

A Singular Perturbation Approach to Control of Flexible Arms in Compliant Motion

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Summary. The problem of controlling the interaction of a flexible link arm with a compliant environment is considered. The arm's tip is required to keep contact with a surface by applying a constant force and maintaining a prescribed position or following a desired path on the surface. Using singular perturbation theory, the system is decomposed into a slow subsystem associated with rigid motion and a fast subsystem associated with link flexible dynamics. A parallel force and position control developed for rigid robots is adopted for the slow subsystem while a fast control action is employed to stabilize the link deflections. Simulation results are presented for a two-link planar arm under gravity in contact with an elastically compliant surface.

1.1 Introduction

Lightweight flexible robots offer many advantages over conventional industrial robots, like high speed, large workspace and high payload-to-arm weight [1]. In fact, they are conveniently employed in a large variety of fields including teleoperation, space robotics, and nuclear waste manipulation.

The dynamics of multilink flexible arms is however much more complex than rigid robot dynamics, due to the distributed flexibility of the links [2]. As a consequence, several challenging problems are still open, regarding both modelling and control aspects.

From the modelling standpoint, the dynamics of a flexible structure is described by infinite dimensional model. Various techniques have been proposed to achieve approximate finite dimensional models, e.g. the assumed modes method, the finite elements method and the Ritz-Kantorovich expansion. In the case of multilink flexible arms, a recursive procedure can be set up for dynamic model computation by using a Lagrangian formulation in conjunction with the assumed mode technique [3].

The inherent difficulty of the control problem can be ascribed to the fact that the number of controlled variables is strictly less than the number of

mechanical degrees of freedom. Moreover, the dynamic relation between the input torques of the joint actuators and the tip position reveals a behavior which is the nonlinear counterpart of the non-minimum phase phenomenon of linear systems. Hence, inversion-based control strategies would normally lead to instability in the closed-loop. See [4] and the references therein for further discussion about modelling and control problems for flexible link arms.

An effective approach to motion control design is based on singular perturbation theory [5]. When the link stiffness is large, a two-time scale model of the flexible arm can be derived [6], consisting in a slow subsystem corresponding to the rigid body motion and a fast subsystem describing the flexible motion. A composite control strategy can then be applied, based on a slow control designed for the equivalent rigid arm and a fast control which stabilizes the fast subsystem. Further developments of perturbation techniques for flexible arms can be found in [7, 8, 9, 10].

When the arm interacts with an external environment, suitable strategies have to be adopted to control both the tip position and the contact force. In fact, during the interaction, the environment sets constraints on the geometric path that can be followed by the end effector, and high contact forces may arise if purely motion strategies are adopted. The higher the environment stiffness and position control accuracy are, the easier an unstable behavior with damage to the robot or to the environment may occur. Hence, interaction control should ensure a suitable compliant behavior to the robotic arm to keep limited the contact forces.

Notice that the intrinsic compliance of a flexible link arm may contribute to reduce the value of the forces that can be generated when the interaction task is executed by a rigid robot. This means that by using flexible robots to perform interaction tasks some benefits may arise, even though the distributed flexibility of the links makes the interaction control problem more complex than for rigid robots.

The most common solution to interaction control is the use of a force/torque sensor, mounted between the last link and the end effector, which provides force measurements that can be suitably exploited by the robot controller.

While several control schemes have been proposed to force and position control of rigid robot manipulators [11], only few papers on interaction control of flexible arms have been published so far.

Early works addressing stability problems in force controlled flexible manipulators are [12, 13]. Models for multilink constrained flexible robots have been developed in [14, 15] where a hybrid position and force control approach is adopted. Hybrid control is used in [16] and [17] to design robust and adaptive control strategies respectively as well as in [18, 19] to control a flexible macro manipulator carrying a rigid micro. In most of these papers ([13, 15, 17]) singular perturbation techniques are exploited to cope with link flexibility; a singular perturbed model for a constrained multilink flexible arm was developed in [20].

The singular perturbation method is adopted in this chapter to design a force and position control for flexible arms based on the parallel approach developed in [21, 22] for rigid robots in contact with compliant environments. As opposed to the hybrid control strategies where force and position are controlled in reciprocal subspaces [23, 24], both force and position variables are used in each subspace without any selection mechanism. This makes parallel controllers suitable to manage contacts with non-perfectly known environments and unplanned collisions, which represent a drawback for hybrid controllers. Moreover, differently from previous works tackling the problem of force and position control of flexible manipulators (e.g., [13, 15, 17, 20]), the equations of the constraint environment have not to be taken into account for control design.

The proposed control scheme guarantees force regulation and position tracking for the slow dynamics. An additional control action is required in both cases to stabilize the fast dynamics related to link flexibility.

Simulation results have been carried out on the model of a two-link planar arm developed in [25]; interaction with an elastically compliant plane have been considered. The numerical case study confirms the results anticipated in theory.

1.2 Modelling

Consider a robot arm composed by a serial chain of n flexible links connected by rigid revolute joints subject only to bending deformations in the plane of motion, without torsional effects. A sketch of a two-link arm is shown in Fig. 1.1 with coordinate frame assignment; the tip of the robotic arm is assumed to be in contact with a planar surface. The rigid motion is described by the joint angles ϑ_i , while $w_i(x_i)$ denotes the transversal deflection of link i at x_i with $0 \leq x_i \leq \ell_i$, being ℓ_i the link length.

1.2.1 Kinematics

Let $\mathbf{p}_i^i(x_i) = [x_i \ w_i(x_i)]^T$ be the position of a point along the deflected link i with respect to frame (X_i, Y_i) and $\mathbf{p}_i$ be the position of the same point in the base frame. Also let $\mathbf{r}_{i+1}^i = \mathbf{p}_i^i(\ell_i)$ be the position of the origin of frame (X_{i+1}, Y_{i+1}) with respect to frame (X_i, Y_i) , and $\mathbf{r}_{i+1}$ its position in the base frame.

The joint (rigid) rotation matrix $\mathbf{R}_i$ and the rotation matrix $\mathbf{E}_i$ of the (flexible) link at the end point are, respectively,

$$\mathbf{R}_i = \begin{bmatrix} \cos \vartheta_i & -\sin \vartheta_i \\ \sin \vartheta_i & \cos \vartheta_i \end{bmatrix} \quad (1.1)$$

and

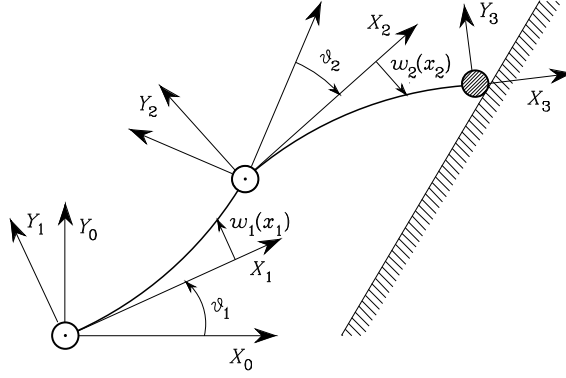


Fig. 1.1. Planar two-link flexible arm in contact with a planar surface

$$\mathbf{E}_i = \begin{bmatrix} 1 & -w'_{ie} \\ w'_{ie} & 1 \end{bmatrix}, \quad (1.2)$$

where $w'_{ie} = (\partial w_i / \partial x_i)|_{x_i=\ell_i}$, and the approximation $\arctan w'_{ie} \simeq w'_{ie}$, in the hypothesis of small deflections, has been made. Hence the above absolute position vectors can be expressed as

$$\mathbf{p}_i = \mathbf{r}_i + \mathbf{W}_i \mathbf{p}_i^i \quad (1.3)$$

and

$$\mathbf{r}_{i+1} = \mathbf{r}_i + \mathbf{W}_i \mathbf{r}_{i+1}^i, \quad (1.4)$$

where $\mathbf{W}_i$ is the global transformation matrix from the base frame to (X_i, Y_i) given by the recursive equation

$$\mathbf{W}_i = \mathbf{W}_{i-1} \mathbf{E}_{i-1} \mathbf{R}_i = \widehat{\mathbf{W}}_{i-1} \mathbf{R}_i, \quad (1.5)$$

with

$$\widehat{\mathbf{W}}_0 = \mathbf{I}. \quad (1.6)$$

On the basis of the above relations, the kinematics of any point along the arm is completely specified as a function of joint angles and link deflections.

A finite-dimensional model (of order m_i) of link flexibility can be obtained by the assumed mode technique [2]. Links are modelled as Euler-Bernoulli beams of uniform density ρ_i and constant flexural rigidity $(EI)_i$, with deflection $w_i(x_i, t)$ satisfying the partial differential equation

$$(EI)_i \frac{\partial^4 w_i(x_i, t)}{\partial x_i^4} + \rho_i \frac{\partial^2 w_i(x_i, t)}{\partial t^2} = 0, \quad i = 1, \dots, n. \quad (1.7)$$

Exploiting separability in time and space of solutions of (1.7), the link deflection $w_i(x_i, t)$ can be expressed as the sum of a finite number of modes

$$w_i(x_i, t) = \sum_{j=1}^{m_i} \phi_{ij}(x_i) \delta_{ij}(t), \quad (1.8)$$

where $\phi_{ij}(x)$ is the shape assumed for the j -th mode of link i , and $\delta_{ij}(t)$ is its time-varying amplitude. The mode shapes have to satisfy proper boundary conditions at the base (clamped) and at the end of each link (mass).

In view of (1.8), a direct kinematics equation can be derived expressing the (2×1) position vector $\mathbf{p}$ of the arm tip point as a function of the $(n \times 1)$ joint variable vector $\boldsymbol{\vartheta} = [\vartheta_1 \dots \vartheta_n]^T$ and the $(m \times 1)$ deflection variable vector $\boldsymbol{\delta} = [\delta_{11} \dots \delta_{1m_1} \dots \delta_{n1} \dots \delta_{nm_n}]^T$ [3, 25], i.e.

$$\mathbf{p} = \mathbf{k}(\boldsymbol{\vartheta}, \boldsymbol{\delta}). \quad (1.9)$$

For later use, also the differential kinematics is needed. The absolute linear velocity of an arm point is

$$\dot{\mathbf{p}}_i = \dot{\mathbf{r}}_i + \dot{\mathbf{W}}_i \mathbf{p}_i^i + \mathbf{W}_i \dot{\mathbf{p}}_i^i, \quad (1.10)$$

with $\dot{\mathbf{r}}_{i+1}^i = \dot{\mathbf{p}}_i^i(\ell_i)$. Since the links are assumed inextensible ($\dot{x}_i = 0$), then $\dot{\mathbf{p}}_i^i(x_i) = [0 \ \dot{w}_i(x_i)]^T$. The computation of (1.10) takes advantage of the recursion

$$\dot{\mathbf{W}}_i = \dot{\widehat{\mathbf{W}}}_{i-1} \mathbf{R}_i + \widehat{\mathbf{W}}_{i-1} \dot{\mathbf{R}}_i \quad (1.11)$$

with

$$\dot{\widehat{\mathbf{W}}}_i = \dot{\mathbf{W}}_i \mathbf{E}_i + \mathbf{W}_i \dot{\mathbf{E}}_i. \quad (1.12)$$

Also, note that

$$\dot{\mathbf{R}}_i = \mathbf{S} \mathbf{R}_i \dot{\vartheta}_i, \quad \dot{\mathbf{E}}_i = \mathbf{S} \dot{w}'_{ie} \quad (1.13)$$

with

$$\mathbf{S} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \quad (1.14)$$

In view of (1.9)–(1.14), it is not difficult to show that the differential kinematics equation expressing the tip velocity $\dot{\mathbf{p}}$ as a function of $\dot{\boldsymbol{\vartheta}}$ and $\dot{\boldsymbol{\delta}}$ can be written in the form

$$\dot{\mathbf{p}} = \mathbf{J}_{\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}, \boldsymbol{\delta}) \dot{\boldsymbol{\vartheta}} + \mathbf{J}_{\boldsymbol{\delta}}(\boldsymbol{\vartheta}, \boldsymbol{\delta}) \dot{\boldsymbol{\delta}}, \quad (1.15)$$

where $\mathbf{J}_{\boldsymbol{\vartheta}} = \partial \mathbf{k} / \partial \boldsymbol{\vartheta}$ and $\mathbf{J}_{\boldsymbol{\delta}} = \partial \mathbf{k} / \partial \boldsymbol{\delta}$.

1.2.2 Dynamics

Using the assumed modes link approximation (1.8), a finite-dimensional Lagrangian dynamic model of the planar arm can be obtained as a function of the $n + m$ vector of generalized coordinates $\mathbf{q} = [\boldsymbol{\vartheta}^T \ \boldsymbol{\delta}^T]^T$ in the form [3, 25]

$$\mathbf{B}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + \begin{bmatrix} \mathbf{0} \\ \mathbf{D} \dot{\boldsymbol{\delta}} + \mathbf{K} \boldsymbol{\delta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\tau} \\ \mathbf{0} \end{bmatrix} \quad (1.16)$$

where $\mathbf{B}$ is the positive definite symmetric inertia matrix, $\mathbf{c}$ is the vector of Coriolis and centrifugal torques, $\mathbf{g}$ is the vector of gravitational torques, $\mathbf{K}$ is the diagonal and positive definite link stiffness matrix, $\mathbf{D}$ is the diagonal and positive semidefinite link damping matrix and $\boldsymbol{\tau}$ is the vector of the input joint torques.

In the case that the arm's tip is in contact with the environment, by virtue of the virtual work principle, the vector $\mathbf{f}$ of the forces exerted by the arm on the environment performing work on $\mathbf{p}$ has to be related to the $(n \times 1)$ vector $\mathbf{J}_\vartheta^T \mathbf{f}$ of joint torques performing work on $\boldsymbol{\vartheta}$ and to the $(m \times 1)$ vector $\mathbf{J}_\delta^T \mathbf{f}$ of the elastic reaction forces performing work on $\boldsymbol{\delta}$. Hence, the dynamic model (1.16) can be rewritten in the form:

$$\begin{aligned} & \begin{bmatrix} \mathbf{B}_{\vartheta\vartheta}(\boldsymbol{\vartheta}, \boldsymbol{\delta}) & \mathbf{B}_{\vartheta\delta}(\boldsymbol{\vartheta}, \boldsymbol{\delta}) \\ \mathbf{B}_{\vartheta\delta}^T(\boldsymbol{\vartheta}, \boldsymbol{\delta}) & \mathbf{B}_{\delta\delta}(\boldsymbol{\vartheta}, \boldsymbol{\delta}) \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{\vartheta}} \\ \ddot{\boldsymbol{\delta}} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_\vartheta(\boldsymbol{\vartheta}, \boldsymbol{\delta}, \dot{\boldsymbol{\vartheta}}, \dot{\boldsymbol{\delta}}) \\ \mathbf{c}_\delta(\boldsymbol{\vartheta}, \boldsymbol{\delta}, \dot{\boldsymbol{\vartheta}}, \dot{\boldsymbol{\delta}}) \end{bmatrix} \\ & + \begin{bmatrix} \mathbf{g}_\vartheta(\boldsymbol{\vartheta}, \boldsymbol{\delta}) \\ \mathbf{g}_\delta(\boldsymbol{\vartheta}, \boldsymbol{\delta}) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{D}\dot{\boldsymbol{\delta}} + \mathbf{K}\boldsymbol{\delta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\tau} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{J}_\vartheta^T(\boldsymbol{\vartheta}, \boldsymbol{\delta})\mathbf{f} \\ \mathbf{J}_\delta^T(\boldsymbol{\vartheta}, \boldsymbol{\delta})\mathbf{f} \end{bmatrix} \quad (1.17) \end{aligned}$$

where the matrix and vectors have been partitioned in blocks according to the rigid and flexible components.

1.3 Singularly perturbed model

When the link stiffness is large, it is reasonable to expect that the dynamics related to link flexibility is much faster than the dynamics associated with the rigid motion of the robot so that the system naturally exhibits a two-time scale dynamic behaviour in terms of rigid and flexible variables. This feature can be conveniently exploited for control design.

Following the approach proposed in [6], the system can be decomposed in a slow and a fast subsystems by using singular perturbation theory; this leads to a composite control strategy for the full system based on separate control designs for the two reduced-order subsystems.

1.3.1 Unconstrained motion

In the absence of contact with the environment, assuming that full-state measurements are available, the joint torques can be conveniently chosen as

$$\boldsymbol{\tau} = \mathbf{g}_\vartheta(\boldsymbol{\vartheta}, \boldsymbol{\delta}) + \mathbf{u}, \quad (1.18)$$

in order to cancel out the effects of the static torques acting on the rigid part of the arm dynamics; the vector $\mathbf{u}$ is the new control input to be designed on the basis of the singular perturbation approach.

The time scale separation between the slow and fast dynamics can be determined by defining the singular perturbation parameter $\epsilon = 1/\sqrt{k_m}$, where k_m is the smallest coefficient of the diagonal stiffness matrix $\mathbf{K}$, and the new variable

$$\mathbf{z} = \mathbf{K}\boldsymbol{\delta} = \frac{1}{\epsilon^2}\hat{\mathbf{K}}\boldsymbol{\delta}, \quad (1.19)$$

corresponds to the elastic force, where $\mathbf{K} = k_m\hat{\mathbf{K}}$. Considering the inverse $\mathbf{H}$ of the inertia matrix $\mathbf{B}$, the dynamic model (1.17), with control law (1.18), can be rewritten in terms of the new variable $\mathbf{z}$ as

$$\begin{aligned} \ddot{\boldsymbol{\vartheta}} = & \mathbf{H}_{\boldsymbol{\vartheta}\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}, \epsilon^2\mathbf{z}) \left(\mathbf{u} - \mathbf{c}_{\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}, \epsilon^2\mathbf{z}, \dot{\boldsymbol{\vartheta}}, \epsilon^2\dot{\mathbf{z}}) \right) \\ & - \mathbf{H}_{\boldsymbol{\vartheta}\boldsymbol{\delta}}(\boldsymbol{\vartheta}, \epsilon^2\mathbf{z}) \left(\mathbf{c}_{\boldsymbol{\delta}}(\boldsymbol{\vartheta}, \epsilon^2\mathbf{z}, \dot{\boldsymbol{\vartheta}}, \epsilon^2\dot{\mathbf{z}}) + \mathbf{g}_{\boldsymbol{\delta}}(\boldsymbol{\vartheta}, \boldsymbol{\delta}) \right. \\ & \left. + \epsilon^2 \mathbf{D}\hat{\mathbf{K}}^{-1}\dot{\mathbf{z}} + \mathbf{z} \right) \end{aligned} \quad (1.20)$$

$$\begin{aligned} \epsilon^2\ddot{\mathbf{z}} = & \hat{\mathbf{K}}\mathbf{H}_{\boldsymbol{\vartheta}\boldsymbol{\delta}}^T(\boldsymbol{\vartheta}, \epsilon^2\mathbf{z}) \left(\mathbf{u} - \mathbf{c}_{\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}, \epsilon^2\mathbf{z}, \dot{\boldsymbol{\vartheta}}, \epsilon^2\dot{\mathbf{z}}) \right) \\ & - \hat{\mathbf{K}}\mathbf{H}_{\boldsymbol{\delta}\boldsymbol{\delta}}(\boldsymbol{\vartheta}, \epsilon^2\mathbf{z}) \left(\mathbf{c}_{\boldsymbol{\delta}}(\boldsymbol{\vartheta}, \epsilon^2\mathbf{z}, \dot{\boldsymbol{\vartheta}}, \epsilon^2\dot{\mathbf{z}}) + \mathbf{g}_{\boldsymbol{\delta}}(\boldsymbol{\vartheta}, \boldsymbol{\delta}) \right. \\ & \left. + \epsilon^2 \mathbf{D}\hat{\mathbf{K}}^{-1}\dot{\mathbf{z}} + \mathbf{z} \right), \end{aligned} \quad (1.21)$$

where a suitable partition of $\mathbf{H}$ has been considered

$$\mathbf{H} = \mathbf{B}^{-1} = \begin{bmatrix} \mathbf{H}_{\boldsymbol{\vartheta}\boldsymbol{\vartheta}} & \mathbf{H}_{\boldsymbol{\vartheta}\boldsymbol{\delta}} \\ \mathbf{H}_{\boldsymbol{\vartheta}\boldsymbol{\delta}}^T & \mathbf{H}_{\boldsymbol{\delta}\boldsymbol{\delta}} \end{bmatrix}. \quad (1.22)$$

Equations (1.20) and (1.21) represent a singularly perturbed form of the flexible arm model; when $\epsilon \rightarrow 0$, the model of an equivalent rigid arm is recovered. In fact, setting $\epsilon = 0$ and solving for $\mathbf{z}$ in (1.21) gives

$$\mathbf{z}_s = \bar{\mathbf{H}}_{\boldsymbol{\delta}\boldsymbol{\delta}}^{-1}(\boldsymbol{\vartheta}_s) \bar{\mathbf{H}}_{\boldsymbol{\vartheta}\boldsymbol{\delta}}^T(\boldsymbol{\vartheta}_s) \left(\mathbf{u}_s - \bar{\mathbf{c}}_{\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}_s, \dot{\boldsymbol{\vartheta}}_s) \right) - \bar{\mathbf{c}}_{\boldsymbol{\delta}}(\boldsymbol{\vartheta}_s, \dot{\boldsymbol{\vartheta}}_s) - \bar{\mathbf{g}}_{\boldsymbol{\delta}}(\boldsymbol{\vartheta}_s), \quad (1.23)$$

where the subscript s indicates that the system is considered in the *slow* time scale and the overbar denotes that a quantity is computed with $\epsilon = 0$. Plugging (1.23) into (1.20) with $\epsilon = 0$ yields

$$\ddot{\boldsymbol{\vartheta}}_s = \bar{\mathbf{B}}_{\boldsymbol{\vartheta},\boldsymbol{\vartheta}}^{-1}(\boldsymbol{\vartheta}_s) \left(\mathbf{u}_s - \bar{\mathbf{c}}_{\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}_s, \dot{\boldsymbol{\vartheta}}_s) \right), \quad (1.24)$$

where the equality

$$\bar{\mathbf{B}}_{\boldsymbol{\vartheta},\boldsymbol{\vartheta}}^{-1}(\boldsymbol{\vartheta}_s) = \left(\bar{\mathbf{H}}_{\boldsymbol{\vartheta}\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}_s) - \bar{\mathbf{H}}_{\boldsymbol{\vartheta}\boldsymbol{\delta}}(\boldsymbol{\vartheta}_s) \bar{\mathbf{H}}_{\boldsymbol{\delta}\boldsymbol{\delta}}^{-1}(\boldsymbol{\vartheta}_s) \bar{\mathbf{H}}_{\boldsymbol{\vartheta}\boldsymbol{\delta}}^T(\boldsymbol{\vartheta}_s) \right) \quad (1.25)$$

has been exploited, being $\bar{\mathbf{B}}_{\boldsymbol{\vartheta}\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}_s)$ the inertia matrix of the equivalent rigid arm and $\bar{\mathbf{c}}_{\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}_s, \dot{\boldsymbol{\vartheta}}_s)$ the vector of the corresponding Coriolis and centrifugal torques.

The dynamics of the system in the *fast* time scale can be obtained by setting $t_f = t/\epsilon$, treating the slow variables as constants in the fast time scale, and introducing the fast variables $\mathbf{z}_f = \mathbf{z} - \mathbf{z}_s$; thus, the fast system of (1.21) is

$$\frac{d^2 \mathbf{z}_f}{dt_f^2} = -\hat{\mathbf{K}} \bar{\mathbf{H}}_{\delta\delta}(\boldsymbol{\vartheta}_s) \mathbf{z}_f + \hat{\mathbf{K}} \bar{\mathbf{H}}_{\vartheta\delta}^T(\boldsymbol{\vartheta}_s) \mathbf{u}_f, \quad (1.26)$$

where the fast control $\mathbf{u}_f = \mathbf{u} - \mathbf{u}_s$ has been introduced accordingly.

On the basis of the above two-time scale model, the design of a feedback controller for the system (1.20) and (1.21) can be performed according to a composite control strategy, i.e.,

$$\mathbf{u} = \mathbf{u}_s(\boldsymbol{\vartheta}_s, \dot{\boldsymbol{\vartheta}}_s) + \mathbf{u}_f(\mathbf{z}_f, d\mathbf{z}_f/dt_f) \quad (1.27)$$

with the constraint that $\mathbf{u}_f(0,0) = 0$, so that $\mathbf{u}_f$ is inactive along the equilibrium manifold specified by (1.23).

Notice that the fast system (1.26) is a marginally stable linear slowly time-varying system that can be stabilized to the equilibrium manifold $\dot{\mathbf{z}}_f = \mathbf{0}$ ($\dot{\mathbf{z}} = \mathbf{0}$) and $\mathbf{z}_f = \mathbf{0}$ ($\mathbf{z} = \mathbf{z}_s$) by a proper choice of the control input $\mathbf{u}_f$. A reasonable way to achieve this goal is to design a state-space control law of the form

$$\mathbf{u}_f = \mathbf{K}_1 \dot{\mathbf{z}}_f + \mathbf{K}_2 \mathbf{z}_f \quad (1.28)$$

where, in principle, the matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ should be tuned for every configuration $\boldsymbol{\vartheta}_s$. However, the computational burden necessary to perform this strategy can be avoided by using constant matrix gains tuned with reference to a given robot configuration [6]; any state-space technique can be used, e.g., based on classical pole placement algorithms.

1.3.2 Constrained motion

When the arm's tip is constrained by the environment, a similar model can be derived provided that the contact force is measured by using a force sensor mounted on the tip.

In detail, in lieu of (1.18), the joint torques can be chosen as

$$\boldsymbol{\tau} = \mathbf{g}_{\boldsymbol{\vartheta}}(\boldsymbol{\vartheta}, \boldsymbol{\delta}) + \mathbf{J}_{\boldsymbol{\vartheta}}^T(\boldsymbol{\vartheta}, \boldsymbol{\delta}) \mathbf{f} + \mathbf{u}, \quad (1.29)$$

in order to cancel out also the effects of the contact force $\mathbf{f}$ acting on the rigid part of the arm dynamics.

By introducing the variable $\mathbf{z}$ as in (1.19) and following the same procedure as in the unconstrained motion case, the rigid robot dynamics for the slow time scale can be achieved in the same form (1.24).

As for the fast dynamics, the same expression (1.26) holds by defining the fast variable as $\mathbf{z}_f = \mathbf{z} - \mathbf{z}'_s$, where

$$\mathbf{z}'_s = \mathbf{z}_s - \bar{\mathbf{J}}_{\boldsymbol{\delta}}^T(\boldsymbol{\vartheta}_s) \mathbf{f}_s \quad (1.30)$$

with $\mathbf{z}_s$ defined in (1.23). Hence the same control law (1.28) can be adopted to stabilize the fast dynamics.

1.4 Compliant motion control

Control of the interaction between a robotic arm and the environment by using a pure motion control strategy is a candidate to fail if the task is not accurately planned. In practice, planning errors may give rise to a contact force causing a deviation of the tip from the desired trajectory. Since motion control reacts to reduce such deviation, the contact force may reach high values that can lead to saturation of joint actuators or breakage of the parts in contact.

The higher environment stiffness and position control accuracy are, the easier a situation like the one just described can occur. This drawback can be overcome if a compliant behavior is ensured during the interaction. This is partially achieved in a passive fashion, due to the flexibility of the arm, but it can be enhanced in active fashion by adopting a suitable compliant control strategy.

In this section, a compliant control strategy that offers the possibility of controlling the contact force to a desired value, as well as the tip position to a desired trajectory assigned along the unconstrained directions, is presented. This strategy is based on the parallel force/position control approach [21], which is especially effective in the case of inaccurate contact modelling. The key feature is that to have a force control loop working in parallel to a position control loop. The logical conflict between the two loops is managed by imposing a dominance of the force control action over the position one, i.e., force regulation is always guaranteed at the expense of a position error along the constrained directions.

1.4.1 Position control

To gain insight into parallel force/position control applied to a flexible arm, the position control loop is first designed. To this purpose, it is useful to derive the slow dynamics corresponding to the tip position. Differentiating Eq. (1.15) gives the tip acceleration

$$\ddot{\mathbf{p}} = \mathbf{J}_{\vartheta}(\vartheta, \delta)\ddot{\vartheta} + \mathbf{J}_{\delta}(\vartheta, \delta)\ddot{\delta} + \mathbf{h}(\vartheta, \delta, \dot{\vartheta}, \dot{\delta}), \quad (1.31)$$

where $\mathbf{h} = \dot{\mathbf{J}}_{\vartheta}\dot{\vartheta} + \dot{\mathbf{J}}_{\delta}\dot{\delta}$; hence the corresponding slow system is

$$\ddot{\mathbf{p}}_s = \bar{\mathbf{J}}_{\vartheta}(\vartheta_s)\bar{\mathbf{B}}_{\vartheta, \vartheta}^{-1}(\vartheta_s) \left(\mathbf{u}_s - \bar{\mathbf{c}}_{\vartheta}(\vartheta_s, \dot{\vartheta}_s) \right) + \bar{\mathbf{h}}(\vartheta_s, \dot{\vartheta}_s), \quad (1.32)$$

where Eq. (1.24) has been used. The slow dynamic models (1.24) and (1.32) enjoy the same notable properties of the rigid robot dynamic models [4], hence the control strategies used for rigid arms can be adopted.

If tracking of a time-varying position $\mathbf{p}_r(t)$ is desired (with an order ϵ approximation), an inverse dynamics motion scheme can be adopted for the slow system, i.e.

$$\mathbf{u}_s = \bar{\mathbf{B}}_{\vartheta, \dot{\vartheta}}(\vartheta_s) \bar{\mathbf{J}}_{\dot{\vartheta}}^{-1}(\vartheta_s) \left(\mathbf{a}_s - \bar{\mathbf{h}}(\vartheta_s, \dot{\vartheta}_s) \right) + \bar{\mathbf{c}}_{\dot{\vartheta}}(\vartheta_s, \dot{\vartheta}_s) \quad (1.33)$$

where $\mathbf{a}_s$ is a new control input and the Jacobian matrix is assumed to be nonsingular.

Folding (1.33) into (1.32) gives

$$\ddot{\mathbf{p}}_s = \mathbf{a}_s; \quad (1.34)$$

hence the control input $\mathbf{a}_s$ can be chosen as

$$\mathbf{a}_s = \ddot{\mathbf{p}}_r + k_D(\dot{\mathbf{p}}_r - \dot{\mathbf{p}}_s) + k_P(\mathbf{p}_r - \mathbf{p}_s), \quad (1.35)$$

giving the closed-loop equation for the slow subsystem

$$\ddot{\mathbf{p}}_r - \ddot{\mathbf{p}}_s + k_D(\dot{\mathbf{p}}_r - \dot{\mathbf{p}}_s) + k_P(\mathbf{p}_r - \mathbf{p}_s) = \mathbf{0}. \quad (1.36)$$

The system (1.36) is exponentially stable for any choice of the positive gains k_D and k_P and thus tracking of $\mathbf{p}_r$ and $\dot{\mathbf{p}}_r$ is ensured for the slow subsystem.

As a further step, the full-order system (1.16) with the composite control law (1.27), (1.33), (1.35) and (1.28) have to be analyzed. By virtue of Tikhonov's theorem, it can be shown that tracking of the reference position $\mathbf{p}_r(t)$ is achieved with an order ϵ approximation.

1.4.2 Parallel force/position control

The interaction of a flexible arm with a compliant environment can be managed by controlling both the contact force and the tip position.

A better insight into the behaviour of the system during the interaction can be achieved by considering a model of the compliant environment. To this purpose, a planar surface is considered, which is locally a good approximation to surfaces of regular curvature, and the model of the contact force is given by

$$\mathbf{f} = k_e \mathbf{n} \mathbf{n}^T (\mathbf{p} - \mathbf{p}_o) \quad (1.37)$$

where $\mathbf{p}_o$ represents the position of any point on the undeformed plane, $\mathbf{n}$ is the unit vector along the normal to the plane, and $k_e > 0$ is the contact stiffness coefficient. For the purpose of this work, it is assumed that the same equation can be established in terms of the slow variables. Such a model shows that the contact force is normal to the plane, and thus a null force error can be obtained only if the desired force $\mathbf{f}_d$ is aligned with $\mathbf{n}$. Also, null position errors can be obtained only on the contact plane while the component of the position along $\mathbf{n}$ has to accommodate the force requirement specified by $\mathbf{f}_d$.

The parallel force/position controller is based on the inverse dynamics law (1.33) and (1.35), where $\mathbf{p}_r$ is chosen as

$$\mathbf{p}_r = \mathbf{p}_d + \mathbf{p}_c \quad (1.38)$$

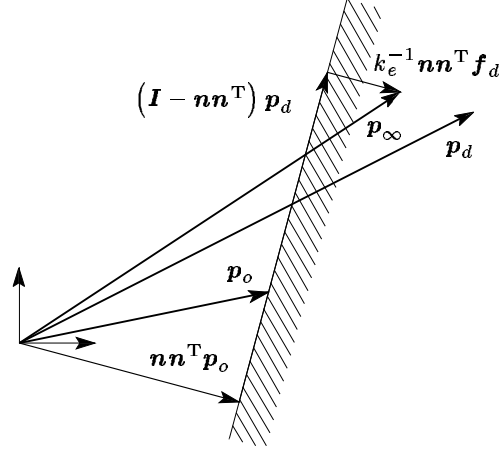


Fig. 1.2. Equilibrium position with parallel force and position control

and $\mathbf{p}_c$ is the solution of the differential equation

$$k_A \ddot{\mathbf{p}}_c + k_V \dot{\mathbf{p}}_c = \mathbf{f}_d - \mathbf{f}_s; \quad (1.39)$$

$k_P, k_D, k_A, k_V > 0$ are suitable feedback gains. It is worth pointing out that $\mathbf{p}_c$ resulting from integration of (1.39) provides an integral control action on the force error.

The stability analysis for the slow system (1.32) with the control law (1.33), (1.35), (1.38) and (1.39) can be carried out with the same arguments used in [11] for the case of rigid robots. In particular, the force/position parallel control scheme ensures regulation of the contact force to the desired set-point $\mathbf{f}_d$ and tracking of the time-varying component of the desired position on the contact plane $(\mathbf{I} - \mathbf{n}\mathbf{n}^T) \mathbf{p}_d(t)$.

To better understand the compliant behavior ensured by parallel control, consider for simplicity the case that $\mathbf{p}_d$ is constant. It can be shown that the closed-loop system has an exponentially stable equilibrium at

$$\mathbf{p}_\infty = (\mathbf{I} - \mathbf{n}\mathbf{n}^T) \mathbf{p}_d + \mathbf{n}\mathbf{n}^T (k_e^{-1} \mathbf{f}_d + \mathbf{p}_o) \quad (1.40)$$

$$\mathbf{f}_\infty = k_e \mathbf{n}\mathbf{n}^T (\mathbf{p}_\infty - \mathbf{p}_o) = \mathbf{f}_d \quad (1.41)$$

where the matrix $(\mathbf{I} - \mathbf{n}\mathbf{n}^T)$ projects the vectors on the contact plane.

The equilibrium position is depicted in Fig. 1.2. It can be recognized that $\mathbf{p}_\infty$ differs from $\mathbf{p}_d$ by a vector aligned along the normal to the contact plane whose magnitude is that necessary to guarantee $\mathbf{f}_\infty = \mathbf{f}_d$ in view of (1.41). Therefore (for the slow system) force regulation is ensured while a null position error is achieved only for the component parallel to the contact plane.

If $\mathbf{f}_d$ is not aligned with $\mathbf{n}$, then it can be found that a drift motion of the arm's tip is generated along the plane; for this reason, if the contact geometry is unknown, it is advisable to set $\mathbf{f}_d = \mathbf{0}$.

As before, Tikhonov's theorem has to be applied to the full-order system (1.17) with the composite control law (1.27), (1.33), (1.35)–(1.39) and (1.28); it can be shown that that force regulation and position tracking are achieved with an order ϵ approximation.

1.5 Simulation

In order to illustrate the effectiveness of the proposed strategy, a planar two-link flexible arm (Fig. 1.1) is considered:

$$\boldsymbol{\vartheta} = [\vartheta_1 \ \vartheta_2]^T$$

and an expansion with two clamped-mass assumed modes is taken for each link:

$$\boldsymbol{\delta} = [\delta_{11} \ \delta_{12} \ \delta_{21} \ \delta_{22}]^T.$$

The following parameters are set up for the links and a payload is assumed to be placed at the arm's tip:

$$\rho_1 = \rho_2 = 1.0 \text{ kg/m (link uniform density)}$$

$$\ell_1 = \ell_2 = 0.5 \text{ m (link length)}$$

$$d_1 = d_2 = 0.25 \text{ m (link center of mass)}$$

$$m_1 = m_2 = 0.5 \text{ m (link mass)}$$

$$m_{h1} = m_{h2} = 1 \text{ kg (hub mass)}$$

$$m_p = 0.1 \text{ kg (payload mass)}$$

$$(EI)_1 = (EI)_2 = 10 \text{ N m}^2 \text{ (flexural link rigidity)}.$$

The stiffness coefficients of the diagonal matrix K are:

$$k_{11} = 38.79 \text{ N} \quad k_{12} = 513.37 \text{ N}$$

$$k_{21} = 536.09 \text{ N} \quad k_{22} = 20792.09 \text{ N}.$$

The dynamic model of the arm and the missing numerical data can be found in [25], while the direct and differential kinematics equations are reported in [26].

The contact surface is a vertical plane, thus the normal vector in (1.37) is $\mathbf{n} = [1 \ 0]^T$; a point of the undeformed plane is

$$\mathbf{p}_o = [0.55 \ 0]^T \text{ m}$$

and the contact stiffness is $k_e = 50 \text{ N/m}$.

The arm was initially placed with the tip in contact with the undeformed plane in the position

$$\mathbf{p}(0) = [0.55 \ -0.55]^T \text{ m}$$

with null contact force; the corresponding generalized coordinates of the arm are:

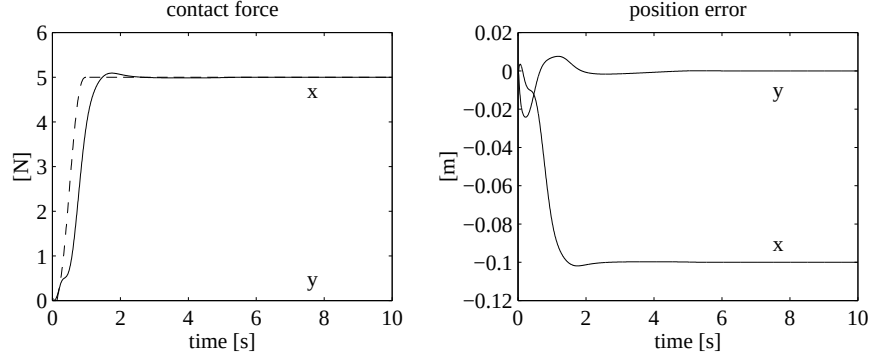


Fig. 1.3. Time histories of contact force and position error

$$\boldsymbol{\vartheta} = [-1.396 \ 1.462]^T \text{ rad}$$

$$\boldsymbol{\delta} = [-0.106 \ 0.001 \ -0.009 \ -0.0001]^T \text{ m}$$

It is desired to reach the tip position

$$\boldsymbol{p}_d = [0.55 \ -0.35]^T \text{ m}$$

and a 5th-order polynomial trajectory with null initial and final velocity and acceleration is imposed from the initial to the final position with a duration of 5 s.

The desired force is taken from zero to the desired value

$$\boldsymbol{f}_d = [5 \ 0]^T \text{ N},$$

according to a 5th-order polynomial trajectory with null initial and final first and second derivative and a duration of 1 s.

The fast control law $\boldsymbol{u}_f$ has been implemented with $\epsilon = 0.1606$. The matrix gains in (1.28) have been tuned by solving an LQ problem for the system (1.26) with the configuration dependent terms computed in the initial configuration of the arm. The matrix weights of the index performance have been chosen so as to preserve the time-scale separation between slow and fast dynamics for both the control schemes. The resulting matrix gains are:

$$\boldsymbol{K}_1 = \begin{bmatrix} -0.0372 & -0.0204 & -0.0375 & 0.1495 \\ 0.0573 & 0.0903 & 0.0080 & -0.7856 \end{bmatrix}$$

$$\boldsymbol{K}_2 = \begin{bmatrix} -0.1033 & -0.0132 & -0.0059 & -0.0053 \\ -0.0882 & 0.0327 & -0.0537 & -0.0217 \end{bmatrix}.$$

Numerical simulations have been performed via MATLAB/Simulink. In order to reproduce a real situation of a continuous-time system with a digital controller, the control laws are discretized with 5 ms sampling time, while the equations of motion are integrated using a variable step Runge-Kutta method with a minimum step size of 1 ms.

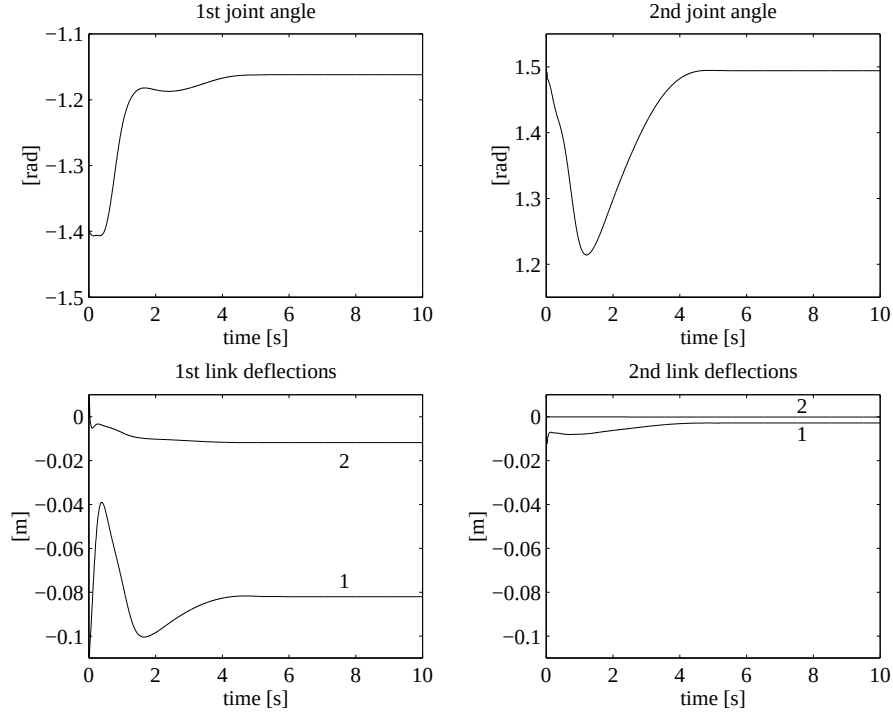


Fig. 1.4. Time histories of contact force and position error

The slow controller (1.33), (1.35)–(1.39) has been used in the composite control law (1.27). The actual force $\mathbf{f}$ and position $\mathbf{p}$ are used in the slow control law instead of the corresponding slow values, assuming that direct force measurement is available and that the tip position is computed from joint angles and link deflection measurements via the direct kinematics equation (1.9). The control gains have been set to $k_P = 100$, $k_D = 22$, $k_A = 0.7813$, $k_V = 13.75$.

In Fig. 1.3 the time histories of the desired (dashed) and actual (solid) contact force are reported, together with the position error. It is easy to see that the contact force remains close to the desired value during the tip motion (notice that the commanded position trajectory has a 5 s duration) and reaches the desired set-point after about 3 s, before the completion of the tip motion. Tracking of the y -component of the position is ensured, while a significant error occurs for the x -component; its (constant) value at steady state is exactly that required to achieve null force error along the same axis, according to the equilibrium equations (1.40) and (1.41).

The time histories of the joint angles and link deflections are reported in Fig. 1.4. It can be recognized that the oscillations of the link deflections are

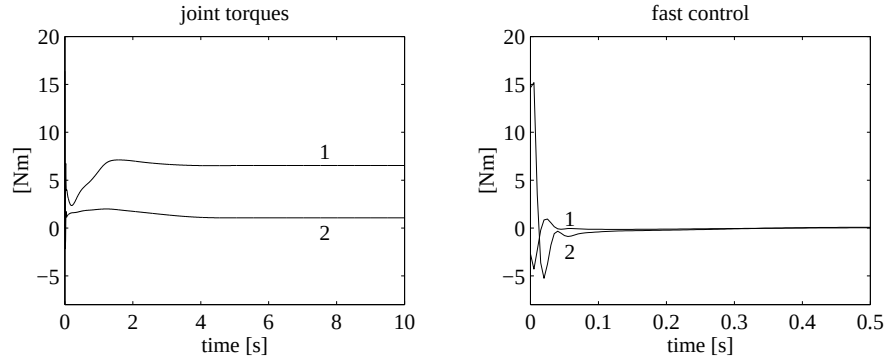


Fig. 1.5. Time histories of contact force and position error

well damped; moreover, because of gravity and contact force, the arm has to bend to reach the desired tip position with the desired contact force.

Figure 1.5 shows the time history of the joint torque $\mathbf{u}$ and the first 0.5 s of the time history of the fast torque $\mathbf{u}_f$. It can be observed that the control effort keeps limited values during task execution; remarkably, the control torque $\mathbf{u}_f$ converges to zero with a transient much faster than the transient of $\mathbf{u}$, as expected.

It is worth pointing out that the simulation of both slow control laws without the fast control action (1.28) has revealed an unstable behaviour; the results have not been reported here for brevity.

1.6 Conclusion

The problem of force and position control for flexible link arms has been considered in this chapter. Because of the presence of structural link flexibility, the additional objective of damping the vibrations that are naturally excited during task execution was considered. By using singular perturbation theory, under the reasonable hypothesis that link stiffness be large, the system has been split into a slow subsystem describing the rigid motion dynamics and a fast subsystem describing the flexible dynamics. Then a force and position parallel control has been adopted for the slow subsystem, while a fast action has been designed for vibration damping. Simulation results have confirmed the feasibility of the proposed approach.

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