (16) yields

$$\dot{s} = n^T e. \quad (19)$$

At this point, consider the (7×1) augmented state vector

$$z = (\dot{x} \quad e \quad s)^T. \quad (20)$$

The augmented system described by Eqs. (1,17,19) under control (9) can be written in the standard compact form:

$$\dot{z} = Fz, \quad (21)$$

where

$$F = \begin{pmatrix} -B^{-1}(C + k_D I) & B^{-1}(k_P I + k'_F k n n^T) & k_I k B^{-1} n \\ -I & O & 0 \\ 0^T & n^T & 0 \end{pmatrix} \quad (22)$$

with $k'_F = 1 + k_F$; O denotes the (3×3) null matrix and 0 the (3×1) null vector. Notice that some handy reductions, using the structural properties of K in Eqs. (8,13) and the definition of s in (18), have been performed to derive (22).

The following result can be stated :

Theorem 1. *If the assumption*

$$\|e(0)\| \leq \Phi < \infty \quad (23)$$

holds, then there exists a choice of feedback gains k_P , k_D , k_F , k_I that makes the origin of the state space for system (21,22) asymptotically stable. ■

Proof. Consider the Lyapunov function candidate

$$V = \frac{1}{2} z^T P z, \quad (24)$$

where

$$P = \begin{pmatrix} B & -\rho B & 0 \\ -\rho B & (k_P + \rho k_D)I + k'_F k n n^T & k_I k n \\ 0^T & k_I k n^T & \rho k_I k \end{pmatrix} \quad (25)$$

with $\rho > 0$. Computing the time derivative of V along the trajectories of system (21,22) gives

$$\dot{V} = -\dot{x}^T (k_D I - \rho B) \dot{x} - \rho e^T C^T \dot{x} - e^T (\rho k_P I + (\rho k'_F - k_I) k n n^T) e, \quad (26)$$

in which the skew-symmetry of the matrix (4) and Eq. (5) have been conveniently exploited.

By virtue of assumption (23) and Eq. (6), the term $-\rho e^T C^T \dot{x}$ in (26) can be upper-bounded as

$$-\rho e^T C^T \dot{x} \leq \rho \Phi k_C \|\dot{x}\|^2. \quad (27)$$

By accounting for (3,19,27), function (26) can be upper-bounded as

$$\dot{V} \leq -(k_D - \rho \lambda_M - \rho \Phi k_C) \|\dot{x}\|^2 - \rho k_P \|e\|^2 - k(\rho k'_F - k_I) \dot{s}^2. \quad (28)$$

On the other hand, the function candidate (24,25), accounting for (19), can be written as

$$V = \frac{1}{2} \dot{x}^T B \dot{x} - \rho \dot{x}^T B e + \frac{1}{2} (k_P + \rho k_D) e^T e + \frac{1}{2} k'_F k \dot{s}^2 + k_I k s \dot{s} + \frac{\rho}{2} k_I k s^2 \quad (29)$$

which, by using (3), can be lower-bounded as

$$V \geq \frac{1}{2} \begin{pmatrix} \dot{x}^T B & e^T \end{pmatrix} \begin{pmatrix} (1/\lambda_M)I & -\rho I \\ -\rho I & (k_P + \rho k_D)I \end{pmatrix} \begin{pmatrix} B \dot{x} \\ e \end{pmatrix} + \frac{k}{2} \begin{pmatrix} \dot{s} & s \end{pmatrix} \begin{pmatrix} k'_F & k_I \\ k_I & \rho k_I \end{pmatrix} \begin{pmatrix} \dot{s} \\ s \end{pmatrix}. \quad (30)$$

Eqs. (28,30) reveal that there exists a choice of k_P , k_D , k_F , k_I , and ρ that makes V positive definite and $\dot{V}$ negative semidefinite. In fact, Eq. (30) gives

$$k_P + \rho k_D > \rho^2 \lambda_M \quad (31)$$

$$\rho k'_F > k_I, \quad (32)$$

while Eq. (28) gives

$$k_D > \rho(\lambda_M + \Phi k_C) \quad (33)$$

plus (32) again. Observing that condition (33) implies (31) for $\dot{x} \neq 0$, it can be concluded that only conditions (32,33) must be satisfied.

Since $\dot{V}$ is negative semidefinite, the inequality $\dot{V} \leq 0$ must be further analyzed to prove asymptotic stability. In particular, it is $\dot{V} < 0$, $\forall \dot{x} \neq 0, e \neq 0, \dot{s} \neq 0$, while $\dot{V} = 0$ implies $\dot{x} = 0, e = 0, \dot{s} = 0$. From the first three equations of (21,22) it can be seen that $\dot{x}$ vanishes only if $k_I k s B^{-1} n = 0$; this implies $s = 0$, too.

In sum, the only equilibrium trajectory satisfying $\dot{V} = 0$ is $z = (0^T \ 0^T \ 0)^T$. Hence, due to LaSalle invariance set theorem, this state is asymptotically stable. ■

About the feedback gains, notice that k_P is not involved in conditions (32,33) and then is available to meet further design requirements. Remarkably ρ is a free parameter that is *not* used in the control law (9), and then allows an opportune choice of the feedback gains k_D , k_F , k_I . Also, by increasing k_D , a larger value of Φ can be tolerated.

Theorem 1 provides design guidelines to ensure asymptotic stability of system (21,22). A further result can be established to give an estimate of the rate of convergence.

Theorem 2. *If the assumption*

$$\|z(0)\| \leq \Theta < \infty \quad (34)$$

holds, then there exists a choice of feedback gains k_P , k_D , k_F , k_I that makes the origin of the state space for system (21,22) exponentially stable.

Proof. Consider the Lyapunov function candidate

$$W = \frac{1}{2} \mathbf{z}^T \mathbf{Q} \mathbf{z}, \quad (35)$$

where

$$\mathbf{Q} = \begin{pmatrix} \mathbf{B} & -\beta \mathbf{B} \\ -\beta \mathbf{B} & (k_P + \beta k_D) \mathbf{I} + k'_F k \mathbf{n} \mathbf{n}^T \\ -\gamma \mathbf{n}^T \mathbf{B} & (k_I k + \gamma k_D) \mathbf{n}^T \\ & -\gamma \mathbf{B} \mathbf{n} \\ & (k_I k + \gamma k_D) \mathbf{n} \\ & \beta k_I k + \gamma(k_P + k'_F k) \end{pmatrix}, \quad (36)$$

with $\beta, \gamma > 0$. Computing the time derivative of W along the trajectories of system (21,22) gives

$$\begin{aligned} \dot{W} = & -\dot{\mathbf{x}}^T (k_D \mathbf{I} - \beta \mathbf{B}) \dot{\mathbf{x}} \\ & - \mathbf{e}^T (\beta k_P \mathbf{I} + (\beta k'_F k - k_I k - \gamma k_D) \mathbf{n} \mathbf{n}^T) \mathbf{e} \\ & - \gamma k_I k s^2 - \gamma \dot{\mathbf{x}}^T \mathbf{B} \mathbf{n} \mathbf{n}^T \mathbf{e} - (\beta \mathbf{e}^T + \gamma s \mathbf{n}^T) \mathbf{C}^T \dot{\mathbf{x}}, \end{aligned} \quad (37)$$

in which the skew-symmetry of the matrix (4) and Eq. (5) have been conveniently exploited.

The function candidate (35,36), accounting for (19) and (3), can be lower-bounded as

$$\begin{aligned} W \geq & \frac{1}{2} (\dot{\mathbf{x}}^T \mathbf{B} \quad \mathbf{e}^T) \begin{pmatrix} (1/2\lambda_M) \mathbf{I} & -\beta \mathbf{I} \\ -\beta \mathbf{I} & (k_P + \beta k_D) \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{B} \dot{\mathbf{x}} \\ \mathbf{e} \end{pmatrix} \\ & + \frac{1}{2} (\dot{\mathbf{x}}^T \mathbf{B} \quad s \mathbf{n}^T) \begin{pmatrix} (1/2\lambda_M) \mathbf{I} & -\gamma \mathbf{I} \\ -\gamma \mathbf{I} & \gamma(k_P + k'_F k) \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{B} \dot{\mathbf{x}} \\ s \mathbf{n} \end{pmatrix} \\ & + \frac{1}{2} \begin{pmatrix} \dot{s} & s \end{pmatrix} \begin{pmatrix} k'_F k & k_I k + \gamma k_D \\ k_I k + \gamma k_D & \beta k_I k \end{pmatrix} \begin{pmatrix} \dot{s} \\ s \end{pmatrix}. \end{aligned} \quad (38)$$

Eq. (38) reveals that there exists a choice of k_P , k_D , k_F , k_I , β , and γ that makes W positive definite, provided that:

$$k_P + \beta k_D > 2\beta^2 \lambda_M \quad (39)$$

$$k_P + k'_F k > 2\gamma \lambda_M \quad (40)$$

$$\beta k'_F k_I k^2 > (k_I k + \gamma k_D)^2. \quad (41)$$

On the other hand, the function candidate (35,36), accounting for (3,19), and the inequalities

$$-\dot{\mathbf{x}}^T \mathbf{B} \mathbf{e} \leq \frac{\lambda_M}{2} \left(\frac{\|\dot{\mathbf{x}}\|^2}{\eta_1} + \eta_1 \|\mathbf{e}\|^2 \right) \quad (42)$$

$$-s \dot{\mathbf{x}}^T \mathbf{B} \mathbf{n} \leq \frac{\lambda_M}{2} \left(\frac{\|\dot{\mathbf{x}}\|^2}{\eta_2} + \eta_2 s^2 \right) \quad (43)$$

$$\dot{s} s \leq \frac{1}{2} \left(\frac{\dot{s}^2}{\eta_3} + \eta_3 s^2 \right) \quad (44)$$

with $\eta_1, \eta_2, \eta_3 > 0$, can be upper-bounded as

$$\begin{aligned} W \leq & H_1 = \\ & \frac{1}{2} \left(1 + \frac{\beta}{\eta_1} + \frac{\gamma}{\eta_2} \right) \lambda_M \|\dot{\mathbf{x}}\|^2 + \frac{1}{2} (k_P + \beta k_D + \beta \eta_1 \lambda_M) \|\mathbf{e}\|^2 \\ & + \frac{1}{2} (\beta k_I k + \gamma(k_P + k'_F k) + \eta_3(k_I k + \gamma k_D) + \gamma \eta_2 \lambda_M) s^2 \\ & + \frac{1}{2} (k'_F k + \frac{1}{\eta_3} (k_I k + \gamma k_D)) \dot{s}^2. \end{aligned} \quad (45)$$

By virtue of assumption (34) and Eq. (6), the last term in (37) can be upper-bounded as

$$-(\beta \mathbf{e}^T + \gamma s \mathbf{n}^T) \mathbf{C}^T \dot{\mathbf{x}} \leq (\beta + \gamma) \Theta k_C \|\dot{\mathbf{x}}\|^2. \quad (46)$$

Taking into account (3,19), and the inequality

$$-\dot{\mathbf{x}}^T \mathbf{B} \mathbf{n} \mathbf{n}^T \mathbf{e} \leq \frac{\lambda_M}{2} \left(\frac{\|\dot{\mathbf{x}}\|^2}{\eta_4} + \eta_4 \|\mathbf{e}\|^2 \right) \quad (47)$$

with $\eta_4 > 0$, the function (37) can be upper-bounded as

$$\begin{aligned} \dot{W} \leq & -H_2 = -(k_D - \beta \lambda_M - \gamma \frac{\lambda_M}{2\eta_4} - (\beta + \gamma) \Theta k_C) \|\dot{\mathbf{x}}\|^2 \\ & - (\beta k_P - \gamma \frac{\eta_4 \lambda_M}{2}) \|\mathbf{e}\|^2 - \gamma k_I k s^2 - (\beta k'_F k - k_I k - \gamma k_D) \dot{s}^2. \end{aligned} \quad (48)$$

By comparison of (48) with (45), it can be recognized that if the following inequalities are satisfied for some $\alpha > 0$:

$$k_D - \beta \lambda_M - \gamma \frac{\lambda_M}{2\eta_4} - (\beta + \gamma) \Theta k_C \geq \quad (49)$$

$$\frac{\alpha}{2} \left(1 + \frac{\beta}{\eta_1} + \frac{\gamma}{\eta_2} \right) \lambda_M$$

$$\beta k_P - \gamma \frac{\eta_4 \lambda_M}{2} \geq \frac{\alpha}{2} (k_P + \beta k_D + \beta \eta_1 \lambda_M) \quad (50)$$

$$\gamma k_I k \geq \quad (51)$$

$$\frac{\alpha}{2} (\beta k_I k + \gamma(k_P + k'_F k) + \eta_3(k_I k + \gamma k_D) + \gamma \eta_2 \lambda_M)$$

$$\beta k'_F k - k_I k - \gamma k_D \geq \frac{\alpha}{2} (k'_F k + \frac{1}{\eta_3} (k_I k + \gamma k_D)), \quad (52)$$

the chain of inequalities holds

$$\dot{W} \leq -H_2 \leq -\alpha H_1 \leq -\alpha W, \quad (53)$$

and then

$$W(t) \leq W(0) \exp(-\alpha t). \quad (54)$$

For a given Θ , it is possible to choose the gains k_P , k_D , k_F , k_I and the parameters β , γ , η_1 , η_2 , η_3 , η_4 so that the relations (39-41) and (49-52) are satisfied for some $\alpha > 0$. By virtue of (54), exponential stability of system (21,22) at the origin of the state space is obtained. Further, α allows to compute a lower bound on the convergence rate toward the equilibrium state in the ball of radius Θ .

About the feedback gains, notice that now k_P is involved in conditions (39,40,50,51). However, by means of the free parameters β , γ , η_1 , η_2 , η_3 , η_4 , which are not used in the control law (9), it is possible to find a set of feedback gains k_P , k_D , k_F , k_I that guarantee exponential stability.

5. Case Study

The proposed force/position controller was tested in a case study on the industrial robot COMAU SMART 6.10R. Only the first three joints were considered, constituting a typical elbow manipulator geometry with zero shoulder offsets. The complete numerical data for

the robot parameters can be found in [14]. Simulations were run in MATLAB on a PC-486/33.

A step motion from $\mathbf{x} = (1.100 \ 0 \ 0)^T$ [m] to the set point $\mathbf{x}_d = (1.120 \ 0 \ 0)^T$ [m] was commanded to the manipulator's tip. The set point $\mathbf{f}_d = 0$ was assigned for the tip force. The geometry of the contact plane is characterized by $\mathbf{n} = (1 \ 0 \ 0)^T$ and $\mathbf{x}_0 = (1.115 \ 0 \ 0)^T$ [m]; different values for the stiffness coefficient k were considered. It can be recognized that an unexpected impact occurs along the normal direction to the plane at a distance of 0.005 m from the target point $\mathbf{x}_d$. For simplicity, the time scale in the simulations is reset ($t = 0$) at the instant of the impact; further, the sole $\mathbf{x}$ -component of position and force vectors are reported, in view of the particular task geometry.

The gains in (9) were set to $k_P = 10^5 \text{ N m}^{-1}$, $k_D = 10^4 \text{ N s m}^{-1}$, $k_F = 4$, $k_I = 55 \text{ s}^{-1}$. On one hand, k_P and k_D were chosen so as to guarantee a well-damped behavior for the unconstrained motion of the manipulator in a large region of the workspace. On the other hand, k_F and k_I were chosen so as to achieve a satisfactory behavior during the constrained motion with an estimate of the stiffness coefficient of $k = 10^5 \text{ N m}^{-1}$. With the above values, the design conditions (32,33) were satisfied with some $\rho > 0$ for available estimates of λ_M and k_C .

In the following, the numerical results of two simulation runs with a sampling time of 2 ms are presented.

In the first run, it is $k = 10^5 \text{ N m}^{-1}$. For the purpose of comparison, both the full force/position controller and the pure position controller ($k_F = 0$, $k_I = 0$) were used. It can be seen that the equilibrium position $\mathbf{x}_\infty (= \mathbf{x}_0$ in this case) is reached (Fig. 1) and the contact force is null at steady-state (Fig. 2), so as desired. Notice also that, without force feedback, finite steady-state errors occur both for position and force. Figure 3 reports the time history of the Lyapunov function $W(t)$ in (35,36), together with the function $W(0)\exp(-\alpha t)$; the value of α was computed via the MATLAB optimization function *CONSTR* applied to the set of conditions (39-41,49-52) with free parameters β , γ , η_1 , η_2 , η_3 , η_4 . It is easy to recognize the exponential stability result established by (54).

In the second run, the robustness of the proposed controller was tested by changing the stiffness coefficient into $k = 10^6 \text{ N m}^{-1}$ but leaving the same feedback gains as above. The results of Figs. 4,5 demonstrate that satisfactory performance is obtained even when tuning of the force feedback gains was done by underestimating the actual stiffness of the environment.

6. Conclusions

A stable force/position controller for robot manipulators in contact with an elastically compliant surface was presented in this work. The controller consists of a PD action on the position loop, a PI action on the force loop, together with gravity compensation and desired contact force feedforward. Differently from most previous schemes, this force/position controller does not require explicit knowledge of the environment and then allows handling of unplanned collisions; the equilibrium

state of the system automatically fits the contact geometry and is stable under the proposed controller.

A case study was developed for an industrial elbow manipulator, whose tip experiences an unexpected impact with the environment on the way toward the target position. The numerical results of two sets of simulations confirmed the theoretical derivation. Exponential stability was verified and robustness of the scheme to imprecise estimate of contact stiffness was successfully tested.

Future work will be devoted to investigate the stability of the scheme in the case of imperfect compensation of the gravity term, by possibly resorting to an adaptation mechanism on the system state.

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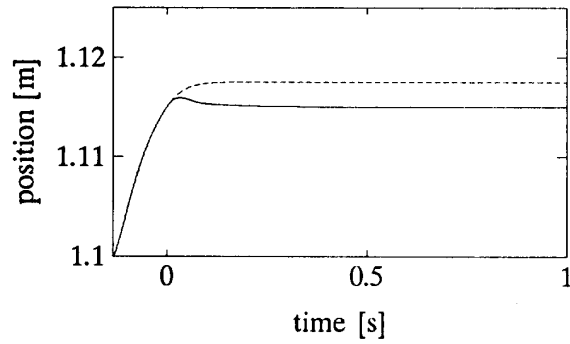


Fig. 1. Time history of the tip position (*solid*: force/position controller, *dashed*: position controller).

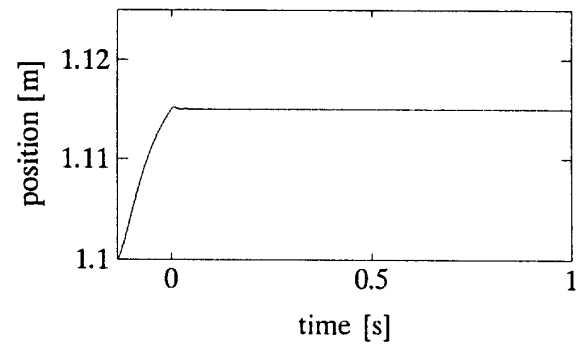


Fig. 4. Time history of the tip position with imprecise stiffness estimate.

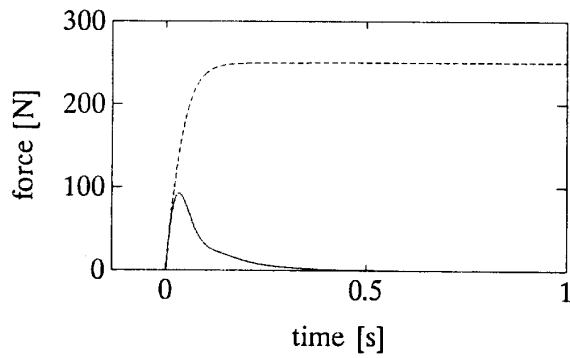


Fig. 2. Time history of the contact force (*solid*: force/position controller, *dashed*: position controller).

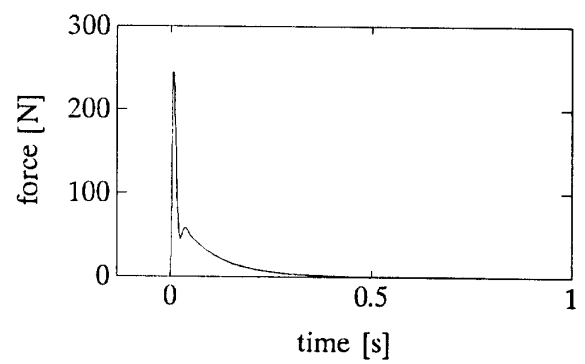


Fig. 5. Time history of the contact force with imprecise stiffness estimate.

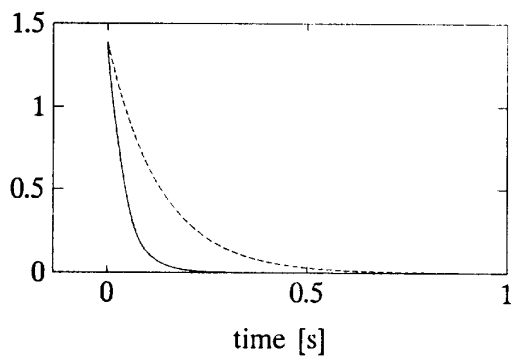


Fig. 3. Time history of the Lyapunov function (40,41) (*solid*) and its bounding exponential function (*dashed*).