

An Object Space Control Scheme for Cooperative Manipulators

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Abstract—This paper presents a new control strategy for a system of two cooperative manipulators tightly grasping a common rigid object. The reference joint trajectories realizing a coordinated motion are assumed to be available. Differently from pure joint space control schemes, the proposed strategy leads to a controller acting in the object space which still uses joint errors. A case study is developed for a system of two planar manipulators.

1. Introduction

There has been an increasing interest in the robotics community towards the use of cooperative robot manipulator systems for special applications where the employment of a single manipulator is deficient for effective task execution.

Finding a suitable control strategy for a system of two manipulators holding a common object is complicated by the mechanical constraints imposed by the contact between each manipulator's end effector and the object. These constraints regard the mapping between the forces and velocities from the contact space to the object space [1].

Earlier control schemes for cooperative arms were of master/slave fashion [2] or of leader/follower type [3]. Most recent schemes were designed to operate directly on object space variables, allowing control of both absolute object motion [4],[5] and internal forces [6]–[8].

More recently, a two-stage control strategy has been proposed [9] that consists in first solving an inverse kinematics problem for the cooperative system and then designing a joint control scheme. The advantage of this approach is that the user is allowed to specify a coordinated motion task in terms of a set of meaningful absolute and relative variables [10]; an inverse kinematics scheme has been developed for the general case of two cooperative spatial manipulators to compute the joint trajectories corresponding to the given motion task [11].

In the framework of a two-stage control strategy, this work is focused on the issues related to the design of a joint control scheme that operates in respect of the kineto-static constraints imposed by cooperation. The ordinary solution is to design a pure joint space controller [8], but this would result in high sensitivity to inaccurate modeling or disturbances.

Following the intuitive scheme proposed in [9], in this paper a new control scheme is proposed which uses joint errors to generate a proportional control action in the object space, i.e. the natural space where cooperation is achieved. In addition, the control law possesses a derivative action in the joint space and compensation of manipulator and object gravitational terms.

A case study for a system of two planar arms is developed to illustrate the performance of the proposed solution in comparison with a pure joint space control in the case of errors on the object dimensions.

2. Cooperative System Modeling

Consider a system of two rigid manipulators grasping a common rigid object and choose a common world frame for the system. The dynamics of the object can be written as

$$M_o \dot{v}_o + C_o v_o + g_o = h_o \quad (1)$$

where v_o is the $(m \times 1)$ vector of (linear and angular) velocity of a frame attached to the object, M_o is the $(m \times m)$ object inertia matrix, $C_o v_o$ is the $(m \times 1)$ vector of Coriolis and centrifugal forces, g_o is the $(m \times 1)$ vector of gravitational forces, and h_o is the $(m \times 1)$ vector of external forces acting on the object.

Assuming that the grasp is tight, each end effector can exert both a force and a moment on the object at the contact point. Since the object is rigid, the composition of forces and moments can be taken directly at the origin of the object frame. The mapping of the $(m \times 1)$ vectors of reported contact force h_j and the object force vector h_o is simply

$$h_o = W h = [I \quad I] \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \quad (2)$$

where W is the $(m \times 2m)$ constant grasp matrix and h is the $(2m \times 1)$ vector of applied forces.

The matrix W possesses an m -dimensional null space; then, for a given h_o , the general solution to (2) is given by

$$h = W^\dagger h_o + V h_i \quad (3)$$

where $W^\dagger$ denotes the $(2m \times m)$ pseudoinverse of W , V is a $(2m \times m)$ matrix spanning the null space of W ,

and $\mathbf{h}_i$ is an $(m \times 1)$ vector of internal forces [12]. Notice that, in view of the expression of $\mathbf{W}$ in (2), it is

$$\mathbf{W}^\dagger = \begin{bmatrix} \frac{1}{2}\mathbf{I} \\ \frac{1}{2}\mathbf{I} \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{I} \end{bmatrix} \quad (4)$$

where a possible expression of $\mathbf{V}$ has been taken.

In view of application of the principle of virtual work, the dual mappings for the velocities are:

$$\mathbf{v}_o = \mathbf{W}^\dagger \mathbf{T} \mathbf{v} \quad (5)$$

and

$$\mathbf{v}_i = \mathbf{V}^T \mathbf{v} \quad (6)$$

where $\mathbf{v} = [\mathbf{v}_1^T \quad \mathbf{v}_2^T]^T$ is the $(2m \times 1)$ stacked vector of tip velocities reported to the origin of the object frame. The rigid object and tight grasp assumption imposes the constraint

$$\mathbf{v}_i = \mathbf{0}. \quad (7)$$

Let $\mathbf{q}_j$ be the $(n_j \times 1)$ vector of joint variables for each manipulator ($j = 1, 2$). Define the stacked joint variable vector as $\mathbf{q} = [\mathbf{q}_1^T \quad \mathbf{q}_2^T]^T$. Then the tip velocities can be expressed in terms of the joint velocities as

$$\dot{\mathbf{v}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \quad (8)$$

where $\mathbf{J}$ is the block-diagonal matrix given by the Jacobians of the two manipulators; in the following a singularity-free Jacobian is assumed.

The dynamic model of the two manipulators expressed in a stacked form can be written as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} - \mathbf{J}^T(\mathbf{q})\mathbf{h} \quad (9)$$

where $\mathbf{M}$ is the positive definite symmetric inertia matrix, $\mathbf{C}\dot{\mathbf{q}}$ is the vector of Coriolis and centrifugal forces, $\mathbf{g}$ is the vector of gravitational forces, and $\boldsymbol{\tau}$ is the vector of joint driving torques.

3. Object Space Control Scheme

The equations presented in the previous section provide a complete description of the kineto-statics and dynamics of a system of two cooperative manipulators. In the framework of a two-stage control strategy, once the coordinated motion problem has been solved in the first stage by means of a suitable inverse kinematics scheme [11], the goal is to design a control scheme that operates on joint variable errors.

The simplest solution is to devise a pure joint control scheme. Let $\mathbf{q}_d$ denote the joint reference set point. With reference to the PD + gravity compensation kind of controller for the single arm [13], the following control law

$$\boldsymbol{\tau} = \mathbf{K}_P(\mathbf{q}_d - \mathbf{q}) - \mathbf{K}_D\dot{\mathbf{q}} + \boldsymbol{\tau}_g \quad (10)$$

has been presented in [8], where $\mathbf{K}_P$, $\mathbf{K}_D$ are positive definite feedback gain matrices and $\boldsymbol{\tau}_g$ is a compensation torque of the gravitational terms.

The dynamics of the system (1),(9) under the control (10) give at the equilibrium ($\dot{\mathbf{q}} \equiv \mathbf{0}, \mathbf{v}_o \equiv \mathbf{0}$)

$$\mathbf{K}_P(\mathbf{q}_d - \mathbf{q}_e) = \mathbf{J}^T(\mathbf{q}_e)\mathbf{h} \quad (11)$$

$$\mathbf{h}_o = \mathbf{0} \quad (12)$$

where $\mathbf{q}_e$ denotes the joint configuration at the equilibrium.

Substituting (3) in (11) and accounting for (12) yields

$$\mathbf{K}_P(\mathbf{q}_d - \mathbf{q}_e) = \mathbf{J}^T(\mathbf{q}_e)\mathbf{V}\mathbf{h}_i \quad (13)$$

revealing that steady-state internal forces exist whenever a steady-state joint error is present. This occurs when the desired set point $\mathbf{q}_d$ violates the kinematic constraint imposed through the object, e.g. because of imperfect modeling in the kinematics of the cooperative system.

The above analysis suggests that the proportional control action should operate on object space variables in such a way that at the equilibrium no internal forces exist when steady-state joint errors occur. This can be achieved by filtering the joint error $\tilde{\mathbf{q}} = \mathbf{q}_d - \mathbf{q}$ as

$$\mathbf{z} = \mathbf{W}^\dagger \mathbf{T} \mathbf{J}(\mathbf{q})\tilde{\mathbf{q}}; \quad (14)$$

which is intuitively motivated by (5),(8). Then the control law can be chosen as

$$\boldsymbol{\tau} = \mathbf{J}^T(\mathbf{q})\mathbf{W}^\dagger \mathbf{K} \mathbf{z} - \mathbf{K}_D\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{J}^T(\mathbf{q})\mathbf{W}^\dagger \mathbf{g}_o \quad (15)$$

where $\mathbf{K}$ is a positive definite gain matrix and the object forces $\mathbf{K} \mathbf{z}$ and $\mathbf{g}_o$ have been mapped back to the joint space via (3),(9).

The dynamics of the system (1),(9) under the control (15) give at the equilibrium ($\dot{\mathbf{q}} \equiv \mathbf{0}, \mathbf{v}_o \equiv \mathbf{0}$)

$$\mathbf{J}^T(\mathbf{q}_e)\mathbf{W}^\dagger \mathbf{K} \mathbf{W}^\dagger \mathbf{T} \mathbf{J}(\mathbf{q}_e)(\mathbf{q}_d - \mathbf{q}_e) = \mathbf{J}^T(\mathbf{q}_e)\mathbf{h} \quad (16)$$

and (12) again. Substituting (3) in (16) and accounting for (12) yields

$$\mathbf{W}^\dagger \mathbf{K} \mathbf{z}_e = \mathbf{V}\mathbf{h}_i \quad (17)$$

where $\mathbf{z}_e$ is given by (14) computed at $\mathbf{q}_e$. By observing that the two terms in (17) are in orthogonal spaces, it can be concluded that at the equilibrium it must be

$$\mathbf{z}_e = \mathbf{0} \quad \mathbf{h}_i = \mathbf{0}. \quad (18)$$

4. Case Study

In order to illustrate the potential of the proposed solution, a case study has been developed for a system of two equal three-degree-of-freedom planar arms holding a common disk-shaped object. Link parameters in SI units are reported in the table below where ℓ_k is the link length, ℓ_{ck} is the distance of the link center of mass

from the joint axis, m_k is the link mass, and I_k is the link inertia.

Link	ℓ_k	ℓ_{ck}	m_k	I_k
1	0.5	0.25	1	0.08
2	0.3	0.15	0.6	0.04
3	0.1	0.05	0.2	0.01

The object has a radius of 0.1 and its parameters are the mass $m_o = 1$ and moment of inertia $I_o = 0.05$.

The world frame is located at the base of the first arm and the base of the second arm is located at a distance of 1 from the reference frame along the x -axis. The initial configuration of the system is $q_1 = [\pi/2 \ -\pi/2 \ 0]^T$, $q_2 = [\pi/2 \ \pi/2 \ 0]^T$; the object frame is located at (0.5, 0.5) and it is initially aligned with the world frame.

The assigned task requires an object motion along a straight line to (0.3, 0.3) with a change of orientation of $\pi/4$. The trajectories for the reference variables are generated by using interpolating polynomials of fifth order with null initial and final velocities and accelerations, and a time duration of 0.5 s. The inverse kinematics scheme proposed in [11] is applied to compute the corresponding joint reference trajectories q_d which are displayed in Fig. 1.

In order to compare the performance of the proposed object space control scheme with that of a pure joint space control scheme, an object of greater radius (0.105) is assumed for the simulated system. For the control law in (11) the proportional action has been chosen as $K_P = \text{diag}(12000, 9000, 5000, 12000, 9000, 5000)$, whereas for the control law in (16) the proportional action has been chosen as $K = \text{diag}(10000, 10000, 10000)$. The derivative action for both control laws has been chosen as $K_D = \text{diag}(25, 20, 15, 25, 20, 15)$; also a feed-forward term $K_D \dot{q}_d$ has been included to enhance the tracking performance of the scheme.

The motion of the cooperative system has been simulated by adopting a joint space model obtained by combining the equations of motion of the two arms and of the object with the kinematic constraints [14]. Simulations have been performed in MATLAB at 1 ms sampling time.

The results in Figs. 2 and 3 demonstrate the superior performance of the object space control scheme over the joint space control scheme. The object location errors are reduced at steady-state. The forces at the tip of the second arm are smaller by an order of magnitude, indicating that smaller internal forces are exerted on the object in spite of the incorrect dimension used in simulated system.

5. Conclusions

A PD + gravity compensation control scheme has been proposed for two cooperative manipulators tightly grasping a common rigid object. Compared to a pure

joint space control scheme, the key feature of the new scheme is the adoption of a proportional action on object space variables which offers good recovering to imperfect modeling in the kinematics of the cooperative system. This is experienced in the form of reduced internal forces on the object, even if the control scheme does not make use of contact force sensors. The case study developed for two planar arms has clearly demonstrated the advantages of the proposed solution. It can be concluded that the scheme can eventually alleviate the efforts of an additional force sensor-based control action aimed at achieving accurate control of internal forces. This issue, together with a stability analysis for the object space control scheme, is currently under investigation.

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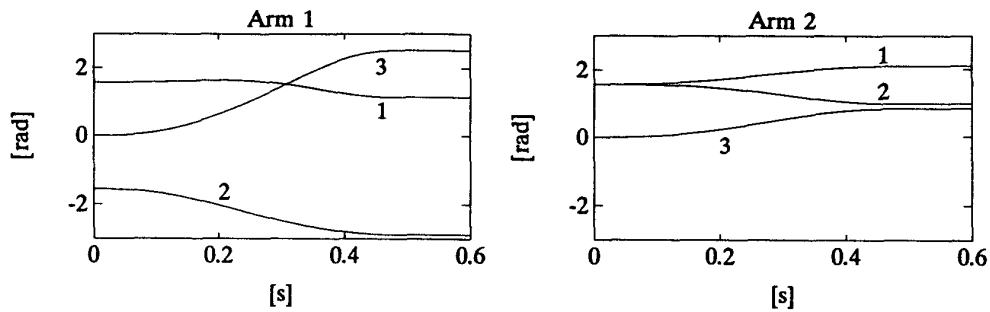


Fig. 1. Joint reference trajectories for the two arms.

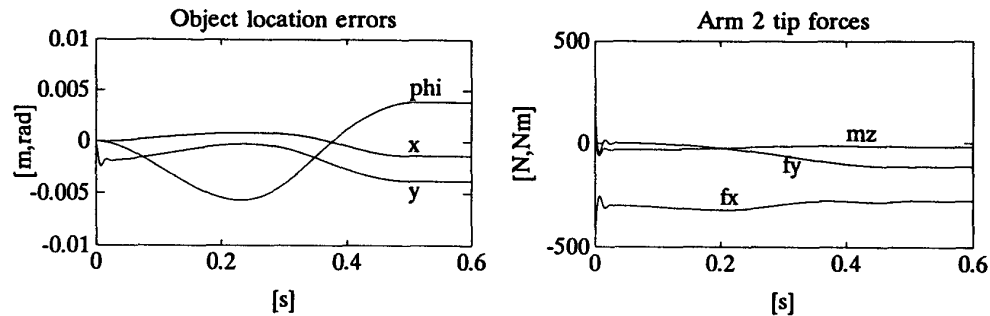


Fig. 2. Time history of object location errors and arm 2 tip forces with the joint space control scheme.

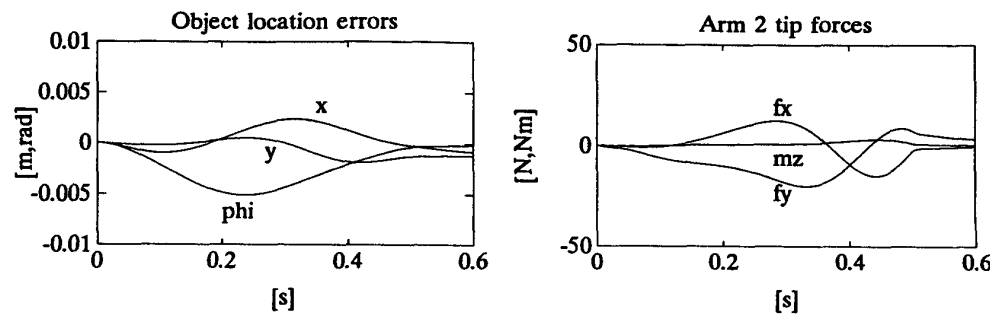


Fig. 3. Time history of object location errors and arm 2 tip forces with the object space control scheme.