

A Simple Control Strategy for a Robot Cart

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1. Introduction

Mobile robots have been receiving a great deal of attention in the latest years by the research community. The key point is the presence of nonholonomic constraints which pose interesting issues both for the planning and for the control problem.

Several works have studied the problem of open-loop motion planning. Strategies have been proposed aimed at finding a bounded sequence of inputs to steer the cart from any initial position to any other arbitrary configuration, e.g. [1],[2].

On the other hand, the controllability and stabilization of non-holonomic mechanical systems like mobile carts have been addressed in [3],[4]. Although the cart model is both locally controllable and reachable, it has been shown that no pure smooth state feedback law exists which can locally stabilize the system [5]. Closed-loop feedback control schemes have been designed to stabilize the cart about an arbitrary point in the state space; some schemes utilize continuous time-varying feedback laws, e.g. [6],[7], whereas others are based on piecewise smooth feedback laws, e.g. [8],[9].

The goal of this paper is to present a simple control strategy for steering a mobile robot cart of unicycle type around a target point.

2. Main Result

The kinematic equations of the mobile cart with two degrees of freedom are given by

$$\begin{aligned}\dot{x} &= v \cos \theta \\ \dot{y} &= v \sin \theta \\ \dot{\theta} &= \omega\end{aligned}\quad (1)$$

where x, y, θ are evidenced in Fig. 1. The control inputs are the linear velocity v of point M along axis x' and the angular velocity ω of the cart.

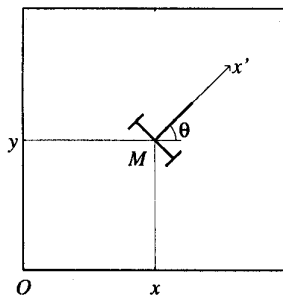


Fig. 1. Kinematics of the mobile cart.

The goal is to find a state feedback control law that steers the system around the origin of the state space $x = 0, y = 0, \theta = 0$.

Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2}(x^2 + y^2 + \theta^2). \quad (2)$$

Differentiating (2) along the trajectories of system (1) yields

$$\begin{aligned}\dot{V}_1 &= x\dot{x} + y\dot{y} + \theta\dot{\theta} \\ &= (x \cos \theta + y \sin \theta)v + \theta\omega.\end{aligned}\quad (3)$$

Choose

$$\begin{aligned}v &= -k_1(x \cos \theta + y \sin \theta) \\ \omega &= -k_2\theta;\end{aligned}\quad (4)$$

plugging (4) in (3) gives

$$\dot{V}_1 = -k_1(x \cos \theta + y \sin \theta)^2 - k_2\theta^2 \quad (5)$$

which is negative semi-definite. In view of LaSalle theorem, it is found that the state of system (1),(4) asymptotically tends to the invariant manifold

$$\begin{aligned}x &= 0 \\ y &\text{ any (but bounded)} \\ \theta &= 0.\end{aligned}\quad (6)$$

In other words, the cart under control law (4) reaches axis y asymptotically (at a finite distance from the origin) with an angle $\theta = 0$.

Differently, the choice of the control law as

$$\begin{aligned}v &= -k_1(x \cos \theta + y \sin \theta) \\ \omega &= -k_2\bar{\theta}\end{aligned}\quad (7)$$

where

$$\bar{\theta} = \theta - \sigma \frac{\pi}{2} \quad (8)$$

with $\sigma = \pm 1$ leads to the invariant manifold

$$\begin{aligned}x &\text{ any (but bounded)} \\ y &= 0 \\ \theta &= \sigma \frac{\pi}{2}.\end{aligned}\quad (9)$$

The Lyapunov function candidate associated with system (1) under control (7),(8) is

$$V_2 = \frac{1}{2}(x^2 + y^2 + \bar{\theta}^2). \quad (10)$$

In this case, the cart under control law (7),(8) reaches axis x asymptotically (at a finite distance from the origin) with an angle $\theta = \sigma\pi/2$.

At this point, a simple control strategy is devised which consists in switching between control laws (4) and (7),(8) depending upon the sign of the difference $|x| - |y|$. It can be found that after a finite number of switches, the states x and y are taken to a circle with center at the origin and arbitrarily small radius ε . By keeping control law (4) inside the circle, the states x and θ are asymptotically driven to zero, whereas the state y reaches a constant value smaller than ε .

The above control strategy is described in the form

$$\begin{aligned} v &= -k_1(x \cos \theta + y \sin \theta) \\ \omega &= -k_2\theta + k_2\sigma(x, y) \frac{\pi}{2} \end{aligned} \quad (11)$$

where

$$\sigma(x, y) = \sigma_1(x, y)\sigma_2(x, y, x_s, y_s) \quad (12)$$

with

$$\begin{aligned} \sigma_1 &= \begin{cases} 0 & \sqrt{x^2 + y^2} < \varepsilon \\ 1 & \sqrt{x^2 + y^2} \geq \varepsilon \end{cases} \\ \sigma_2 &= \begin{cases} 0 & |x| \geq |y| \\ -\operatorname{sgn}(x_s)\operatorname{sgn}(y_s) & |x| < |y| \end{cases} \end{aligned} \quad (13)$$

x_s and y_s denote the values of the states at the switching time.

3. Simulation

In the remainder the simulation results regarding two case studies are shown with $\varepsilon = 10^{-4}$.

In the first one, the cart is initially placed at $x = \sqrt{3}/2$, $y = 0.5$, $\theta = \pi$. The plots in Fig. 2 show the actual path of the cart in the plane and the time histories of the state variables. It is easy to see that the target is satisfactorily reached within 4 switchings.

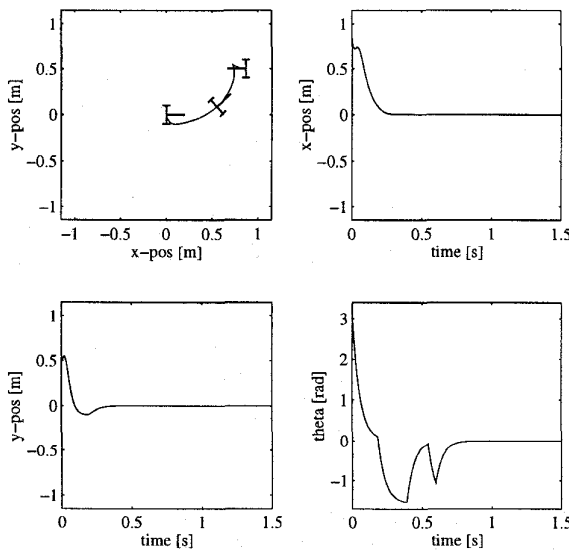


Fig. 2. Results for the first case study.

In the second one, the cart is initially placed at $x = 0$, $y = 1$, $\theta = 0$, and thus the task is to achieve parallel parking of the cart. The plots in Fig. 3 show that the target is satisfactorily reached within 7 switchings.

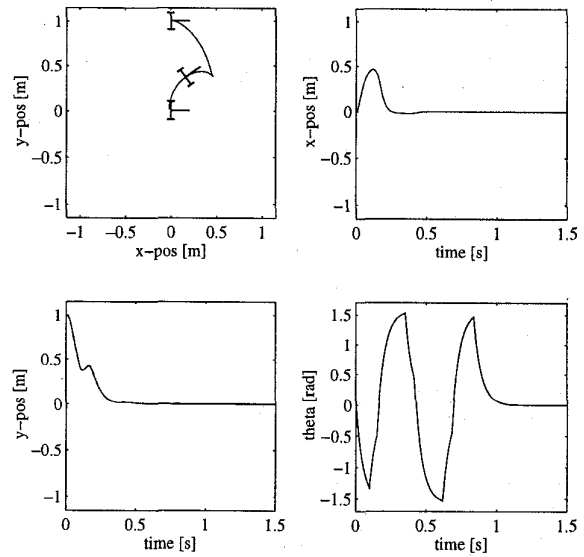


Fig. 3. Results for the second case study.

Other simulations have been run to verify the effectiveness of the proposed strategy.

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