

A LYAPUNOV-STABLE ADAPTIVE SCHEME FOR FORCE REGULATION AND MOTION CONTROL OF ROBOT MANIPULATORS

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Abstract. An adaptive force/position controller for robot manipulators in contact with a compliant surface is presented in this paper. The control law is designed in the task space and contains a nonlinear model-based term and a linear compensator action. This is obtained as a linear combination of the position error, the velocity error and the integral of the force error. The scheme is adaptive with respect to the dynamic parameters of the model of the robot manipulator. By using the classical Lyapunov method it is demonstrated that the proposed control law ensures tracking of the unconstrained components of the desired end-effector trajectory with regulation of the desired contact force along the constrained direction. Numerical case studies are developed for an industrial robot manipulator.

Keywords. Robots; force control; position control; stability; Lyapunov function; adaptive control.

1. INTRODUCTION

In recent years several strategies for controlling both force and position of the end effector of a robot manipulator in contact with the environment have been proposed. The most widely adopted one is the hybrid force/position control strategy (Raibert and Craig, 1981; Khatib, 1987; Yoshikawa, 1987) where either a position or a force is controlled along each task space direction through the use of proper selection matrices. Adaptive hybrid force/position control laws have also been developed (Seraji, 1987; Lozano and Brogliato, 1992; Arimoto, Liu and Naniwa, 1993).

A different approach is the parallel control (Chiaverini and Sciavicco, 1988) where force and position variables are used along each task space direction without any selection mechanism. The conflict between force and position along the constrained task directions is resolved by ensuring the dominance of the force control loop above the position control loop.

The performance of a parallel controller with inverse dynamics compensation in the case of contact with a compliant frictionless surface was studied by Chiaverini and Sciavicco (1993). Recently a parallel regulator with gravity compensation has been proposed (Chiaverini and Siciliano, 1991; Chiaverini, Siciliano and Villani, 1994) which guarantees that the force error is driven to zero at the expense of a steady-state position error along the constrained task direction. An adaptive version with respect to gravity parameters has also been developed (Siciliano and Villani, 1993).

In Siciliano and Villani (1994) a passivity-based parallel control scheme ensuring tracking of the uncon-

strained components of the desired end-effector trajectory with regulation of the desired contact force along the constrained direction has been proposed. The scheme is naturally made adaptive with respect to the dynamic model parameters of the manipulator. The stability analysis based on the passivity of the manipulator dynamics does not allow establishing stability in the sense of Lyapunov (Ortega and Spong, 1989; Spong, Ortega and Kelly, 1990).

This work presents a complete stability proof based on the Lyapunov direct method; only boundedness of the desired force, position, velocity and acceleration are required to guarantee convergence to zero of the force, position and velocity errors.

An industrial robot manipulator is considered to develop numerical examples aimed at analyzing the performance of the proposed scheme both in the known parameter case and when an unknown payload mass is considered.

2. MODELLING

The dynamics of a rigid robot manipulator are described in the task space (Khatib, 1987) by the equation

$$B_x(x)\ddot{x} + C_x(x, \dot{x})\dot{x} + g_x(x) = u - f, \quad (1)$$

where x is the $(m \times 1)$ vector of task variables (usually the end-effector location), B_x is the $(m \times m)$ symmetric inertia matrix, $C_x\dot{x}$ is the $(m \times 1)$ vector of Coriolis and centrifugal generalized forces, g_x is the $(m \times 1)$ vector of gravitational generalized forces, u is the $(m \times 1)$ vector of driving generalized forces, and

f is the $(m \times 1)$ vector of contact generalized forces exerted by the manipulator on the environment; all task space quantities are expressed in a common reference frame.

The $(n \times 1)$ vector τ of joint actuating generalized forces can be computed as

$$\tau = J^T(q)u, \quad (2)$$

where q is the $(n \times 1)$ vector of joint variables and J is the $(m \times n)$ manipulator Jacobian matrix.

In this work, the case of non-redundant ($m = n$) non-singular manipulators is treated. Then the vector x represents a set of Lagrangian generalized coordinates and the matrix B_x is positive definite.

Two notable properties of the dynamic model (1) can be established (Slotine and Li, 1987):

- There exists a choice of the matrix C_x such that the matrix

$$S(x, \dot{x}) = \dot{B}_x(x) - 2C_x(x, \dot{x}) \quad (3)$$

is skew-symmetric.

- The dynamic model (1) is linear in terms of a suitable set of manipulator and load constant parameters, i.e.

$$B_x(x)\ddot{x} + C_x(x, \dot{x})\dot{x} + g_x(x) = Y_x(x, \dot{x}, \ddot{x})\theta \quad (4)$$

where $Y_x(x, \dot{x}, \ddot{x})$ is an $(m \times p)$ matrix and θ is a $(p \times 1)$ vector of manipulator and load parameters.

3. CONTROL DESIGN

Consider the following control law

$$u = \hat{B}_x(x)\dot{r} + \hat{C}_x(x, \dot{x})r + \hat{g}_x(x) - k_D(\dot{x} - r) + f, \quad (5)$$

where $\hat{B}_x$, $\hat{C}_x$, $\hat{g}_x$ are the estimates of B_x , C_x , g_x respectively, f is the measured contact force, r is an $(m \times 1)$ reference vector and $k_D > 0$.

Assuming that $\hat{B}_x$, $\hat{C}_x$, $\hat{g}_x$ have the same functional form of B_x , C_x , g_x with a $(p \times 1)$ vector of estimated parameters $\hat{\theta}$, the control law (5) can be written as

$$u = Y_x(x, \dot{x}, r, \dot{r})\hat{\theta} - k_D(\dot{x} - r) + f \quad (6)$$

where the property expressed by (4) has been used. Notice that the matrix $\hat{C}_x$ in (5) must satisfy a property analogous to the skew-symmetry of the matrix in (3).

Combining (1) with (6) gives

$$B_x(x)\dot{e} + C_x(x, \dot{x})e + k_De = Y_x(\cdot)\tilde{\theta} \quad (7)$$

where $\tilde{\theta} = \hat{\theta} - \theta$ and

$$r = \dot{x} - e \quad (8)$$

$$\dot{r} = \ddot{x} - \dot{e}. \quad (9)$$

To obtain a force/position controller, the error vector e should be properly related to the force and position errors. The parallel control strategy (Chiaverini and Sciavicco, 1988) suggests to relate the error vector in (8) to both a position and a force error, without any selection mechanism as in (Slotine and Li, 1987; Lozano and Brogliato, 1992) instead. Along the constrained task directions the conflict between the position and force actions must be managed by imposing dominance of the force action above the position one.

Let x_d , $\dot{x}_d$, $\ddot{x}_d$ denote the time-varying desired end-effector position, velocity and acceleration respectively. Let also f_d denote the constant desired force. Then indicate by $\Delta x = x - x_d$ the error between the actual and desired end-effector position, and by $\Delta f = f - f_d$ the error between the actual and desired contact force.

A keen choice for the error vector in (8) is

$$e = \Delta \dot{x} + \lambda_1 \Delta x + \lambda_2 \int_0^t \Delta f d\sigma \quad (10)$$

where $\lambda_1, \lambda_2 > 0$.

By virtue of this choice, from (8) and (9) the reference vector r and its derivative become

$$r = \dot{x} - e = \dot{x}_d - \lambda_1 \Delta x - \lambda_2 \int_0^t \Delta f d\sigma \quad (11)$$

$$\dot{r} = \ddot{x} - \dot{e} = \ddot{x}_d - \lambda_1 \Delta \dot{x} - \lambda_2 \Delta f. \quad (12)$$

These expressions reveal that the control law (5) requires only joint position and velocity and end-effector contact force measurements.

It is worth noticing that Δx and Δf are not independent since they are constrained by the contact with the environment. Without loss of generality, the case of $m = n = 3$ is taken, i.e. only translational motion and force components are considered. Also the environment is thought of as a frictionless, elastically compliant plane. Hence, the model of the contact force takes on the simple form

$$f = K(x - x_0), \quad (13)$$

where x is the position of the contact point, x_0 is a point of the plane at rest, and K is the (3×3) constant symmetric stiffness matrix that can be expressed as

$$K = k n n^T \quad (14)$$

where $k > 0$ is the stiffness coefficient and n is the unit vector orthogonal to the contact plane which are assumed to be constant.

The elastic contact model (13),(14) reveals that a null force error can be obtained only if the vector of desired force f_d is aligned with n . If no information about the geometry of the environment is available, i.e. n is unknown, it is advisable to choose $f_d = 0$. Analogously, it can be recognized that null position errors can be obtained only on the contact plane, while the component of x along n has to accommodate the force

requirement specified by f_d ; thus x_d can be freely reached only along the unconstrained directions of the operational space. In the remainder it is assumed that a constant component of x_d along n is assigned; then $n^T \dot{x}_d = 0$, $n^T \ddot{x}_d = 0$. If n is unknown, x_d is assumed to be a constant vector and $\dot{x}_d = 0$, $\ddot{x}_d = 0$.

As a further assumption, it is supposed that the contact between the manipulator and the environment is not lost.

In view of the above consideration and of (13),(14), the following equilibrium trajectory is obtained:

$$x_e = (I - nn^T)x_d + nn^T \left(\frac{1}{k} f_d + x_0 \right) \quad (15)$$

$$\dot{x}_e = (I - nn^T)\dot{x}_d = \dot{x}_d \quad (16)$$

$$\ddot{x}_e = (I - nn^T)\ddot{x}_d = \ddot{x}_d \quad (17)$$

$$f_e = knn^T(x_e - x_0) = f_d. \quad (18)$$

4. STABILITY PROOF

In order to study stability of the system (7),(10),(13), (14) around the equilibrium trajectory (15)–(18) with a proper parameter estimate update law, define

$$s = x - x_e \quad (19)$$

which, by virtue of (15), can be also written as

$$s = \Delta x - d \quad (20)$$

where

$$d = nn^T \left(\frac{1}{k} f_d + x_0 - x_d \right) \quad (21)$$

is a constant vector. Then, from (20) it is

$$\dot{s} = \Delta \dot{x}. \quad (22)$$

In view of (10) and (20),(21), Equation (22) can be written in the form

$$\dot{s} = e - \lambda_1 s - \lambda_2 h n \quad (23)$$

where

$$h = n^T \left(\int_0^t \Delta f d\sigma + \frac{\lambda_1}{\lambda_2} d \right). \quad (24)$$

Differentiating (24) with respect to time and taking into account (13),(14),(18),(19) yields

$$\dot{h} = n^T \Delta f = kn^T s. \quad (25)$$

At this point, consider the (7×1) state vector

$$z = (e^T \quad s^T \quad h)^T. \quad (26)$$

The system described by Eqs. (7),(23),(25) can be written in the standard compact form:

$$\dot{z} = \begin{pmatrix} -B_x^{-1}(C_x + k_D I) & O & 0 \\ I & -\lambda_1 I & -\lambda_2 n \\ 0^T & kn^T & 0 \end{pmatrix} z + \begin{pmatrix} B_x^{-1} Y_x \tilde{\theta} \\ 0 \\ 0 \end{pmatrix} \quad (27)$$

where O denotes the (3×3) null matrix and 0 the (3×1) null vector.

Notice that an equivalent representation of the system (7),(23),(25) can be obtained in terms of the (7×1) state vector

$$w = (\dot{s}^T \quad s^T \quad h)^T \quad (28)$$

through a non-singular state space transformation $w = Tz$ where

$$T = \begin{pmatrix} I & -\lambda_1 I & -\lambda_2 n \\ O & I & 0 \\ 0^T & 0^T & 1 \end{pmatrix}. \quad (29)$$

The following result can be stated:

Theorem. Consider the parameter estimate update law

$$\dot{\tilde{\theta}} = -\Gamma^{-1} Y_x^T(\cdot) e \quad (30)$$

where Γ is a $(p \times p)$ symmetric, positive definite matrix. There exists a choice of feedback gains λ_1 and λ_2 that makes the origin of the state space for the augmented system (27),(30) stable. Moreover if x_d , $\dot{x}_d$, $\ddot{x}_d$ are bounded, then $z \rightarrow 0$ as $t \rightarrow \infty$.

Proof. Consider the Lyapunov function candidate

$$V = \frac{1}{2} z^T \begin{pmatrix} B_x & O & 0 \\ O & 2\lambda_1 k_D I & 2\lambda_2 k_D n \\ 0^T & 2\lambda_2 k_D n^T & 2\lambda_1 \lambda_2 k_D k^{-1} \end{pmatrix} z + \frac{1}{2} \tilde{\theta}^T \Gamma \tilde{\theta}. \quad (31)$$

Computing the time derivative of V along the trajectories of the system (27),(30) gives

$$\dot{V} = -k_D z^T T^T \begin{pmatrix} I & O & 0 \\ O & \lambda_1^2 I - 2\lambda_2 k n n^T & 0 \\ 0^T & 0^T & \lambda_2^2 \end{pmatrix} T z \quad (32)$$

in which the skew-symmetry of the matrix $\dot{B}_x - 2C_x$ has been conveniently exploited.

Since the matrix T given in (29) is non-singular, from (32) it follows that $\dot{V}$ is negative semidefinite if $\lambda_1^2 > 2\lambda_2 k$. This choice makes V in (31) positive definite. Therefore V is a Lyapunov function and the system (27),(30) is stable. Moreover z and $\tilde{\theta}$ are bounded and $e, s \in \mathcal{L}_2^3, h \in \mathcal{L}_2$.

Equations (23),(25) imply respectively that $\dot{s}$ and $\dot{h}$ are bounded. It is easy to show that also $\dot{e}$ is bounded if $x_d, \dot{x}_d, \ddot{x}_d$ are bounded. It follows that e, s and h are uniformly continuous and $z \rightarrow 0$ as $t \rightarrow \infty$.

By virtue of the state-space transformation given by (29), the result $z \rightarrow 0$ as $t \rightarrow \infty$ is equivalent to $w \rightarrow 0$ as $t \rightarrow \infty$. Then from (19),(22) it follows

that $x \rightarrow x_e$ and $\dot{x} \rightarrow \dot{x}_d$ as $t \rightarrow \infty$; from (25) $f \rightarrow f_d$ as $t \rightarrow \infty$.

If the dynamic parameters of the manipulator are known i.e. $\hat{\theta} = \theta$, a stronger result can be stated.

Corollary. There exists a choice of feedback gains λ_1 and λ_2 that makes the origin of the state space for the system (27) with $\tilde{\theta} = 0$ uniformly asymptotically stable.

5. JOINT SPACE IMPLEMENTATION

The above force/position control scheme has been derived in the task space, which is the natural space to describe the interaction of the end-effector with the environment. Since control actions are to be realized by the joint actuators, the implementation of the proposed control law shall be carried out in the joint space.

By using the relationship between the dynamics in the joint space and the dynamics in the task space for non-redundant manipulators (Khatib, 1987), the counterpart of the control law (5) in the joint space is given by

$$\begin{aligned} \tau &= \hat{B}(q)\dot{\sigma} + \hat{C}(q, \dot{q})\sigma + \hat{g}(q) \\ &\quad - k_D J^T(q)J(q)(\dot{q} - \dot{\sigma}) + J^T(q)f \\ &= Y(q, \dot{q}, \sigma, \dot{\sigma})\hat{\theta} \\ &\quad - k_D J^T(q)J(q)(\dot{q} - \dot{\sigma}) + J^T(q)f \end{aligned} \quad (33)$$

where

$$\sigma = J^{-1}(q)r \quad (34)$$

$$\dot{\sigma} = J^{-1}(q)(\dot{r} - \dot{J}(q, \dot{q})\sigma). \quad (35)$$

The parameter estimate update law in the joint space is given by

$$\dot{\hat{\theta}} = -\Gamma^{-1}Y^T(q, \dot{q}, \sigma, \dot{\sigma})(\dot{q} - \sigma). \quad (36)$$

6. CASE STUDY

The proposed control scheme was tested in simulation on the industrial robot COMAU SMART-3 6.12R. Only the first three joints were considered, constituting an anthropomorphic manipulator geometry with zero shoulder offsets.

The Denavit-Hartenberg parameters are listed in the table below:

Link	a_i	α_i	d_i	q_i
1	0	$-\pi/2$	0	q_1
2	a_2	0	0	q_2
3	a_3	$\pi/2$	0	q_3

Let m_i denote the mass of the augmented link i which for $i = 1, 2$ includes the link i and the motor driving the joint $i + 1$; whereas for $i = 3$ includes link 3, wrist and payload. The 3 first moments and the 6 elements of

the inertia tensor of the augmented link i with respect to the origin of frame $i - 1$ are denoted respectively by $m_i \ell_{C_i x}, m_i \ell_{C_i y}, m_i \ell_{C_i z}, \hat{I}_{3xx}, \hat{I}_{3xy}, \hat{I}_{3xz}, \hat{I}_{3yy}, \hat{I}_{3yz}, \hat{I}_{3zz}$. Let I_{mi} be the inertia moment of rotor i and k_{ri} be the gear ratio, $i = 1, 2, 3$. A minimum set of dynamic parameters for the manipulator is:

$$\begin{aligned} \pi_1 &= \hat{I}_{1yy} + \hat{I}_{2yy} + \hat{I}_{3zz} + a_2^2 m_3 + k_{r1}^2 I_{m1} \\ \pi_2 &= m_2 \ell_{C_2 x} + a_2 m_3 \\ \pi_3 &= \hat{I}_{2xx} - \hat{I}_{2yy} - a_2^2 m_3 \\ \pi_4 &= \hat{I}_{2xz} - a_2 m_3 \ell_{C_3 y} \\ \pi_5 &= \hat{I}_{2zz} + k_{r2}^2 I_{m2} + a_2^2 m_3 \\ \pi_6 &= \hat{I}_{3xx} - \hat{I}_{3zz} \\ \pi_7 &= m_2 \ell_{C_2 y} \\ \pi_8 &= \hat{I}_{2xy} \\ \pi_9 &= \hat{I}_{2yz} \\ \pi_{10} &= m_3 \ell_{C_3 x} \\ \pi_{11} &= m_3 \ell_{C_3 z} \\ \pi_{12} &= \hat{I}_{3xy} \\ \pi_{13} &= \hat{I}_{3xz} \\ \pi_{14} &= \hat{I}_{3yy} \\ \pi_{15} &= \hat{I}_{3yz} \\ \pi_{16} &= I_{m3}. \end{aligned}$$

The elements of the (3×16) regressor matrix $Y(q, \dot{q}, \sigma, \dot{\sigma})$ are:

$$\begin{aligned} Y_{1,1} &= \dot{\sigma}_1 \\ Y_{1,2} &= 0 \\ Y_{1,3} &= (\sigma_2 \dot{q}_1 + \sigma_1 \dot{q}_2) c_2 s_2 + \dot{\sigma}_1 s_2^2 \\ Y_{1,4} &= -\sigma_2 \dot{q}_2 c_2 - \dot{\sigma}_2 s_2 \\ Y_{1,5} &= 0 \\ Y_{1,6} &= \dot{\sigma}_1 s_{23}^2 + ((\sigma_2 + \sigma_3) \dot{q}_1 \\ &\quad + \sigma_1 (\dot{q}_2 + \dot{q}_3)) s_{23} c_{23} \\ Y_{1,7} &= 0 \\ Y_{1,8} &= (\sigma_1 \dot{q}_2 + \sigma_2 \dot{q}_1) (c_2^2 - s_2^2) + 2\dot{\sigma}_1 c_2 s_2 \\ Y_{1,9} &= -\dot{\sigma}_2 c_2 + \sigma_2 \dot{q}_2 s_2 \\ Y_{1,10} &= 2a_2 \dot{\sigma}_1 c_2 c_{23} - a_2 (\sigma_3 \dot{q}_1 + \sigma_1 \dot{q}_3) c_2 s_{23} \\ &\quad - a_2 (\sigma_2 \dot{q}_1 + \sigma_1 \dot{q}_2) (c_{23} s_2 + c_2 s_{23}) \\ Y_{1,11} &= a_2 (\sigma_2 \dot{q}_1 + \sigma_1 \dot{q}_2) (c_2 c_{23} - s_2 s_{23}) \\ &\quad + a_2 (\sigma_3 \dot{q}_1 + \sigma_1 \dot{q}_3) c_2 c_{23} + 2a_2 \dot{\sigma}_1 c_2 s_{23} \\ Y_{1,12} &= -(\sigma_2 + \sigma_3) (\dot{q}_3 + \dot{q}_2) c_{23} - (\dot{\sigma}_2 + \dot{\sigma}_3) s_{23} \\ Y_{1,13} &= -((\sigma_2 + \sigma_3) \dot{q}_1 + \sigma_1 (\dot{q}_2 + \dot{q}_3)) (c_{23}^2 - s_{23}^2) \\ &\quad - 2\dot{\sigma}_1 c_{23} s_{23} \\ Y_{1,14} &= 0 \\ Y_{1,15} &= (\dot{\sigma}_2 + \dot{\sigma}_3) c_{23} - (\sigma_2 + \sigma_3) (\dot{q}_2 + \dot{q}_3) s_{23} \\ Y_{1,16} &= 0 \end{aligned}$$

$$\begin{aligned}
Y_{2,1} &= 0 \\
Y_{2,2} &= -gc_2 \\
Y_{2,3} &= -\sigma_1 \dot{q}_1 c_2 s_2 \\
Y_{2,4} &= -\dot{\sigma}_1 s_2 \\
Y_{2,5} &= \dot{\sigma}_2 \\
Y_{2,6} &= -\sigma_1 \dot{q}_1 s_{23} c_{23} \\
Y_{2,7} &= gs_2 \\
Y_{2,8} &= \sigma_1 \dot{q}_1 (s_2^2 - c_2^2) \\
Y_{2,9} &= -\dot{\sigma}_1 c_2 \\
Y_{2,10} &= a_2(2\dot{\sigma}_2 + \dot{\sigma}_3)c_3 + a_2\sigma_1 \dot{q}_1 (c_{23}s_2 + c_2s_{23}) \\
&\quad - a_2(\sigma_2 \dot{q}_3 + \sigma_3(\dot{q}_2 + \dot{q}_3))s_3 - gc_{23} \\
Y_{2,11} &= a_2(\sigma_3(\dot{q}_2 + \dot{q}_3) + \sigma_2 \dot{q}_3)c_3 \\
&\quad + a_2(2\dot{\sigma}_2 + \dot{\sigma}_3)s_3 \\
&\quad - a_2\sigma_1 \dot{q}_1 (c_2 c_{23} - s_2 s_{23}) + gs_{23} \\
Y_{2,12} &= -\dot{\sigma}_1 s_{23} \\
Y_{2,13} &= -\sigma_1 \dot{q}_1 (s_{23}^2 - c_{23}^2) \\
Y_{2,14} &= \dot{\sigma}_2 + \dot{\sigma}_3 \\
Y_{2,15} &= \dot{\sigma}_1 c_{23} \\
Y_{2,16} &= \dot{\sigma}_3 k_{r3} \\
\\
Y_{3,1} &= 0 \\
Y_{3,2} &= 0 \\
Y_{3,3} &= 0 \\
Y_{3,4} &= 0 \\
Y_{3,5} &= 0 \\
Y_{3,6} &= -\sigma_1 \dot{q}_1 s_{23} c_{23} \\
Y_{3,7} &= 0 \\
Y_{3,8} &= 0 \\
Y_{3,9} &= 0 \\
Y_{3,10} &= a_2(\dot{\sigma}_2 c_3 + \sigma_2 \dot{q}_2 s_3 + \sigma_1 \dot{q}_1 c_2 s_{23}) - gc_{23} \\
Y_{3,11} &= -a_2(\sigma_2 \dot{q}_2 c_3 + \sigma_1 \dot{q}_1 c_2 c_{23} - \dot{\sigma}_2 s_3) + gs_{23} \\
Y_{3,12} &= -\dot{\sigma}_1 s_{23} \\
Y_{3,13} &= -\sigma_1 \dot{q}_1 (s_{23}^2 - c_{23}^2) \\
Y_{3,14} &= \dot{\sigma}_2 + \dot{\sigma}_3 \\
Y_{3,15} &= \dot{\sigma}_1 c_{23} \\
Y_{3,16} &= k_{r3}(\dot{\sigma}_2 + \dot{\sigma}_3 k_{r3})
\end{aligned}$$

where the standard abbreviations $s_{i\dots j}$ and $c_{i\dots j}$ have been used for $\sin(q_i + \dots + q_j)$ and $\cos(q_i + \dots + q_j)$, respectively.

The numerical data of robot parameters are taken from Caccavale and Chiacchio (1994). A load was added at the end-effector constituted by a cylinder of 12 kg mass and moments of inertia about central axes of 0.0475, 0.0475, 0.015 kg m². Simulations were run in MATLAB at a sampling time of 2 ms.

The geometry of the contact plane is characterized by $n = [1 \ 0 \ 0]^T$ and $x_0 = [0.97 \ 0 \ 0]^T$ m. The gains in (10) and (5) were set to $\lambda_1 = 200 \text{ s}^{-1}$, $\lambda_2 = 0.15 \text{ kg}^{-1}$ and $k_D = 400 \text{ kg s}^{-1}$. This choice guarantees a well-damped behaviour both for the un-

constrained and constrained motion with an estimate of the stiffness coefficient of the plane $k = 10^5 \text{ N m}^{-1}$.

For simplicity a bidimensional task geometry in the xz -plane was considered; thus the sole x - and z -components of the position vector and the x -component of the force vector are reported.

A stable contact of the end effector with the elastic plane was first sought, then a motion on the contact surface was commanded by requiring a constant force of 30 N along the normal direction. The orientation of the contact surface was supposed to be known, but the position of the plane at rest was supposed to be affected by an uncertainty of 0.03 m. To ensure contact starting from $x_{di} = [0.9 \ 0 \ 0]^T$ m, the approach phase was managed by imposing a straight line path towards $x_{dm} = [1 \ 0 \ 0]^T$ m. A trapezoidal velocity profile was assigned with maximum velocity of 0.12 m s^{-1} and time duration of 1.5 s. The force set point was initially set to $f_d = 0$ and switched to $f_d = [30 \ 0 \ 0]^T$ N as soon as a non-null contact force was sensed. After a lapse of 0.4 s, by preserving the force set point, a straight line path from $x_{dm} = [1 \ 0 \ 0]^T$ m to $x_{df} = [1 \ 0 \ 0.03]^T$ m was commanded. A trapezoidal velocity profile was assigned with maximum velocity of 0.1 m s^{-1} and time duration of 0.6 s.

Three sets of simulations were carried out using the following controllers:

- control law as in (33),(34),(35),(11),(12) without adaptation law and with compensated load mass;
- control law as in (33),(34),(35),(11),(12) without adaptation law and with non-compensated load mass;
- control law as in (33),(34),(35),(11),(12) with non-compensated load mass and adaptive law (36).

The numerical results are illustrated in terms of the time history of the contact force, the path of the end-effector in the xz -plane, the time history of the z -component of the position tracking error, and the time history of the norm of the parameter error.

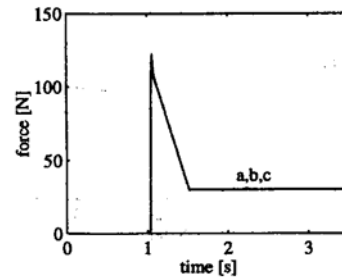


Fig. 1. Time history of contact force.

Figure 1 shows that the contact force is practically the same in the three cases thanks to the integral action on the force error in the control law.

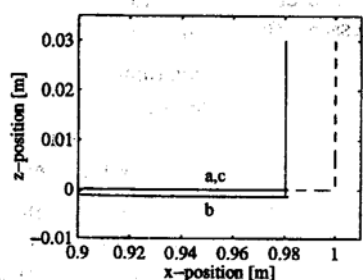


Fig. 2. Desired and actual paths of end-effector position in the xz -plane.

Figure 2 shows the actual paths in the three cases (solid lines) with respect to the desired path (dashed line). It can be seen that a deviation along the z direction occurs in case (b) which is caused by the non-compensated load mass, whereas good reproduction of the desired path is obtained in the other two cases. Notice also that the actual path along the z -direction is constrained by the presence of the contact plane and thus a deviation from the desired path occurs which is determined by the amount of desired contact force and plane stiffness.

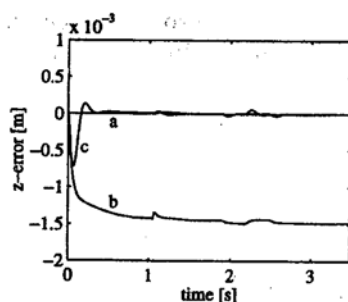


Fig. 3. Time history of z -component of position error.

Figure 3 shows how the adaptation mechanism, case (c), allows recovering the tracking position error which affected the behaviour of the control law in case (b), and the performance favourably compares with that of the controller with perfect compensation, case (a).

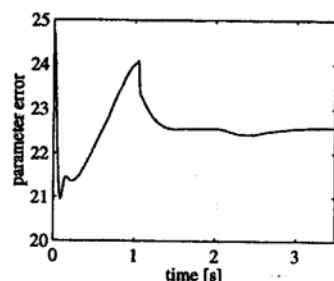


Fig. 4. Time history of parameter error norm.

Finally, Figure 4 shows that the norm of the parameter error remains bounded along the trajectory, as anticipated in theory.

6. CONCLUSIONS

A force/position control scheme for robot manipulators in contact with an elastically compliant surface has been analyzed in this work. The stability of the controller has been proved by means of the Lyapunov direct method. A simple condition involving the feedback gains and the surface stiffness guarantees the stability of the controlled system in the sense of Lyapunov. Simulation results on an industrial robot manipulator have demonstrated the effectiveness of the control scheme when the end-effector carries an unknown load mass.

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