

Two-Time Scale Stabilization of a Flexible Arm With Output Feedback

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Abstract

This paper presents an approach to designing a control system with output feedback for a lightweight flexible arm. A two-time scale dynamic model is obtained, which allows the adoption of a composite control strategy. First a slow control can be designed for the slow (rigid) subsystem, then a fast stabilising control for the fast (flexible) subsystem. The problem of the lack of full state measurements concerned with the fast control design is solved. An output feedback low order dynamic compensator is designed, whose optimal gains are computed via a convergent numerical algorithm. Also, a more robust design can be achieved if a loop transfer recovery procedure is introduced. The design procedure is finally tested by means of nonlinear simulation results for the flexible arm.

1. Introduction

Flexible arms may offer long term solutions to manipulation problems in applications where high performance is not guaranteed by the use of today's rigid manipulator arms. Lower arm cost, higher motion speed, better energy efficiency, safer operation and improved mobility are only some of the benefits which are potentially achievable with lighter arms [1]. The structural flexibility of a lightweight mechanical system, however, complicates the modeling and control problem.

A solution to the modeling problem is given by the Lagrangian-assumed mode method established by Book [2] where the flexible motion of a single link is obtained via a truncated series of assumed mode shapes [3]. The result is an extended number of generalized coordinates, and then state variables, to handle for control purposes.

Previous research efforts aimed at designing linear control systems for active control of flexible vibrations have been produced by Hastings and Book [4], Cannon Jr. and Schmitz [5], Meldrum and Balas [6], Fukuda [7], Sakawa *et al.* [8], Bayo [9], and Siciliano

et al. [10]. Nonlinear control strategies, instead, have been proposed by Singh and Schy [11], and De Luca and Siciliano [12].

The approach pursued here is based on a two-time scale model of the flexible arm, as derived by Siciliano and Book [13]. This allows the definition of a slow subsystem corresponding to the rigid body motion, and a fast subsystem describing the flexible motion. A composite control strategy [14] is then applied. First a slow control is designed for the slow subsystem as it would be done for an equivalent rigid arm, then a fast control stabilises the fast subsystem.

The main contribution of this work is to address the problem of the lack of full state availability, as the rates of the flexible variables cannot be directly measured. Preliminary work can be found in Siciliano *et al.* [15], where a fixed order dynamic compensator for the fast subsystem is designed [16] and a convergent numerical algorithm for calculating LQ optimal output feedback gains [17] is used. Here, a loop transfer recovery formulation [18] is shown to lead to a straightforward design procedure, with excellent robustness properties. An example is finally worked out for the flexible arm in the Flexible Automation Laboratory at the Georgia Institute of Technology and nonlinear simulation results are presented.

2. Two-Time Scale Dynamic Model

The one link flexible arm of Fig. 1 is considered in this work. The arm moves on the horizontal plane and is stiff with respect to torsional effects. This flexible beam can be also regarded as the last member of an open kinematic structure whose previous links are rigid. It is believed that analysing first the simple one link case is the necessary step towards the goal of designing an effective control system for multi-link flexible manipulators.

The derivation of the dynamic model follows from Siciliano and Book [13] and is briefly summarised below. The link is modeled as an Euler-Bernoulli beam with uniform density. A solution to

the flexible motion of the link can be obtained assuming separability in time and space [3], i.e.

$$y(\eta, t) = \sum_{i=1}^{\infty} \delta_i(t) \phi_i(\eta) \quad (1)$$

where ϕ_i is the admissible eigenfunction expressing the displacement of the i -th assumed mode of link deflection and δ_i is the time-varying amplitude of the i -th mode. The boundary conditions will then determine the eigenfrequencies of the beam and the expression of the eigenfunctions.

By considering a finite number m of modal terms, the dynamic equations for the one-link flexible arm of Fig. 1 can be written in compact matrix form as

$$\mathbf{M}(\theta, \mathbf{d}) \begin{bmatrix} \ddot{\theta} \\ \ddot{\mathbf{d}} \end{bmatrix} + \begin{bmatrix} f(\dot{\theta}, \mathbf{d}, \dot{\mathbf{d}}) \\ \mathbf{g}(\dot{\theta}, \mathbf{d}) \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{K}\mathbf{d} \end{bmatrix} = \mathbf{F}\mathbf{u} \quad (2)$$

where θ is the joint variable, $\mathbf{d} = (\delta_1 \dots \delta_m)^T$, $\mathbf{u}$ is the control torque at the joint location, $\mathbf{M}$ is the inertia matrix, f and $\mathbf{g}$ are nonlinear terms, $\mathbf{K}$ is the diagonal spring constant matrix, and $\mathbf{F}$ is the constant input matrix. The expressions of these quantities can be found in [13], where clamped-mass boundary conditions are assumed. In that case, there is no input force in the equations for the flexible dynamics and the input matrix takes on the particular form $\mathbf{F} = \text{diag}(1 \ 0 \dots 0)$ [5].

Since the inertia matrix in (2) can be shown to be positive definite, it can be inverted and partitioned as follows:

$$\mathbf{M}^{-1} = \begin{bmatrix} s & \mathbf{t}^T \\ \mathbf{t} & \mathbf{H} \end{bmatrix} \quad (3)$$

with $\mathbf{H}$ of order $m \times m$. Eq. (2) then becomes

$$\ddot{\theta} = -sf - \mathbf{t}^T \mathbf{g} - \mathbf{t}^T \mathbf{K}\mathbf{d} + s\mathbf{u} \quad (4)$$

$$\ddot{\mathbf{d}} = -\mathbf{t}f - \mathbf{H}\mathbf{g} - \mathbf{H}\mathbf{K}\mathbf{d} + \mathbf{t}\mathbf{u}. \quad (5)$$

In order to put the system (4-5) in two-time scale form, the smallest spring constant of $\mathbf{K}$ in (2), say k_1 , can be regarded as the inverse of a perturbation parameter, i.e. $\mu = 1/k_1$. Note that for a given arm geometry, the limit $\mu \rightarrow 0$ corresponds to the case of a rigid arm. Then, factoring $\mathbf{K}$ as $\mathbf{K} = \tilde{\mathbf{K}}/\mu$ and defining the new variables

$$\mathbf{z} := \frac{1}{\mu} \tilde{\mathbf{K}}\mathbf{d} \quad (6)$$

yields the equations of the system in two-time scale form — singularly perturbed system — i.e.

$$\ddot{\theta} = -sf - \mathbf{t}^T \mathbf{g} - \mathbf{t}^T \mathbf{z} + s\mathbf{u} \quad (7)$$

$$\mu \ddot{\mathbf{z}} = -\mathbf{t}f - \mathbf{H}\mathbf{g} - \mathbf{H}\mathbf{z} + \mathbf{t}\mathbf{u}. \quad (8)$$

where it is understood that, without loss of generality, the terms on the right side of (8) have been scaled by $\tilde{\mathbf{K}}$, by virtue of the definition in (6).

At this point the typical steps of a singular perturbation formulation can be taken [14]. Because of the presence of μ , the system (7-8) exhibits a boundary layer phenomenon in the fast variables $\mathbf{z}$. Formally setting $\mu = 0$ accomplishes a model order reduction from $m+1$ to 1. The differential equations (8) degenerate into the algebraic transcendental equations

$$0 = -\bar{\mathbf{H}}\mathbf{z} + \bar{\mathbf{t}}\mathbf{u} \quad (9)$$

where the upper bar is used to indicate that the variables belong to the so-called slow subsystem with $\mu = 0$. It can be seen [13] that $f(\mu=0) = 0$ and $\mathbf{g}(\mu=0) = 0$. Also in (9) $\bar{\mathbf{H}} = \mathbf{H}(\mu=0)$ and $\bar{\mathbf{t}} = \mathbf{t}(\mu=0)$. Since $\bar{\mathbf{H}}$ is positive definite, it is possible to find the distinct quasi steady-state solution $\mathbf{z}$ to (9),

$$\mathbf{z} = \bar{\mathbf{H}}^{-1} \bar{\mathbf{t}}\mathbf{u}. \quad (10)$$

Using this solution in (7) formally yields the slow subsystem

$$\ddot{\theta} = (-\bar{\mathbf{t}}^T \bar{\mathbf{H}}^{-1} \bar{\mathbf{t}} + s)\mathbf{u} = \frac{1}{m_{11}}\mathbf{u} \quad (11)$$

which turns out to be the equivalent rigid system; $m_{11} = m_{11}(\mu=0)$.

To derive the fast (boundary layer) subsystem, the slow variables are treated as constant in the boundary layer. Defining the fast state variables $\mathbf{z}_f = \mathbf{z} - \mathbf{z}$, and the fast control $\mathbf{u}_f = \mathbf{u} - \bar{\mathbf{u}}$, the fast subsystem of (7-8) becomes

$$\frac{d^2 \mathbf{z}_f}{d\tau^2} = -\bar{\mathbf{H}}\mathbf{z}_f + \bar{\mathbf{t}}\mathbf{u}_f \quad (12)$$

where $\tau = t/\sqrt{\mu}$ is the fast time scale.

3. Composite Control Design

Under the results of the previous section, the design of a feedback control for the system (7-8) can be performed on the basis of a composite control strategy as [14]

$$\mathbf{u} = \bar{\mathbf{u}}(\bar{\theta}, \dot{\bar{\theta}}) + \mathbf{u}_f(\mathbf{z}_f, \frac{d\mathbf{z}_f}{d\tau}) \quad (13)$$

with the constraint that $\mathbf{u}_f(0, 0) = 0$, so that $\mathbf{u}_f$ is inactive along the solution (10).

Since the slow subsystem coincides with the equivalent rigid linear system (11), it is straightforward to design the slow control as it is usually done for rigid arms, i.e.

$$\bar{\mathbf{u}} = m_{11}[\ddot{\bar{\theta}} + k_v(\dot{\bar{\theta}} - \dot{\theta}) + k_p(\bar{\theta} - \theta)] \quad (14)$$

which allows the system (11) to track a reference trajectory specified by $(\bar{\theta}, \dot{\bar{\theta}}, \ddot{\bar{\theta}})$.

At this point singular perturbation theory requires that the fast time scale subsystem (12) be uniformly stable along the equilibrium trajectory $\mathbf{z}$ given in (10). A state space representation of the subsystem (10) can be simply obtained as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u_f \quad (15a)$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{H} & \mathbf{O} \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix} \quad (15b)$$

where $\mathbf{x}^T = [\mathbf{z}_f^T \quad (d\mathbf{z}_f/d\tau)^T]$, $\mathbf{O}$ is the $(m \times m)$ null matrix, $\mathbf{0}$ is the $(m \times 1)$ null vector and $\mathbf{I}$ is the $(m \times m)$ identity matrix. This is a linear system with $m/2$ couples of poles on the imaginary axis in the s -plane. Since the pair $(\mathbf{A}, \mathbf{b})$ is completely controllable, a fast state feedback control of the type

$$u_f = \mathbf{k}_p^T \mathbf{z}_f + \mathbf{k}_v^T \frac{d\mathbf{z}_f}{d\tau} \quad (16)$$

can be used to arbitrarily place the poles of the system (15). The synthesis of the control (16), however, would require full state feedback. In practice, Hastings and Book [4] showed that the deflection variables $\mathbf{d}$ can be reconstructed, by a simple low cost method, from strain gage measurements $\mathbf{e}$ via

$$\mathbf{e} = \mathbf{E}\mathbf{d} \quad (17)$$

where $\mathbf{E}$ is an $(m \times m)$ invertible matrix which depends on the mode shapes ϕ_i . Then $\mathbf{z}$ can be computed from (6) and $\mathbf{z}$ from (10), so that $\mathbf{z}_f$ is available. The rates of the deflection variables, instead, cannot be measured so that $d\mathbf{z}_f/d\tau$ is not available. In order to overcome this drawback, in the following an output feedback controller for the system (15) is designed.

4. Output Fast Feedback Stabilization

It is well known that LQR synthesis methods have guaranteed stability margins. Unfortunately, this property holds only in the case of full state feedback. As pointed out above, instead, the stabilising controller for the fast subsystem (15) must be based on the sole measurements of flexible variables $\mathbf{z}_f$. To this purpose, the fixed order dynamic compensator formulation introduced by Kramer and Calise [16] is adopted in the following. This is based on a canonical form for the compensator representation which provides a minimal parameterisation. It also excludes the use of direct feedthrough of the output, which is undesirable both from the point of view of sensor noise reduction and robustness.

The output equation for the fast subsystem (15) can be expressed as

$$\mathbf{y} = \mathbf{C}\mathbf{x} \quad (18a)$$

$$\mathbf{C} = [\mathbf{I} \quad \mathbf{O}] \quad (18b)$$

where the $(m \times m)$ identity matrix $\mathbf{I}$ and the $(m \times m)$ null matrix $\mathbf{O}$ simply account for the fact that the variables $\mathbf{z}_f$ are available, whereas their rates are not. A fixed order compensator without direct feedthrough of the output can be formulated in observer canonical form as

$$u_f = -\mathbf{h}^o \mathbf{w} \quad (19a)$$

$$\dot{\mathbf{w}} = \mathbf{P}^o \mathbf{w} + \mathbf{I}u_p \quad (19b)$$

$$u_p = \mathbf{p}_w u_f - \mathbf{N}\mathbf{y} \quad (19c)$$

where $\mathbf{I}$ is the $(p \times p)$ identity matrix, $\mathbf{h}^o$ is the $(1 \times p)$ row vector

$$\mathbf{h}^o = [0 \quad \dots \quad 0 \quad 1] \quad (20)$$

and $\mathbf{P}^o$ is the $(p \times p)$ matrix

$$\mathbf{P}^o = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \quad (21)$$

In (19c) $\mathbf{N}$ and $\mathbf{p}_w$ are respectively a $(p \times m)$ matrix and a $(p \times 1)$ vector of free parameters.

Defining $\tilde{\mathbf{x}}^T = [\mathbf{x}^T \quad \mathbf{w}^T]$, $\tilde{\mathbf{y}}^T = [\mathbf{y}^T \quad -u_f]$ and $\tilde{\mathbf{u}} = u_p$, the dynamic compensator design can be expressed in terms of a standard output feedback problem for the augmented system

$$\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{A}}\tilde{\mathbf{x}} + \tilde{\mathbf{B}}\tilde{\mathbf{u}} \quad (22a)$$

$$\tilde{\mathbf{y}} = \tilde{\mathbf{C}}\tilde{\mathbf{x}} \quad (22b)$$

$$\tilde{\mathbf{u}} = -\mathbf{G}\tilde{\mathbf{y}} \quad (22c)$$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & -\mathbf{b}\mathbf{h}^o \mathbf{T} \\ \mathbf{O} & \mathbf{P}^o \end{bmatrix} \quad \tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{O} \\ \mathbf{I} \end{bmatrix} \quad (22d)$$

$$\tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C} & \mathbf{O} \\ \mathbf{O} & \mathbf{h}^o \mathbf{T} \end{bmatrix} \quad \mathbf{G} = [\mathbf{N} \quad \mathbf{p}_w] \quad (22e)$$

where the number of free parameters is minimised and no zero elements in $\mathbf{G}$ appear.

The output feedback problem is then optimized according to the following augmented quadratic performance index

$$J = E_{\tilde{\mathbf{x}}_0} \left[\int_0^\infty (\tilde{\mathbf{x}}^T \mathbf{Q} \tilde{\mathbf{x}} + \tilde{\mathbf{u}}^T \mathbf{R} \tilde{\mathbf{u}}) dt \right] \quad (23)$$

where $\mathbf{Q} \geq \mathbf{O}$, $\mathbf{R} > \mathbf{O}$ and the expectation is taken over some initial distribution on $\tilde{\mathbf{x}}_0$.

The necessary conditions for optimality require the solution of the triple $(\mathbf{G}, \mathbf{P}, \mathbf{L})$ satisfying:

$$\mathbf{A}_c^T \mathbf{P} + \mathbf{P} \mathbf{A}_c + \mathbf{Q} + \tilde{\mathbf{C}}^T \mathbf{G}^T \mathbf{R} \mathbf{G} \tilde{\mathbf{C}} = \mathbf{O} \quad (24a)$$

$$\mathbf{A}_c \mathbf{L} + \mathbf{L} \mathbf{A}_c^T + \tilde{\mathbf{X}}_0 = \mathbf{O} \quad (24b)$$

$$\mathbf{R}\mathbf{G}\tilde{\mathbf{C}}\tilde{\mathbf{L}}\tilde{\mathbf{C}}^T - \tilde{\mathbf{B}}^T\mathbf{P}\tilde{\mathbf{L}}\tilde{\mathbf{C}}^T = \mathbf{0} \quad (24c)$$

for a stable closed loop system matrix

$$\mathbf{A}_c = \tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{G}\tilde{\mathbf{C}}. \quad (25)$$

In (24b), $\tilde{\mathbf{X}}_0 = \mathbf{E}[\tilde{\mathbf{x}}_0\tilde{\mathbf{x}}_0^T]$ is the variance matrix associated with the distribution assumed for the initial conditions. A convergent algorithm for solving (24a-24c) can be found in Moerder and Calise [17].

Full state feedback design is often used as a first step in designing an output feedback controller. The most popular approach is LQR design, which also yields guaranteed gain and phase margins when measured at the plant input. If the fixed order compensator is designed to approximate the loop transfer properties of the full state design, then the closed loop system should contain a set of eigenvalues and eigenvectors that approximate those of the full state design. More importantly, the multivariable gain and phase margin properties should also be approximated. The return signal in the case of full state design is $-\mathbf{k}^*T\mathbf{x}$, where $\mathbf{k}^*$ is the optimal gain vector. Referring to (19a), the return signal in the case of fixed order compensator design is $-\mathbf{h}^{\sigma T}\mathbf{w}$. Thus, as in Calise and Prasad [18], the objective in designing the compensator should be to minimize

$$y_1 = \mathbf{k}^*T\mathbf{x} - \mathbf{h}^{\sigma T}\mathbf{w} \quad (26)$$

for a suitably chosen input and for zero initial conditions. This naturally leads to selecting the following index of performance:

$$J = \mathbf{E}_{\mathbf{u}_0} \left[\int_0^{\infty} (y_1^2 + \tilde{\mathbf{u}}^T\tilde{\mathbf{u}})dt \right]. \quad (27)$$

Substituting for y_1 from (26), and rewriting (27) in the form of (23) leads to the following expressions for the weighting matrices:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{k}^*\mathbf{k}^{*T} & -\mathbf{k}^*\mathbf{h}^{\sigma T} \\ -\mathbf{h}^{\sigma}\mathbf{k}^{*T} & \mathbf{h}^{\sigma}\mathbf{h}^{\sigma T} \end{bmatrix} \quad \mathbf{R} = \rho\mathbf{I} \quad (28)$$

Selecting the input waveforms as impulses with magnitudes uniformly distributed on the unit sphere, results in the following expression for $\tilde{\mathbf{X}}_0$:

$$\tilde{\mathbf{X}}_0 = \begin{bmatrix} \mathbf{b}\mathbf{b}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \quad (29)$$

Eqs. (28) and (29) uniquely define the structure of the weighting matrices needed for the fixed order compensator design. Note that, unlike the design of a full order observer, the design of a fixed order compensator depends on the gain matrix from the full state design step. Moreover, this gain matrix is not implemented as a part of the final controller.

5. Numerical Results

A case study is carried out to illustrate the effectiveness of the design procedure. The actual flexible arm is the prototype in the Flexible Automation Laboratory at Georgia Tech. The modal series (1) is truncated at $m = 2$ and clamped-mass boundary conditions are imposed. The dynamic parameters of the model (2) can be found in [13]. It is worth reporting here that the first two modal frequencies of the beam are at 2.12 and 14.3 Hz, respectively. Also the perturbation parameter in (6) is $\mu = 0.012$ which justifies the two-time scale separation.

According to the results of Section 3, a composite control is designed first [13], composed of a slow control as in (14) and a fast full state feedback control as in (16). The linear feedback gains are chosen as $k_p = 6.25$ and $k_v = 5$, corresponding to a double pole at -2.5 for the error dynamics of the rigid model. This choice preserves the time scale separation between the slow and the fast subsystems. Then, a full state feedback design is carried out to damp the two fast modes. In the open loop, both modes have zero damping with natural frequencies at 0.94 and 6.07 Hz in the fast time scale. A standard LQR design is undertaken with the following feedback gains:

$$\mathbf{k}_{pf}^T = [0.37 \quad -5.68] \quad \mathbf{k}_{vf}^T = [-2.47 \quad -1.10]$$

which result in damping ratios of 0.4 for both modes.

Next, an output fast feedback design with second order compensation — $p = 2$ in (19) — is carried out using the procedure described in Section 4. The above full state feedback gains are used to compute $\mathbf{Q}$ in (28), while $\rho = 1.0 \times 10^{-8}$. The design results in a damping ratio of 0.35 for both modes, with the gain matrix in (22e) being

$$\mathbf{G} = \begin{bmatrix} -2.98 \times 10^4 & -7.72 \times 10^4 & 5.98 \times 10^3 \\ 8.07 \times 10^3 & 2.91 \times 10^3 & 7.13 \times 10^1 \end{bmatrix}.$$

Figs. 2 and 3 illustrate the recovery of the full state design by means of ordinary magnitude and phase Bode plots. These have been obtained by taking the return signals $\mathbf{k}^*T\mathbf{x}$ and $\mathbf{h}^{\sigma T}\mathbf{w}$ as outputs for the two designs respectively. By comparing the two figures, it can be recognized that the output feedback design attempts to recover the robustness of the full state design. In particular, the full state design has near 90° of phase margin. The output feedback controller has approximately 70° of phase margin at high frequency, and 9 dB of gain margin at low frequency.

In the following, two sets of simulations are presented. In the first set a step change from $\hat{\theta} = 0^\circ$ to $\hat{\theta} = 90^\circ$ is assigned, whereas in the second set a joint motion is commanded from $\hat{\theta}(0) = 0^\circ$ to $\hat{\theta}(T) = 90^\circ$ with $\hat{\theta}(t) = 90[1 + \sin(360t/T - 90)] [^\circ/s]$, $0 \leq t \leq T$,

where $T = 2$ s. In order to test the robustness of the two controllers designed above, the whole nonlinear model has been simulated. A fifth order Runge-Kutta-Merson method has been implemented to integrate the nonlinear differential equations (2) at a sampling rate of 5×10^{-3} s. The results are shown in Figs. 4 through 9, where (#1) refers to full state feedback design and (#2) to output feedback design. The tip deflection reported in Figs. 5 and 8 has been computed via eq. (1) evaluated at $\eta = L$. It can be recognized that the performance of the output feedback controller favorably compares with the performance of the full state feedback controller.

In the case of step response, the steady-state joint position is reached by both controllers in a time of 2 s and the tip deflection damps out in about the same time. A high frequency component appears in the control torque for the output feedback controller. This effect can be attributed to the high value of the natural frequency of the second order compensator. In practice, decreasing the design parameter ρ in (28) provides increased damping of the flexible modes, at the expense of increasing the natural frequency of the compensator [18], so that a trade-off must be sought.

On the other hand, in the case of trajectory response, the tracking is achieved by both controllers with satisfactory damping of the tip deflection. Therefore, it can be concluded that the above design procedure represents an effective tool to design output feedback controllers for flexible arms, and can be also extended to multi-link arms. It can be shown, indeed, that the fast subsystem resulting from a two-time scale formulation always turns out to be a linear system parameterised in the slow state variables [13], for which an output feedback dynamic compensator can be designed in the same formal way as illustrated in this work.

6. Summary

A two-time scale approach has been developed for the control of a flexible arm. The main drawback concerned with this kind of composite control, namely the rates of flexible modal deflections are not measurable, has been successfully overcome by designing a fixed low order dynamic compensator in an output feedback setting. A more robust design is achieved if a loop transfer recovery procedure is adopted. The simulated example with a nonlinear model of a flexible arm illustrates the effectiveness of the formulation and overall design methodology.

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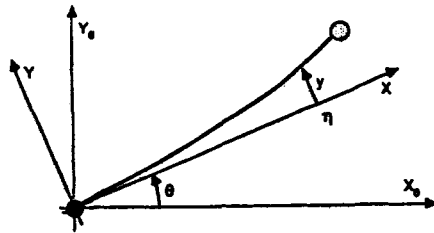


Fig. 1. The flexible arm.

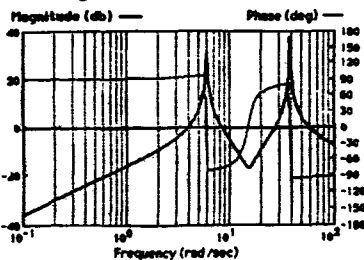


Fig. 2. Bode plots for the full state design.

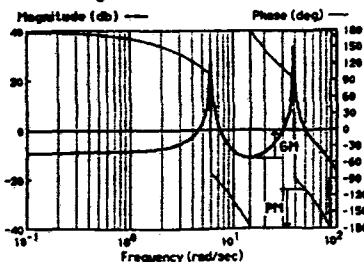


Fig. 3. Bode plots for the output feedback design.

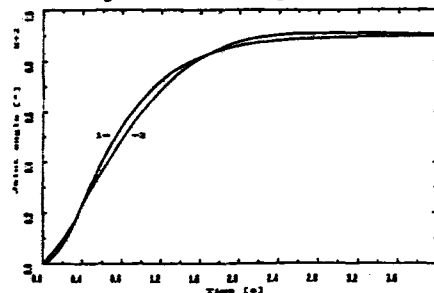


Fig. 4. Joint angle for step response.

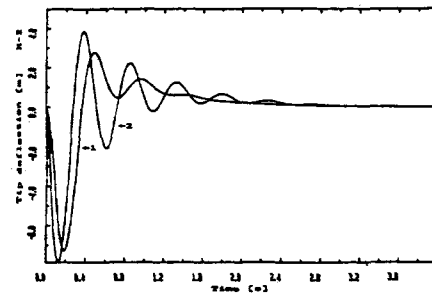


Fig. 5. Tip deflection for step response.

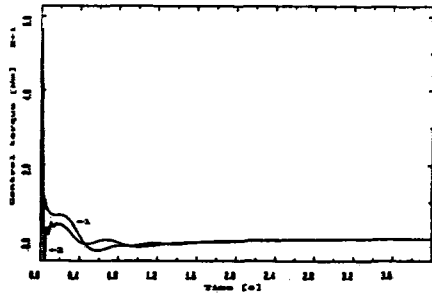


Fig. 6. Control torque for step response.

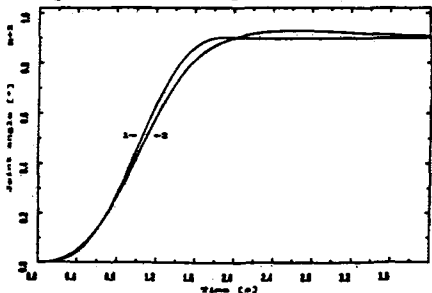


Fig. 7. Joint angle for trajectory response.

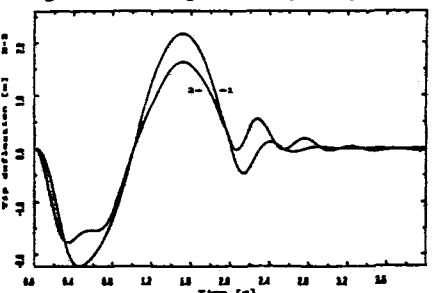


Fig. 8. Tip deflection for trajectory response.

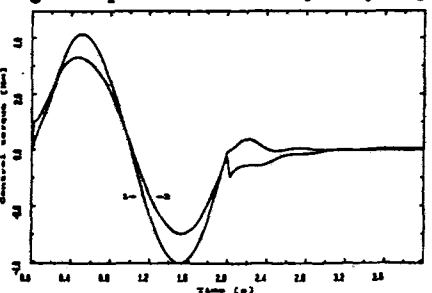


Fig. 9. Control torque for trajectory response.